CIE International A Level Physics

Topic 24

Medical Physics

Revision Booklet

This booklet covers:

- Ultrasound: Generation, Detection and Imaging
- Acoustic Impedance and Reflection
- Attenuation of Ultrasound
- Production and Use of X-Rays
- Attenuation of X-Rays and CT Scanning
- PET Scanning and Annihilation

Production and Use of Ultrasound

Ultrasound

Ultrasound is sound with a frequency above the upper limit of human hearing ($>$ 20 kHz). In medical imaging, frequencies of 1–20 MHz are typical.

Piezoelectric Transducer

A **piezoelectric crystal** exhibits two related effects:

- When a **p.d. is applied** across the crystal, it changes shape (contracts or expands). Applying an alternating p.d. at the crystal's resonant frequency causes it to **vibrate and emit ultrasound**.
- Conversely, when the crystal's shape **changes** (e.g. due to an incoming pressure wave), it **generates an e.m.f.** — it acts as a detector.

The same transducer can therefore act as both **emitter and receiver**.

A-Scan Imaging (Pulse-Echo)

- A short pulse of ultrasound is emitted into the body.
- At each **boundary between tissues** of different acoustic impedance, part of the pulse is **reflected** (echo) and part is **transmitted**.
- The time delay between emission and detection of each echo gives the **depth** of the boundary: $d = \frac{1}{2}vt$.
- The amplitude of each echo gives information about the nature of the boundary.

Acoustic Impedance and Reflection

Specific Acoustic Impedance

The **specific acoustic impedance** Z of a medium is defined as:

$$Z = \rho c$$

Z = specific acoustic impedance (kg m^{-2} s^{-1})
ρ = density of the medium (kg m^{-3})
c = speed of sound in the medium (m s^{-1})

Intensity Reflection Coefficient

The fraction of intensity reflected at a boundary between two media with impedances Z_1 and Z_2:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

- If $Z_1 = Z_2$ (matched impedances): $I_R/I_0 = 0$ — no reflection, all transmitted.
- If $Z_1 \gg Z_2$ or $Z_1 \ll Z_2$ (large mismatch): $I_R/I_0 \approx 1$ — almost all reflected.
- Air–tissue boundary: huge impedance mismatch $\Rightarrow$ almost complete reflection.

Coupling Gel

Because the acoustic impedance of air is much lower than that of tissue, a large fraction of ultrasound would be reflected at the skin–air boundary if no gel were used. A **coupling gel** (with impedance close to that of tissue) is applied between the transducer and the skin to **minimise reflection** and allow ultrasound to enter the body efficiently.

Attenuation of Ultrasound

Attenuation of Ultrasound in Matter

As ultrasound travels through a medium, its intensity decreases exponentially:

$$I = I_0\, e^{-\mu x}$$

I = intensity at depth x (W m^{-2})
I_0 = initial intensity (W m^{-2})
μ = **absorption (attenuation) coefficient** of the medium (m^{-1})
x = distance travelled in the medium (m)

A larger μ means the medium absorbs ultrasound more strongly. Higher frequency ultrasound has a larger μ (greater attenuation) but better resolution.

Resolution vs Penetration Trade-off

- **Higher frequency**: shorter wavelength $\Rightarrow$ better resolution, but higher attenuation $\Rightarrow$ less depth penetration.
- **Lower frequency**: greater penetration but poorer resolution.
- The choice of frequency is a compromise depending on the depth of the structure being imaged.

Production and Use of X-Rays

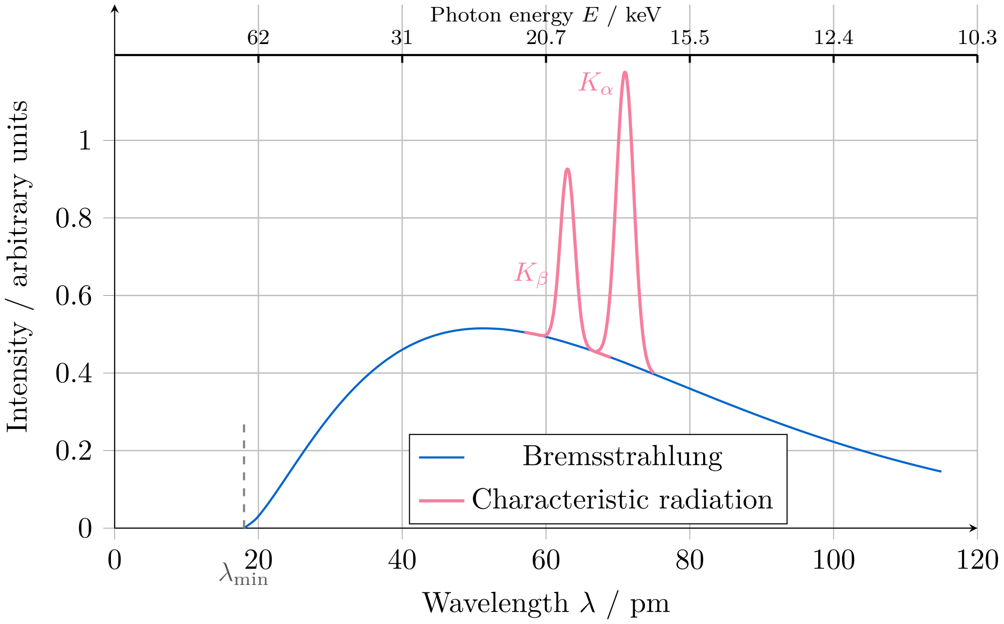
Production of X-Rays

X-rays are produced in an X-ray tube when fast-moving electrons are rapidly decelerated by a metal target (anode).

- Electrons are accelerated from a heated cathode through a high potential difference V.
- On striking the target, electrons lose kinetic energy, producing X-rays by two processes:

- **Bremsstrahlung** (braking radiation): continuous spectrum from deceleration.
- **Characteristic radiation**: discrete lines from inner-shell electron transitions in target atoms.

X-ray Emission Spectrum (Tungsten Target, 70 kV)



Minimum X-Ray Wavelength

The maximum photon energy (minimum wavelength) occurs when an electron gives all its kinetic energy to a single photon:

$$eV = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$$

$$\lambda_{\text{min}} = \frac{hc}{eV}$$

Increasing the accelerating voltage V decreases λ_{min} and increases the penetrating power of the X-rays.

Contrast in X-Ray Imaging

Contrast refers to the difference in image intensity between adjacent structures, allowing them to be distinguished.

- Dense materials (e.g. bone, containing calcium) absorb X-rays strongly ⇒ appear **white** on the image.
- Soft tissues absorb less ⇒ appear **grey**.
- Air absorbs very little ⇒ appears **black**.

- **Contrast agents** (e.g. barium meal, iodine compounds) can be introduced to increase contrast for soft tissue structures such as the gut or blood vessels.

Attenuation of X-Rays in Matter

$$I = I_0 \, e^{-\mu x}$$

The same exponential attenuation law applies as for ultrasound, where μ is now the **linear attenuation coefficient** for X-rays in the medium. Dense materials have larger μ.

CT Scanning (Computed Tomography)

A **CT scanner** produces a three-dimensional image of internal structures:

- Multiple X-ray images of the **same cross-sectional slice** are taken from **different angles**.
- These are combined computationally to produce a **2D image of one slice**.
- The process is **repeated along the body's axis**, producing multiple 2D slice images.
- The 2D slice images are **combined** to build a full **3D image**.

CT gives much better contrast for soft tissues than a plain X-ray, but involves a significantly higher radiation dose.

PET Scanning

Radioactive Tracers

A **tracer** is a substance containing radioactive nuclei that is introduced into the body and absorbed by the tissue under investigation. The emitted radiation is detected externally to produce an image of the tracer distribution.

- In PET scanning, a tracer that decays by β^+ **(positron) emission** is used.
- A common example is fluorine-18 labelled glucose (^{18}F-FDG), which is preferentially absorbed by metabolically active tissue (e.g. tumours).

Annihilation

Annihilation occurs when a particle meets its **antiparticle**: both are destroyed and their combined mass–energy is converted entirely into radiation.

- A positron (β^+) emitted by the tracer quickly meets an electron in the surrounding tissue.
- Both are annihilated, producing **two gamma-ray photons** travelling in **exactly opposite directions** (to conserve momentum).

- **Conservation laws**: mass–energy and momentum are both conserved in the process.

Energy of Annihilation Photons

Each photon carries energy equal to the rest-mass energy of one electron (the two particles have negligible kinetic energy at annihilation):

$$E_\gamma = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}\ \text{J} = \mathbf{0.511}\ \text{MeV}$$

Both photons have this energy (total energy released $= 2m_e c^2 = 1.02$ MeV).

How PET Produces an Image

- Detectors arranged around the patient detect the **coincident arrival** of the two gamma photons.
- Because the photons travel in opposite directions, the annihilation event must have occurred **somewhere along the line** joining the two detectors.
- By processing the **arrival times** from many such coincidences, the positions of annihilation events are reconstructed, producing a map of **tracer concentration** in the tissue.
- High tracer uptake indicates high metabolic activity — useful for identifying tumours, heart disease, and neurological conditions.

PET vs CT vs Ultrasound

- **Ultrasound**: no ionising radiation; good for soft tissue and real-time imaging (e.g. foetal scans); cannot penetrate bone or air.
- **X-ray / CT**: high contrast for bone; CT gives 3D information; ionising radiation dose (CT significantly higher than plain X-ray).
- **PET**: images *function* (metabolic activity), not just structure; requires a cyclotron to produce the short-lived tracer; relatively high radiation dose.

Formula Summary Sheet

Formula	Quantity	Units
$Z = \rho c$	Specific acoustic impedance	kg m^{-2} s^{-1}
$I_R/I_0 = (Z_1 - Z_2)^2/(Z_1 + Z_2)^2$	Intensity reflection coefficient	—
$I = I_0 e^{-\mu x}$	Attenuation (ultrasound or X-ray)	W m^{-2}
$d = \frac{1}{2}vt$	Depth from echo time	m
$\lambda_{\text{min}} = hc/eV$	Minimum X-ray wavelength	m
$E_\gamma = m_e c^2$	Energy of annihilation photon	J

Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

Exam Technique and Problem-Solving Strategy

Key Strategies

1. For **attenuation** questions: identify I_0, μ and x; substitute into $I = I_0 e^{-\mu x}$.
2. For **reflection coefficient**: identify Z_1 and Z_2; substitute directly — the formula is symmetric in Z_1 and Z_2.
3. For **echo depth**: $d = \frac{1}{2}vt$ (factor of $\frac{1}{2}$ because pulse travels to the boundary *and back*).
4. For **annihilation photon energy**: always 0.511 MeV per photon; quote this or calculate from $m_e c^2$.

Common Errors — Avoid These!

- Forgetting the **factor of 2** in $d = vt/2$ for pulse-echo depth calculations.
- Confusing the **attenuation coefficient** μ with decay constant λ from Topic 23 — both appear in exponential decay equations but refer to completely different physical processes.
- Stating that annihilation produces **one** photon — it must produce **two** travelling in opposite directions to conserve momentum.
- Confusing **CT** (multiple X-ray angles ⇒ 3D image) with a plain X-ray (single image, 2D projection).

- Applying the reflection coefficient formula with **intensities** instead of **impedances**.

Worked Examples

Example 1 — Acoustic Impedance and Reflection

Question: The specific acoustic impedance of muscle is 1.70×10^6 kg m^{-2}s^{-1} and of bone is 7.80×10^6 kg m^{-2}s^{-1}. Calculate the intensity reflection coefficient at a muscle–bone boundary.

Solution

Solution:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2} = \frac{(1.70 \times 10^6 - 7.80 \times 10^6)^2}{(1.70 \times 10^6 + 7.80 \times 10^6)^2}$$

$$= \frac{(-6.10 \times 10^6)^2}{(9.50 \times 10^6)^2} = \frac{3.72 \times 10^{13}}{9.02 \times 10^{13}} = \mathbf{0.413}$$

About 41% of the ultrasound intensity is reflected at this boundary.

Example 2 — Ultrasound Attenuation

Question: Ultrasound with an initial intensity of 250 W m^{-2} passes through 4.0 cm of tissue with attenuation coefficient $\mu = 23$ m^{-1}. Calculate the transmitted intensity.

Solution

Solution:

$$I = I_0 e^{-\mu x} = 250 \times e^{-23 \times 0.040} = 250 \times e^{-0.92}$$

$$I = 250 \times 0.399 = \mathbf{99.7}\ \text{W m}^{\mathbf{-2}}$$

Example 3 — PET Scanning Annihilation Energy

Question: In a PET scan, a positron emitted by the tracer annihilates with an electron. Calculate the energy and frequency of each gamma-ray photon produced.

Solution

Solution:
Energy of each photon:

$$E = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}\ \text{J}$$

$$= 8.20 \times 10^{-14} / 1.60 \times 10^{-13} = \mathbf{0.511}\ \text{MeV}$$

Frequency:

$$f = \frac{E}{h} = \frac{8.20 \times 10^{-14}}{6.63 \times 10^{-34}} = \mathbf{1.24 \times 10^{20}}\ \text{Hz}$$

(This lies in the gamma-ray region of the electromagnetic spectrum.)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Describe how a piezoelectric transducer can act as both an emitter and a detector of ultrasound.

[4 marks]

Q2. The specific acoustic impedance of soft tissue is 1.63×10^6 kg m^{-2}s^{-1} and of air is 430 kg m^{-2}s^{-1}. Show that almost all ultrasound is reflected at an air–tissue boundary.

[3 marks]

Q3. An ultrasound pulse is emitted and an echo is received 85 μs later. The speed of ultrasound in tissue is 1500 m s^{-1}. Calculate the depth of the reflecting boundary.

[2 marks]

Q4. An X-ray beam of initial intensity I_0 passes through 8.0 cm of tissue with linear attenuation coefficient $\mu = 12$ m^{-1}. Calculate the ratio I/I_0.

[2 marks]

Q5. State **two** conservation laws that apply during electron–positron annihilation in PET scanning.

[2 marks]

Section B — Longer Structured Questions

Q6. A medical ultrasound system uses a piezoelectric transducer operating at 5.0 MHz.

(a) Explain why a coupling gel is applied between the transducer and the patient's skin.

[3 marks]

(b) The attenuation coefficient of soft tissue at this frequency is 40 m^{-1}. Calculate the depth at which the intensity has fallen to 5.0% of its initial value.

[3 marks]

(c) Suggest why a lower frequency might be chosen when imaging deep structures, and state one disadvantage.

[2 marks]

Q7. PET scanning uses a tracer that emits positrons.

(a) Explain what happens when a positron emitted by the tracer encounters an electron in the body tissue.

[3 marks]

(b) Explain why two detectors placed on opposite sides of the patient must detect photons simultaneously for the event to be recorded.

[2 marks]

(c) Calculate the wavelength of the gamma-ray photons produced in the annihilation.

[2 marks]

Mark Scheme and Answers

Q1. Emitter: an alternating p.d. at the resonant frequency is applied across the crystal [1]; the crystal vibrates at that frequency [1]; emitting ultrasound waves. Detector: incoming pressure wave causes the crystal to change shape [1]; this generates an e.m.f. which is detected as an electrical signal [1].

Q2. $I_R/I_0 = (1.63 \times 10^6 - 430)^2/(1.63 \times 10^6 + 430)^2$ [1] $\approx (1.63 \times 10^6)^2/(1.63 \times 10^6)^2$ [1] ≈ 0.9995 (i.e. ≈ 1, almost total reflection) [1].

Q3. $d = \frac{1}{2}vt = \frac{1}{2} \times 1500 \times 85 \times 10^{-6} = \mathbf{6.4 \times 10^{-2}}$ m (= 6.4 cm) [2].

Q4. $I/I_0 = e^{-\mu x} = e^{-12\times0.080} = e^{-0.96} = \mathbf{0.383}$ [2].

Q5. Any two of: conservation of mass–energy [1]; conservation of momentum [1]; conservation of charge [1].

Q6(a). The acoustic impedance of air is much lower than that of tissue [1]; this large mismatch means nearly all ultrasound would be reflected at the skin–air interface [1]; the gel has impedance close to that of tissue, minimising reflection and allowing ultrasound to enter the body [1].

Q6(b). $I/I_0 = 0.050$, so $e^{-40x} = 0.050$ [1]; $-40x = \ln(0.050) = -3.00$; $x = 3.00/40 = \mathbf{0.075}$ m (= 7.5 cm) [2].

Q6(c). Lower frequency has a smaller attenuation coefficient, so ultrasound penetrates more deeply [1]; disadvantage: lower frequency has a longer wavelength, giving **poorer spatial resolution** [1].

Q7(a). The positron meets an electron [1]; both are annihilated (annihilation) [1]; two gamma-ray photons are produced travelling in exactly opposite directions [1].

Q7(b). The two photons travel in exactly opposite directions (to conserve momentum) [1]; simultaneous detection at opposite detectors confirms the annihilation occurred on the line joining the two detectors [1].

Q7(c). $E = m_e c^2 = 8.20 \times 10^{-14}$ J; $\lambda = hc/E = (6.63 \times 10^{-34} \times 3.00 \times 10^8)/8.20 \times 10^{-14} = \mathbf{2.42 \times 10^{-12}}$ m [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	**Confidence (1–3)**
☐	Explain the piezoelectric effect in both directions (emitter and detector)	
☐	Describe how pulse-echo ultrasound produces diagnostic information	
☐	Define specific acoustic impedance using $Z = \rho c$	
☐	Use the intensity reflection coefficient formula	
☐	Explain the purpose of coupling gel	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of ultrasound	
☐	Explain the resolution vs penetration trade-off for ultrasound frequency	
☐	Explain how X-rays are produced (bremsstrahlung and characteristic)	
☐	Use $\lambda_{\text{min}} = hc/eV$ for X-ray tube calculations	
☐	Explain contrast in X-ray imaging	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of X-rays	
☐	Describe how CT scanning builds a 3D image from multiple 2D slices	
☐	Explain what a radioactive tracer is and why a β^+ emitter is used in PET	
☐	Describe electron–positron annihilation and apply conservation laws	
☐	Calculate the energy of annihilation photons using $E = m_e c^2$	
☐	Explain how coincidence detection produces a PET image	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Medical physics is where fundamental physics saves lives. The same exponential attenuation, the same $E = mc^2$, the same wave properties — seen in a new and important context. Make sure you can explain *why* each technique works, not just apply the formulas.