CIE International A Level Physics

Topic 23

Nuclear Physics

Revision Booklet

This booklet covers:

- Mass–Energy Equivalence: $E = mc^2$
- Mass Defect and Binding Energy
- Binding Energy per Nucleon and Nuclear Stability
- Nuclear Fission and Fusion
- Radioactive Decay: Activity and Decay Constant
- Half-Life and Exponential Decay

Mass–Energy Equivalence

Einstein's Mass–Energy Relation

Einstein's special theory of relativity establishes that mass and energy are equivalent. A mass m at rest has an intrinsic energy given by:

$$E = mc^2$$

- $c = 3.00 \times 10^8$ m s^{-1} (speed of light in free space)
- A small mass corresponds to an enormous amount of energy.
- In nuclear reactions, small changes in mass Δm release measurable amounts of energy.

Atomic Mass Unit

The **unified atomic mass unit** (u) is defined as one-twelfth of the mass of a carbon-12 atom.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

The energy equivalent of 1 u:

$$E = mc^2 = 1.661 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$$

So 1 u $\equiv 931.5$ MeV/c^2.

Nuclear Notation and Equations

A nuclide is written ^A_ZX, where A is the **nucleon number** (mass number) and Z is the **proton number** (atomic number).

Nuclear equations must conserve:

- **Nucleon number** A (top numbers balance)
- **Proton number** Z (bottom numbers balance)
- **Mass–energy** (energy is released or absorbed)
- **Charge** and **momentum**

Example: $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$

Mass Defect and Binding Energy

Mass Defect

The **mass defect** Δm of a nucleus is the difference between the total mass of the separate constituent nucleons and the actual mass of the nucleus.

$$\Delta m = Z\, m_p + (A - Z)\, m_n - m_{\text{nucleus}}$$

m_p = mass of a proton = 1.6726×10^{-27} kg
m_n = mass of a neutron = 1.6749×10^{-27} kg
m_{nucleus} = actual measured mass of the nucleus

The mass defect is always **positive**: the nucleus is always less massive than its parts.

Binding Energy

The **binding energy** of a nucleus is the energy required to completely separate a nucleus into its constituent protons and neutrons (i.e. to infinity).

$$E_B = \Delta m \cdot c^2$$

Equivalently, it is the energy *released* when the nucleus is assembled from separate nucleons.

Energy Released in a Nuclear Reaction

$$E = c^2\, \Delta m$$

where Δm is the difference between the total mass of reactants and the total mass of products.
If $\Delta m > 0$ (reactants heavier than products): energy is **released**.
If $\Delta m < 0$: energy must be **supplied**.

Common Mistake — Mass Defect vs Binding Energy

Students often confuse the *sign convention.* The mass defect is always defined as a positive quantity (how much mass is "missing"). The binding energy is the energy equivalent of this missing mass. A **larger** binding energy means a **more stable** nucleus — it takes more energy to pull it apart.

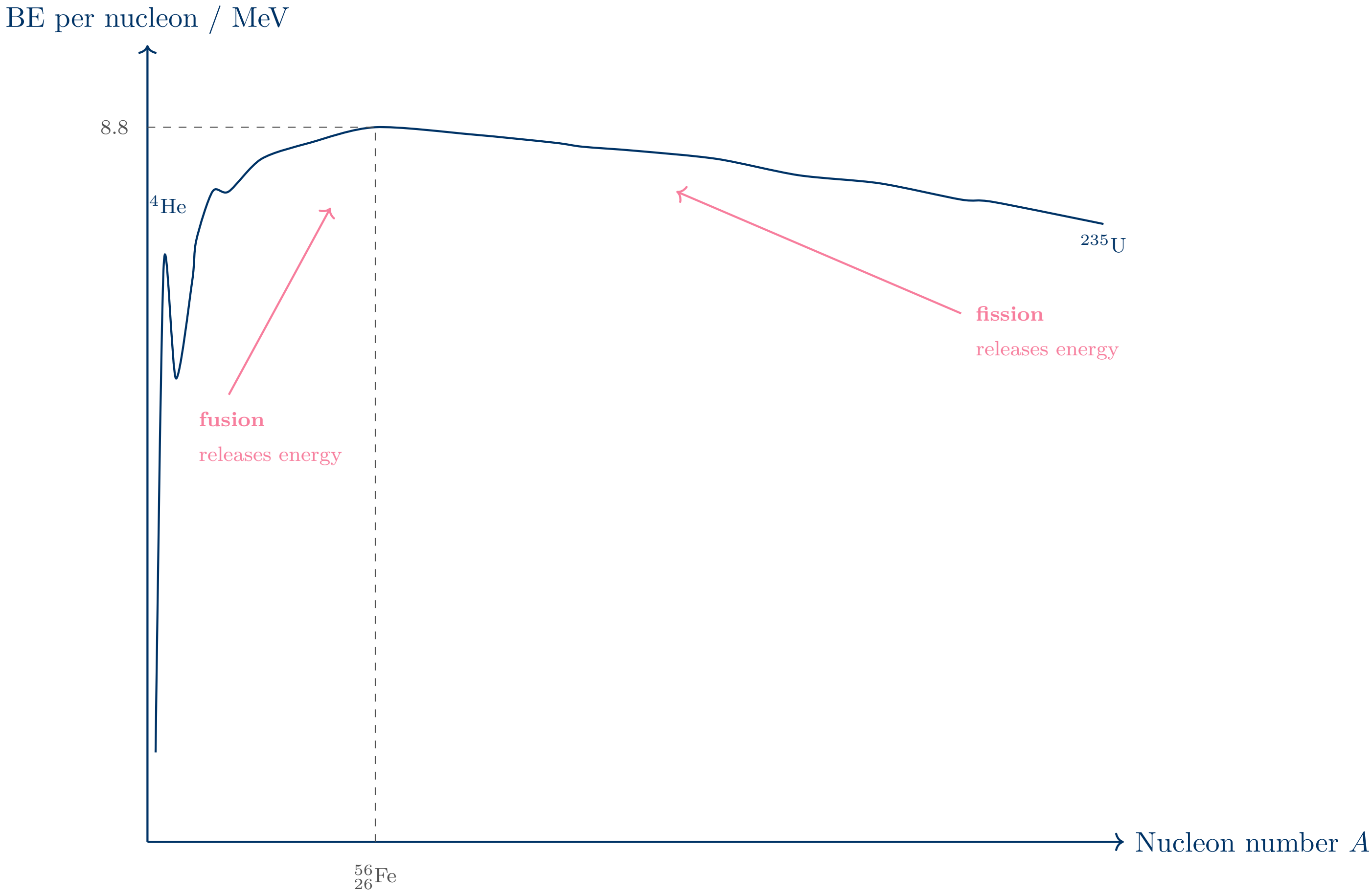
Binding Energy per Nucleon

Binding Energy per Nucleon

The **binding energy per nucleon** is the total binding energy of a nucleus divided by its nucleon number A. It is a measure of nuclear stability: the higher the value, the more stable the nucleus.

$$\text{BE per nucleon} = \frac{E_B}{A} = \frac{c^2 \Delta m}{A}$$

Variation of Binding Energy per Nucleon with Nucleon Number



Key Features of the Graph

- The curve **rises steeply** for light nuclei, peaks near $^{56}_{26}$Fe at approximately 8.8 MeV per nucleon — the most stable nucleus.
- The curve **decreases gradually** for heavy nuclei ($A > 56$).
- **Fusion** of light nuclei (left of peak) moves up the curve ⇒ products are more stable ⇒ energy is released.
- **Fission** of heavy nuclei (right of peak) also moves up the curve ⇒ products are more stable ⇒ energy is released.
- $^{4}_{2}$He (helium-4) lies notably *above* the curve — it is exceptionally stable for its mass number.

Nuclear Fission and Fusion

Nuclear Fission

Nuclear fission is the splitting of a large, unstable nucleus into two smaller (daughter) nuclei of roughly equal mass, accompanied by the release of neutrons and energy.

- Induced fission: a slow (thermal) neutron is absorbed by a heavy nucleus (e.g. ^{235}U), which then splits.
- The products have greater binding energy per nucleon than the original nucleus $\Rightarrow$ energy is released.
- Typically releases 2–3 fast neutrons which can trigger further fissions (**chain reaction**).

Example: $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$

Nuclear Fusion

Nuclear fusion is the combining of two light nuclei to form a heavier nucleus, releasing energy.

- The product nucleus has greater binding energy per nucleon than the reactants $\Rightarrow$ energy is released.
- Requires **extremely high temperatures** ($\sim 10^7$ K) to overcome electrostatic repulsion between nuclei.
- Powers stars; the basis of proposed fusion reactors.

Example: $^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$

Why These Reactions Release Energy

In both fission and fusion, the total mass of the **products** is less than the total mass of the **reactants**. This mass difference Δm is converted to kinetic energy of the products via $E = c^2\Delta m$. The reactions move nuclei *towards* the peak of the BE per nucleon curve (towards ^{56}Fe).

Radioactive Decay

Spontaneous and Random Decay

Radioactive decay is the spontaneous emission of radiation from an unstable nucleus.

- **Spontaneous**: the decay is not triggered by external conditions (temperature, pressure, chemical state); it cannot be predicted or controlled.
- **Random**: it is impossible to predict *when* any particular nucleus will decay. Each nucleus has the same probability of decaying per unit time.
- **Evidence for randomness**: fluctuations (statistical variation) in the measured count rate from a radioactive source.

Activity and Decay Constant

The **activity** A of a source is the number of nuclei that decay per unit time.

$$A = \lambda N$$

A = activity (Bq, where 1 Bq = 1 decay s^{-1})
λ = **decay constant** — the probability of decay of a nucleus per unit time (s^{-1})
N = number of undecayed nuclei present

Half-Life

The **half-life** $t_{1/2}$ is the time taken for the number of undecayed nuclei (or the activity) of a radioactive sample to fall to half its initial value.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

Half-life is constant for a given isotope — it does not depend on the number of nuclei present or external conditions.

Exponential Decay

Exponential Decay Equations

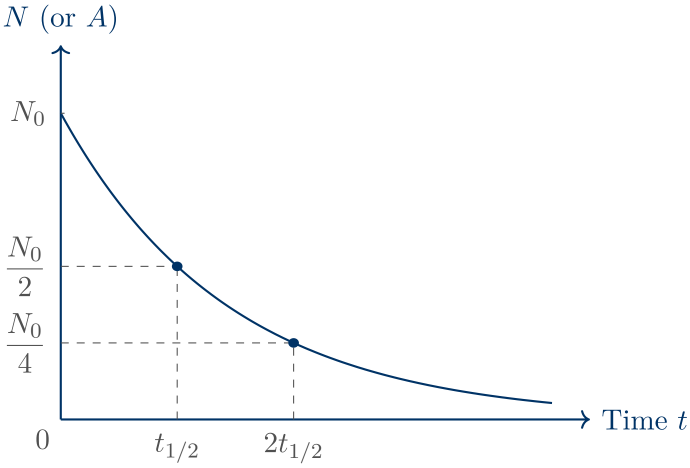
$$x = x_0\, e^{-\lambda t}$$

where x can represent:

- N = number of undecayed nuclei: $N = N_0\, e^{-\lambda t}$
- A = activity of the source: $A = A_0\, e^{-\lambda t}$
- C = received count rate: $C = C_0\, e^{-\lambda t}$

x_0 = initial value at $t = 0$
λ = decay constant (s^{-1})
t = time elapsed (s)

Graph of N against t



Linearising the Decay Equation

Taking the natural logarithm of $N = N_0 e^{-\lambda t}$:

$$\ln N = \ln N_0 - \lambda t$$

A graph of $\ln N$ (or $\ln A$) against t gives a **straight line** with:

- Gradient $= -\lambda$
- y-intercept $= \ln N_0$

This is the standard experimental method to determine λ and hence $t_{1/2}$.

Common Errors with Decay Calculations

- Using $t_{1/2}$ directly in the exponential formula — you must use λ, not $t_{1/2}$. Convert first: $\lambda = 0.693/t_{1/2}$.

- Forgetting to convert time units: if $t_{1/2}$ is in days, convert to seconds before finding λ in s^{-1}.
- Confusing **activity** $A = \lambda N$ (Bq) with **count rate** — the count rate is always less than activity due to detector efficiency and geometry.
- Applying the exponential formula to something that *increases* over time — it only applies to N, A, or count rate, which all decay.

Formula Summary Sheet

Formula	**Quantity**	**Units**
$E = mc^2$	Mass–energy equivalence	J
$E = c^2\Delta m$	Energy from mass change	J
$\Delta m = Zm_p + (A-Z)m_n - m_{\text{nuc}}$	Mass defect	kg
$E_B = \Delta m\, c^2$	Binding energy	J
$A = \lambda N$	Activity	Bq
$\lambda = 0.693/t_{1/2}$	Decay constant from half-life	s^{-1}
$N = N_0 e^{-\lambda t}$	Number of undecayed nuclei	—
$A = A_0 e^{-\lambda t}$	Activity	Bq
$\ln N = \ln N_0 - \lambda t$	Linearised decay	—

Constants: $c = 3.00 \times 10^8$ m s^{-1}, 1 u $= 1.661 \times 10^{-27}$ kg $\equiv 931.5$ MeV, $m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Nuclear Calculations

1. **Balance the equation** — check nucleon numbers and proton numbers sum correctly on both sides.
2. **Find** Δm — total reactant mass minus total product mass; work in kg or u.
3. **Apply** $E = c^2\Delta m$ to find energy released (convert u to kg if needed, or use 1 u = 931.5 MeV).
4. For decay: **convert** $t_{1/2}$ to seconds, find $\lambda = 0.693/t_{1/2}$, then apply $x = x_0e^{-\lambda t}$.

Common Errors — Avoid These!

- Using **atomic masses** (including electron masses) rather than nuclear masses without accounting for electron masses — take care with data provided in exam questions.
- Forgetting that Δm must be in **kg** when using $E = mc^2$ in SI units.
- Confusing **binding energy** with **ionisation energy** — binding energy refers to the nucleus.
- Stating that a higher binding energy per nucleon means **less** stable — it means **more** stable.
- Mixing up the direction of fusion and fission on the BE/nucleon graph.

Worked Examples

Example 1 — Mass Defect and Binding Energy

Question: Calculate the mass defect and binding energy of a helium-4 nucleus ($^4_2$He).
($m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg, $m_{^4\text{He}} = 6.6447 \times 10^{-27}$ kg)

Solution

Solution:
Mass of constituents: $2m_p + 2m_n = 2(1.6726) + 2(1.6749) = 6.6950 \times 10^{-27}$ kg
Mass defect:

$$\Delta m = 6.6950 \times 10^{-27} - 6.6447 \times 10^{-27} = \mathbf{5.03 \times 10^{-29}} \text{ kg}$$

Binding energy:

$$E_B = \Delta m\, c^2 = 5.03 \times 10^{-29} \times (3.00 \times 10^8)^2 = \mathbf{4.53 \times 10^{-12}} \text{ J } (= 28.3 \text{ MeV})$$

Binding energy per nucleon: $4.53 \times 10^{-12}/4 = 1.13 \times 10^{-12}$ J $= 7.07$ MeV per nucleon

Example 2 — Energy Released in Fission

Question: In a fission reaction, the total mass of products is 3.09×10^{-28} kg less than the total mass of reactants. Calculate the energy released in MeV.

Solution

Solution:

$$E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.09 \times 10^{-28} = 2.78 \times 10^{-11} \text{ J}$$

$$E = \frac{2.78 \times 10^{-11}}{1.60 \times 10^{-13}} = \mathbf{174} \text{ MeV}$$

Example 3 — Radioactive Decay Calculation

Question: A radioactive isotope has a half-life of 12.0 hours. A sample initially contains 8.00×10^{20} undecayed nuclei. Calculate (a) the decay constant, (b) the initial activity, (c) the number of undecayed nuclei after 30.0 hours.

Solution

Solution:
(a) Convert: $t_{1/2} = 12.0 \times 3600 = 4.32 \times 10^4$ s

$$\lambda = \frac{0.693}{4.32 \times 10^4} = \mathbf{1.60 \times 10^{-5}} \text{ s}^{-1}$$

(b) $A_0 = \lambda N_0 = 1.60 \times 10^{-5} \times 8.00 \times 10^{20} = \mathbf{1.28 \times 10^{16}}$ Bq
(c) $t = 30.0 \times 3600 = 1.08 \times 10^5$ s

$$N = N_0\, e^{-\lambda t} = 8.00 \times 10^{20} \times e^{-1.60 \times 10^{-5} \times 1.08 \times 10^5}$$

$$N = 8.00 \times 10^{20} \times e^{-1.728} = 8.00 \times 10^{20} \times 0.178 = \mathbf{1.42 \times 10^{20}}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the terms *mass defect* and *binding energy* of a nucleus.

[4 marks]

Q2. State two features of the binding energy per nucleon graph that explain why both nuclear fusion and nuclear fission can release energy.

[2 marks]

Q3. A radioactive source has an activity of 6.4×10^5 Bq and a decay constant of 2.0×10^{-3} s^{-1}. Calculate the number of undecayed nuclei present and the half-life of the source.

[4 marks]

Q4. Explain what is meant by saying that radioactive decay is *spontaneous* and *random*.

[2 marks]

Q5. Complete and balance the following nuclear equation, identifying the unknown particle X:

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + \text{X}$$

[2 marks]

Section B — Longer Structured Questions

Q6. The fusion reaction between deuterium and tritium is:

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$

Relevant masses: $m(^2\text{H}) = 2.01410$ u, $m(^3\text{H}) = 3.01605$ u, $m(^4\text{He}) = 4.00260$ u, $m_n = 1.00867$ u.

(a) Calculate the mass defect of this reaction in kg.

[3 marks]

(b) Calculate the energy released in this reaction in joules and in MeV.

[2 marks]

(c) Explain, with reference to the binding energy per nucleon graph, why this reaction releases energy.

[3 marks]

Q7. The isotope iodine-131 ($^{131}_{53}\text{I}$) is used in medical treatment. It has a half-life of 8.04 days.

(a) Calculate the decay constant of iodine-131 in s^{-1}.

[2 marks]

(b) A patient is given a dose with an initial activity of 4.0×10^8 Bq. Calculate the activity after 24 days.

[2 marks]

(c) Sketch a graph of $\ln A$ against t for this source, labelling the y-intercept and stating the gradient in terms of λ.

[3 marks]

Mark Scheme and Answers

Q1. *Mass defect*: the difference between the total mass of the separate nucleons (protons and neutrons) and the actual mass of the nucleus [2]. *Binding energy*: the energy required to completely separate a nucleus into its constituent protons and neutrons [2].

Q2. The curve rises steeply for light nuclei — fusion of light nuclei produces a product with greater BE/nucleon, so energy is released [1]. The curve falls for heavy nuclei — fission of a heavy nucleus produces fragments with greater BE/nucleon, so energy is released [1].

Q3. $N = A/\lambda = 6.4 \times 10^5/2.0 \times 10^{-3} = \mathbf{3.2 \times 10^8}$ [2]. $t_{1/2} = 0.693/\lambda = 0.693/2.0 \times 10^{-3} = \mathbf{347}$ s [2].

Q4. *Spontaneous*: the decay is not triggered or affected by external conditions; it cannot be induced or prevented [1]. *Random*: it is impossible to predict which nucleus will decay next, or when; each nucleus has the same fixed probability of decaying per unit time [1].

Q5. Nucleon: $238 = 234 + A \Rightarrow A = 4$; proton: $92 = 90 + Z \Rightarrow Z = 2$. X is ${}^4_2$He (an alpha particle) [2].

Q6(a). $\Delta m = (2.01410 + 3.01605) - (4.00260 + 1.00867) = 5.03015 - 5.01127 = 0.01888$ u [1]; $= 0.01888 \times 1.661 \times 10^{-27} = \mathbf{3.14 \times 10^{-29}}$ kg [2].

Q6(b). $E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.14 \times 10^{-29} = \mathbf{2.82 \times 10^{-12}}$ J [1]; $= 2.82 \times 10^{-12}/1.60 \times 10^{-13} = \mathbf{17.6}$ MeV [1].

Q6(c). The reactants (^{2}H and ^{3}H) lie to the left of the peak of the BE/nucleon curve [1]; the product ^{4}He has a higher binding energy per nucleon than the reactants [1]; since the products are more tightly bound, mass is converted to energy and released [1].

Q7(a). $t_{1/2} = 8.04 \times 24 \times 3600 = 6.95 \times 10^5$ s; $\lambda = 0.693/6.95 \times 10^5 = \mathbf{9.97 \times 10^{-7}}$ $\mathbf{s^{-1}}$ [2].

Q7(b). 24 days $= 3 \times t_{1/2}$; $A = 4.0 \times 10^8 \times (1/2)^3 = 4.0 \times 10^8/8 = \mathbf{5.0 \times 10^7}$ Bq [2].

Q7(c). Straight line [1]; y-intercept at $\ln(4.0 \times 10^8) = 19.8$ labelled $\ln A_0$ [1]; gradient $= -\lambda$ (negative slope) [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ State and apply Einstein's mass–energy relation $E = mc^2$	
☐ Write and balance nuclear equations, conserving A and Z	
☐ Define mass defect and calculate it from nuclear masses	
☐ Define binding energy and use $E_B = \Delta m\, c^2$	
☐ Sketch the binding energy per nucleon vs nucleon number graph	
☐ Identify the most stable nucleus and the peak of the curve	
☐ Explain using the graph why fusion of light nuclei releases energy	
☐ Explain using the graph why fission of heavy nuclei releases energy	
☐ Calculate energy released in a nuclear reaction using $E = c^2 \Delta m$	
☐ Explain what is meant by spontaneous and random decay	
☐ Define activity and decay constant; use $A = \lambda N$	
☐ Define half-life and use $\lambda = 0.693/t_{1/2}$	
☐ Apply $x = x_0 e^{-\lambda t}$ to N, A, or count rate	
☐ Linearise the decay equation and interpret the graph of $\ln N$ vs t	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Nuclear physics connects the very small to the very large — from the stability of a single nucleus to the energy source of stars. Master the binding energy curve and the exponential decay equation and most of this topic follows naturally.