CIE International A Level Physics

Topic 25

Astronomy and Cosmology

Revision Booklet

This booklet covers:

- Luminosity and Radiant Flux Intensity
- Standard Candles and Distance Measurement
- Wien's Displacement Law and Stellar Temperature
- The Stefan–Boltzmann Law and Stellar Radii
- Redshift and the Expanding Universe
- Hubble's Law and the Big Bang

Luminosity and Radiant Flux Intensity

Luminosity

The **luminosity** L of a star is the total power of electromagnetic radiation emitted by the star in all directions.

$$L \quad \text{units: W (watts)}$$

Luminosity depends on the star's **surface temperature** and **surface area** — not on its distance from us.

Radiant Flux Intensity

The **radiant flux intensity** F (sometimes called apparent brightness) is the power of radiation received per unit area at a detector (e.g. a telescope on Earth).

$$F \quad \text{units: W m}^{-2}$$

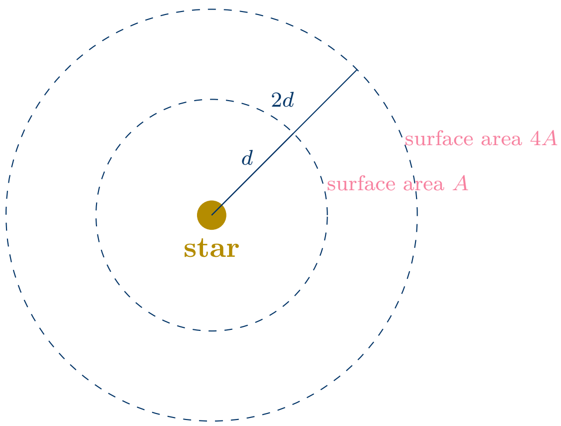
F depends on both the luminosity of the source and its distance d.

Inverse Square Law for Radiant Flux

Assuming the star radiates uniformly in all directions and there is no absorption by the intervening medium, the radiation spreads over a sphere of area $4\pi d^2$:

$$F = \frac{L}{4\pi d^2}$$

F = radiant flux intensity at distance d (W m^{-2})
L = luminosity of the star (W)
d = distance from the star (m)



$F \propto 1/d^2$: double the distance, quarter the flux

Standard Candles

Standard Candle

A **standard candle** is an astronomical object whose **luminosity is known** (or can be determined independently of distance). By measuring the radiant flux intensity F received from the object and using $F = L/(4\pi d^2)$, its distance d can be calculated:

$$d = \sqrt{\frac{L}{4\pi F}}$$

Standard Candles Used in Practice

- **Cepheid variable stars**: pulsating stars whose *period of variation* is related to their luminosity (period–luminosity relation). Measure the period $\Rightarrow$ know $L \Rightarrow$ measure $F \Rightarrow$ find d.
- **Type Ia supernovae**: thermonuclear explosions of white dwarf stars that all reach approximately the same peak luminosity. Visible across enormous distances (billions of light-years), making them useful for measuring distances to distant galaxies.

Assumptions and Limitations

The inverse square law assumes:

- No absorption of radiation between source and detector (no dust or gas in the way).
- The source radiates **isotropically** (equally in all directions).

Dust absorption causes stars to appear **dimmer** than expected, which would lead to an **overestimate** of distance if uncorrected.

Wien's Displacement Law and Stellar Temperature

Black-Body Radiation

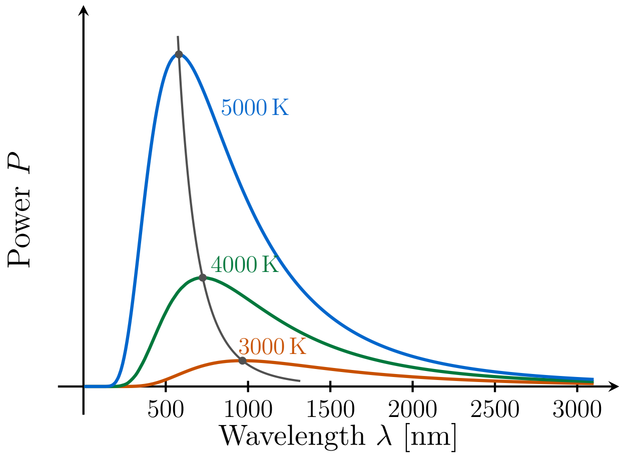
Stars approximate **black bodies** — objects that absorb all incident radiation and emit a characteristic continuous spectrum that depends only on temperature. The peak wavelength of this spectrum shifts with temperature.

Wien's Displacement Law

$$\lambda_{\text{max}} \propto \frac{1}{T} \qquad \text{equivalently} \qquad \lambda_{\text{max}} T = b$$

λ_{max} = wavelength at peak intensity of the spectrum (m)
T = surface temperature of the star (K)
b = Wien's constant $= 2.90 \times 10^{-3}$ m K

A **hotter** star has its peak at a **shorter** wavelength (bluer colour). A **cooler** star peaks at a **longer** wavelength (redder colour).



The Stefan–Boltzmann Law and Stellar Radii

Stefan–Boltzmann Law

For a black-body sphere (approximating a star) of radius r and surface temperature T:

$$L = 4\pi r^2 \sigma T^4$$

L = luminosity (W)
r = radius of the star (m)
σ = Stefan–Boltzmann constant = 5.67×10^{-8} W m^{-2}K^{-4}
T = surface temperature (K)

Estimating Stellar Radius

By combining Wien's law and the Stefan–Boltzmann law:

1. Measure λ_{max} from the star's spectrum $\Rightarrow$ use Wien's law to find T.
2. Measure F (radiant flux intensity) and find d (e.g. via a standard candle or parallax).
3. Find L from $L = 4\pi d^2 F$.
4. Rearrange Stefan–Boltzmann: $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Using Ratios

Exam questions often ask you to *compare* two stars rather than calculate absolute values. In that case, form a ratio to cancel constants:

$$\frac{L_1}{L_2} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4}$$

This avoids large numbers and reduces the risk of errors.

Redshift and the Expanding Universe

Redshift

When a source of light moves **away** from an observer, the observed wavelength is **longer** (shifted towards the red end of the spectrum) than the wavelength emitted. This is the **Doppler effect** applied to light.
The lines in the emission or absorption spectrum of a distant galaxy are observed at **longer wavelengths** than those of the same element measured in the laboratory.

Redshift Formula

For a source moving at speed $v \ll c$ relative to an observer:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

z = redshift (dimensionless)
$\Delta\lambda$ = shift in wavelength (= observed λ − emitted λ) (m)
λ = emitted (rest-frame) wavelength (m)
Δf = shift in frequency (Hz)
v = recession speed of the source (m s^{-1})

Redshift as Evidence for Expansion

- Observations of distant galaxies show that their spectral lines are **all redshifted**.
- The greater the distance of a galaxy, the greater its redshift.
- This indicates that distant galaxies are moving **away** from us, and the further away they are, the faster they recede.
- This is consistent with the **Universe expanding**: it is space itself that is stretching, carrying galaxies apart, rather than the galaxies moving through space.

Hubble's Law and the Big Bang Theory

Hubble's Law

$$v \approx H_0\, d$$

v = recession speed of a galaxy (m s^{-1})
H_0 = Hubble constant (s^{-1}, though often quoted in km s^{-1} Mpc^{-1})
d = distance of the galaxy from Earth (m)

In CIE examinations, SI units are used: H_0 in s^{-1} and d in metres.

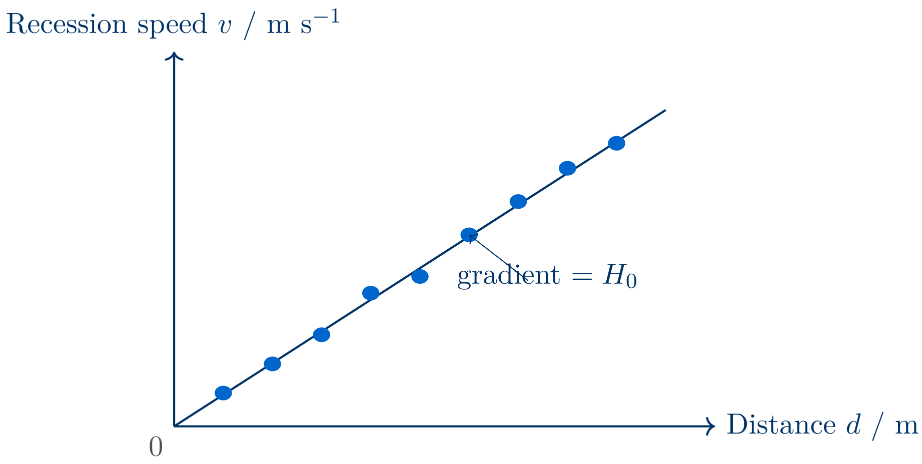
Hubble's Law and the Big Bang Theory

- Hubble's law states that the recession speed of a galaxy is proportional to its distance.
- If all galaxies are currently moving apart, then in the past they must have been **closer together**.
- Extrapolating back in time, all matter was once concentrated in an extremely hot, dense state — the **Big Bang**.
- An estimate of the **age of the Universe** can be obtained from:

$$t \approx \frac{1}{H_0}$$

(This assumes a constant rate of expansion, which is a simplification.)

Hubble Plot: Recession Speed vs Distance



Determining H_0 from a Graph

A graph of recession speed v against distance d gives a straight line through the origin. The **gradient** of this line is the Hubble constant H_0. In practice, there is significant scatter due to the difficulty of measuring distances to distant galaxies accurately.

Formula Summary Sheet

Formula	Quantity	Units
$F = L/(4\pi d^2)$	Inverse square law / flux	W m^{-2}
$\lambda_{\max}T = b$	Wien's displacement law	m K
$L = 4\pi r^2\sigma T^4$	Stefan–Boltzmann law	W
$\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$	Redshift formula	—
$v \approx H_0 d$	Hubble's law	m s^{-1}, m
$t \approx 1/H_0$	Age of Universe estimate	s

Constants: $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4}, $b = 2.90 \times 10^{-3}$ m K, $c = 3.00 \times 10^{8}$ m s^{-1}

Exam Technique and Problem-Solving Strategy

Step-by-Step for Stellar Radius Problems

1. Find T: measure $\lambda_{\max}$ from spectrum $\Rightarrow T = b/\lambda_{\max}$.
2. Find L: use standard candle or parallax to get d; then $L = 4\pi d^2 F$.
3. Find r: rearrange Stefan–Boltzmann: $r = \sqrt{L/(4\pi\sigma T^4)}$.

Common Errors — Avoid These!

- Forgetting the 4π in both $F = L/4\pi d^2$ and $L = 4\pi r^2\sigma T^4$.
- Confusing **luminosity** (intrinsic power, independent of distance) with **flux intensity** (observed brightness, depends on distance).
- Using $\Delta\lambda = \lambda_{\text{obs}} - \lambda_{\text{emitted}}$ but getting the **sign wrong**: for a receding source, $\Delta\lambda > 0$ (observed wavelength is longer).
- Applying $v = H_0 d$ with d in Mpc and H_0 in km s^{-1} Mpc^{-1} — in CIE exams **always convert to SI** (metres and s^{-1}).
- Stating that the Big Bang means galaxies are moving **through space** — more accurately, **space itself is expanding**.

Worked Examples

Example 1 — Stellar Radius from Wien and Stefan–Boltzmann

Question: A star has a peak emission wavelength of 480 nm and a luminosity of 5.2×10^{26} W. Estimate its radius. ($b = 2.90 \times 10^{-3}$ m K, $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4})

Solution

Solution:
Step 1 — Surface temperature:

$$T = \frac{b}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-3}}{480 \times 10^{-9}} = 6040 \text{ K}$$

Step 2 — Rearrange Stefan–Boltzmann:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times (6040)^4}}$$

$$T^4 = (6.04 \times 10^3)^4 = 1.33 \times 10^{15} \text{ K}^4$$

$$r = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times 1.33 \times 10^{15}}} = \sqrt{\frac{5.2 \times 10^{26}}{9.47 \times 10^{8}}} = \sqrt{5.49 \times 10^{17}} = \mathbf{7.4 \times 10^{8}} \text{ m}$$

Example 2 — Distance Using Flux and Luminosity

Question: A type Ia supernova has a peak luminosity of 2.0×10^{36} W. It is observed with a radiant flux intensity of 3.5×10^{-14} W m^{-2}. Calculate its distance.

Solution

Solution:

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{2.0 \times 10^{36}}{4\pi \times 3.5 \times 10^{-14}}}$$

$$d = \sqrt{\frac{2.0 \times 10^{36}}{4.40 \times 10^{-13}}} = \sqrt{4.55 \times 10^{48}} = \mathbf{2.1 \times 10^{24}} \text{ m}$$

Example 3 — Recession Speed and Age of Universe

Question: A galaxy shows a spectral line at 656 nm that is observed at 689 nm. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}. Calculate (a) the recession speed of the galaxy, (b) its distance, and (c) an estimate of the age of the Universe.

Solution

Solution:
(a) $\dfrac{\Delta\lambda}{\lambda} = \dfrac{689 - 656}{656} = \dfrac{33}{656} = 0.0503$
$v = 0.0503 \times c = 0.0503 \times 3.00 \times 10^8 = \mathbf{1.51 \times 10^{7}}$ m s^{-1}

(b) $d = v/H_0 = 1.51 \times 10^7 / 2.2 \times 10^{-18} = \mathbf{6.9 \times 10^{24}}$ m
(c) $t \approx 1/H_0 = 1/(2.2 \times 10^{-18}) = \mathbf{4.5 \times 10^{17}}$ s ($\approx$ 14 billion years)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) luminosity and (b) radiant flux intensity. State the relationship between them and the distance d to the source.

[4 marks]

Q2. The Sun has a surface temperature of 5800 K. Calculate the peak wavelength of its emission spectrum and state what colour this corresponds to.

[2 marks]

Q3. Explain what is meant by a *standard candle* and describe one example of a standard candle used in astronomy.

[3 marks]

Q4. A hydrogen spectral line has a rest wavelength of 434 nm. In the spectrum of a distant galaxy it is observed at 461 nm. Calculate the recession speed of the galaxy.

[3 marks]

Q5. Explain how observations of redshift from distant galaxies lead to the conclusion that the Universe is expanding.

[3 marks]

Section B — Longer Structured Questions

Q6. Star A has surface temperature 12 000 K and radius 3.5×10^9 m. Star B has the same luminosity as Star A but a surface temperature of 4500 K.

(a) Calculate the luminosity of Star A.

[2 marks]

(b) Calculate the radius of Star B.

[3 marks]

(c) State which star would appear bluer and explain why.

[2 marks]

Q7. A galaxy is observed to have a recession speed of 4.8×10^6 m s^{-1}. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}.

(a) Calculate the distance of the galaxy.

[2 marks]

(b) A spectral line in the galaxy's spectrum has an emitted wavelength of 589 nm. Calculate the observed wavelength.

[2 marks]

(c) Explain how Hubble's law provides evidence for the Big Bang theory.

[3 marks]

Mark Scheme and Answers

Q1. (a) Luminosity: the total power of electromagnetic radiation emitted by a star (in all directions) [2]. (b) Radiant flux intensity: the power of radiation received per unit area at the observer's location [1]; $F = L/(4\pi d^2)$ [1].

Q2. $\lambda_{\text{max}} = b/T = 2.90 \times 10^{-3}/5800 = \mathbf{500}$ nm [1]; this corresponds to green light (near the centre of the visible spectrum) [1].

Q3. A standard candle is an astronomical object of known luminosity [1]; example: type Ia supernova (all reach the same peak luminosity) [1]; or Cepheid variable (period of brightness variation gives luminosity via period–luminosity relation) [1].

Q4. $\Delta\lambda = 461 - 434 = 27$ nm [1]; $v = c\Delta\lambda/\lambda = 3.00 \times 10^8 \times 27/434$ [1] $= \mathbf{1.87 \times 10^7}$ m s^{-1} [1].

Q5. Spectral lines from distant galaxies are observed at longer wavelengths (redshifted) than expected [1]; this indicates the galaxies are moving away from us [1]; the greater the distance, the greater the redshift/recession speed — consistent with all space expanding uniformly [1].

Q6(a). $L = 4\pi r^2 \sigma T^4 = 4\pi \times (3.5 \times 10^9)^2 \times 5.67 \times 10^{-8} \times (12000)^4$ [1]; $= 4\pi \times 1.225 \times 10^{19} \times 5.67 \times 10^{-8} \times 2.07 \times 10^{16} = \mathbf{1.80 \times 10^{29}}$ W [1].

Q6(b). Same L; $r_B^2 T_B^4 = r_A^2 T_A^4$ [1]; $r_B = r_A(T_A/T_B)^2 = 3.5 \times 10^9 \times (12000/4500)^2 = 3.5 \times 10^9 \times 7.11$ [1] $= \mathbf{2.49 \times 10^{10}}$ m [1].

Q6(c). Star A is bluer [1]; it has a higher surface temperature, so by Wien's law its peak wavelength is shorter (towards the blue end of the spectrum) [1].

Q7(a). $d = v/H_0 = 4.8 \times 10^6 / 2.2 \times 10^{-18} = \mathbf{2.18 \times 10^{24}}$ m [2].

Q7(b). $\Delta\lambda = \lambda v/c = 589 \times 10^{-9} \times 4.8 \times 10^6 / 3.00 \times 10^8 = 9.42 \times 10^{-9}$ m [1]; observed $\lambda = 589 + 9.4 = \mathbf{598}$ nm [1].

Q7(c). Hubble's law shows recession speed is proportional to distance — the further away a galaxy, the faster it recedes [1]; this means all galaxies are moving apart from one another, implying the Universe is expanding [1]; extrapolating back in time, all matter must have originated from a single point — the Big Bang [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define luminosity and radiant flux intensity	
☐	Use the inverse square law $F = L/(4\pi d^2)$	
☐	Define a standard candle and explain how it is used to find distance	
☐	Give examples of standard candles (Cepheid variables, Type Ia supernovae)	
☐	State and apply Wien's displacement law $\lambda_{\text{max}} T = b$	
☐	State and apply the Stefan–Boltzmann law $L = 4\pi r^2 \sigma T^4$	
☐	Combine Wien and Stefan–Boltzmann to estimate stellar radius	
☐	Explain what redshift is and how it is observed in galaxy spectra	
☐	Use $\Delta\lambda/\lambda \approx v/c$ to find recession speeds	
☐	Explain how redshift provides evidence for an expanding Universe	
☐	State and apply Hubble's law $v \approx H_0 d$ (SI units)	
☐	Use $t \approx 1/H_0$ as an estimate for the age of the Universe	
☐	Explain how Hubble's law leads to the Big Bang theory	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Cosmology asks the biggest questions in physics — and answers them with the same tools you've used all year. A few formulas, careful unit conversions, and the ability to interpret a graph are all you need to explore the scale of the Universe itself.