

# Topic 22

## Quantum Physics

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Revision Booklet

**This booklet covers:**

- The Photoelectric Effect
- Photon Energy and the Planck Relation
- Einstein's Photoelectric Equation
- Wave-Particle Duality & de Broglie Wavelength
- Energy Levels and Line Spectra

## Core Concepts and Definitions

### The Quantum Model of Light

Classical wave theory cannot explain certain phenomena involving light and matter. Quantum theory proposes that electromagnetic radiation is emitted and absorbed in discrete packets of energy called **photons**.

- A **photon** is a quantum of electromagnetic energy.
- The energy of a photon depends only on its **frequency**, not its intensity.
- At a fixed frequency, intensity is proportional to the **number of photons** per unit time. Increasing intensity does **not** increase the energy of individual photons.

### Photon Energy — The Planck Relation

$$E = hf = \frac{hc}{\lambda}$$

- $E$  = energy of one photon (J)  
 $h$  = Planck's constant =  $6.63 \times 10^{-34}$  J s  
 $f$  = frequency of the radiation (Hz)  
 $c$  = speed of light =  $3.00 \times 10^8$  m s<sup>-1</sup>  
 $\lambda$  = wavelength (m)

### The Electronvolt

The **electronvolt** (eV) is a convenient unit of energy at the atomic scale.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

To convert eV → J: multiply by  $1.60 \times 10^{-19}$ .

To convert J → eV: divide by  $1.60 \times 10^{-19}$ .

## The Photoelectric Effect

### The Photoelectric Effect

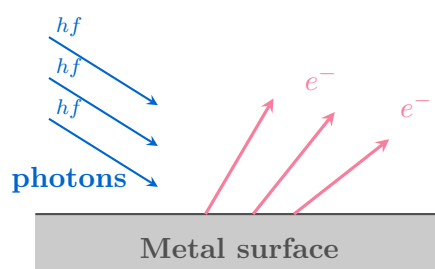
The **photoelectric effect** is the emission of electrons from a metal surface when electromagnetic radiation of sufficiently high frequency is incident on it.

- Electrons emitted are called **photoelectrons**.
- Emission occurs **instantaneously** if the frequency is above a threshold — there is no time delay.
- Below the **threshold frequency**  $f_0$ , no electrons are emitted regardless of intensity.
- Above  $f_0$ , increasing the intensity increases the **number** of photoelectrons per second, not their energy.

### Why Classical Wave Theory Fails

- Wave theory predicts that any frequency of sufficient intensity should eventually eject electrons — **not observed**.
- Wave theory predicts a time delay before emission as energy builds up — **not observed** (emission is instantaneous).
- Wave theory predicts that intensity should increase the electron's kinetic energy — **not observed**.

These failures led Einstein to propose the photon model.



### Einstein's Photoelectric Equation

$$hf = \phi + \frac{1}{2}mv_{\max}^2$$

$hf$  = energy of the incident photon (J)

$\phi$  = **work function** — minimum energy to remove an electron from the surface (J)

$\frac{1}{2}mv_{\max}^2$  = maximum kinetic energy of emitted photoelectron (J)

The **work function**  $\phi = hf_0$ , where  $f_0$  is the threshold frequency.

Maximum KE can also be written as  $eV_s$ , where  $V_s$  is the **stopping potential**.

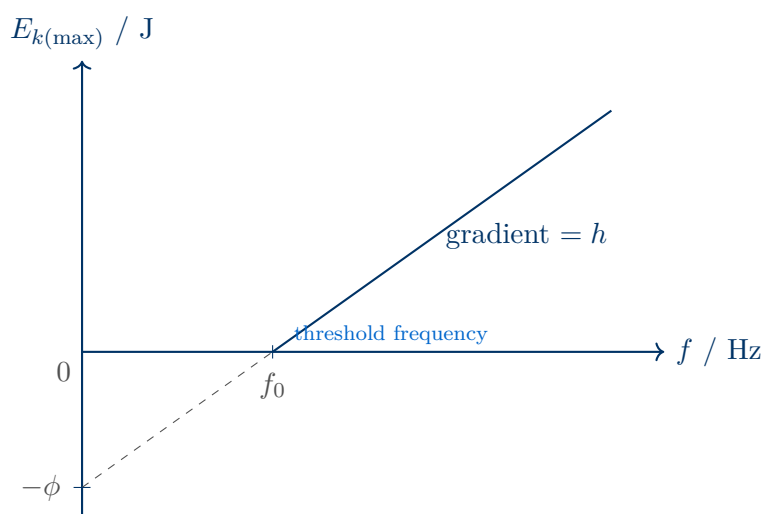
### Stopping Potential $V_s$

The **stopping potential** is the minimum opposing potential difference required to prevent all photoelectrons from reaching the collecting electrode.

$$eV_s = \frac{1}{2}mv_{\max}^2 = hf - \phi$$

Measuring  $V_s$  at different frequencies allows  $h$  and  $\phi$  to be determined experimentally.

## Graph of Maximum KE against Frequency



## Reading the Graph

- The **gradient** of the  $E_{k(\max)}$  vs  $f$  graph equals Planck's constant  $h$ .
- The **x-intercept** gives the threshold frequency  $f_0$ .
- The **y-intercept** (extrapolated) gives  $-\phi$  (the negative of the work function).
- Different metals give **parallel lines** — same gradient ( $h$ ), different intercepts ( $\phi$ ).

## Wave–Particle Duality

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**Wave–particle duality** is the property of matter and radiation by which they exhibit both wave-like and particle-like behaviour depending on the experimental context.

- Light behaves as a **wave**: diffraction, interference, polarisation.
- Light behaves as **particles (photons)**: photoelectric effect.
- Electrons (and other particles) also show **wave behaviour**: electron diffraction.

## de Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$\lambda$  = de Broglie wavelength (m)

$h$  = Planck's constant ( $6.63 \times 10^{-34}$  J s)

$p$  = momentum of the particle ( $\text{kg m s}^{-1}$ )

$m$  = mass of the particle (kg)

$v$  = speed of the particle ( $\text{m s}^{-1}$ )

### Electron Diffraction

When electrons are accelerated through a potential difference  $V$  and directed at a thin crystal or graphite film, a **diffraction pattern** of rings is observed — confirming their wave nature.

- The kinetic energy gained:  $\frac{1}{2}mv^2 = eV$ , giving  $v = \sqrt{2eV/m}$ .
- Therefore:  $\lambda = \frac{h}{\sqrt{2meV}}$
- Increasing  $V$  increases  $p$ , decreasing  $\lambda \Rightarrow$  rings become **smaller**.
- The wavelength must be comparable to the atomic spacing for diffraction to occur ( $\sim 10^{-10}$  m).

### Common Mistake — Confusing Photon and Particle Momentum

For a **photon** (massless):  $p = E/c = hf/c = h/\lambda$ .

For a **particle with mass**:  $p = mv = h/\lambda$  (de Broglie).

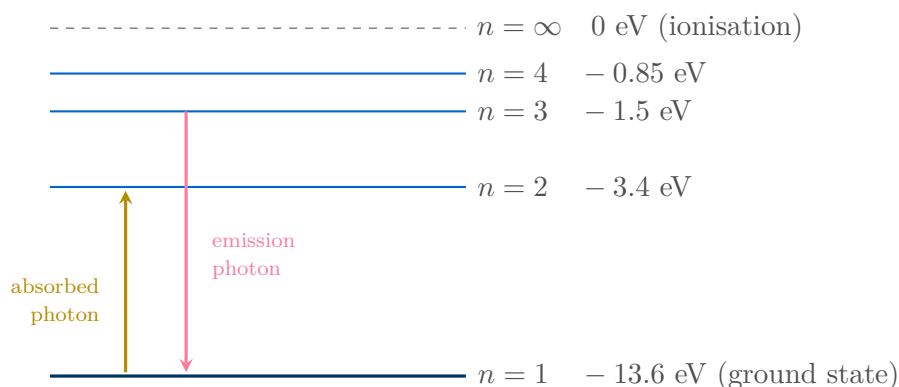
Both use  $\lambda = h/p$ , but the relationship between energy and momentum differs. Do not use  $E = hf$  for material particles.

## Energy Levels and Line Spectra

### Atomic Energy Levels

Electrons in an atom can only occupy discrete **energy levels**. These are quantised — only specific energies are allowed.

- The **ground state** is the lowest energy level (most stable; most negative value).
- **Excited states** have higher (less negative) energy.
- The **ionisation energy** is the energy required to remove an electron from the ground state to infinity ( $E = 0$ ).
- Energy levels are usually expressed in electronvolts (eV) and are **negative** (bound states).



Hydrogen Energy Level Diagram (schematic)

### Photon Energy from a Transition

When an electron moves between energy levels  $E_1$  (lower) and  $E_2$  (higher):

$$hf = E_2 - E_1$$

- **Emission:** electron falls to lower level  $\Rightarrow$  photon **emitted** with  $hf = E_2 - E_1$ .
- **Absorption:** electron promoted to higher level  $\Rightarrow$  photon **absorbed** with  $hf = E_2 - E_1$ .

### Emission and Absorption Spectra

#### Line Spectra

- **Emission spectrum:** a series of **bright coloured lines** on a dark background. Each line corresponds to a specific photon frequency emitted during a downward electron transition. Produced by excited gases.
- **Absorption spectrum:** a **continuous spectrum** crossed by dark lines at the same frequencies as the emission lines. Cool gas in front of a broad-spectrum source absorbs specific photon energies.
- The **line positions are unique** to each element — used for identification (spectroscopy).

#### Why Line Spectra Prove Quantisation

Because electrons can only occupy discrete energy levels, only photons of specific energies (frequencies) can be absorbed or emitted. This produces a **line** spectrum rather than a continuous one. A continuous spectrum would imply electrons can have any energy — which they cannot.

## Formula Summary Sheet

Formula	Quantity	Units
$E = hf$	Photon energy	J
$E = hc/\lambda$	Photon energy from wavelength	J
$hf = \phi + \frac{1}{2}mv_{\max}^2$	Einstein's photoelectric equation	J
$\phi = hf_0$	Work function	J
$eV_s = \frac{1}{2}mv_{\max}^2$	Stopping potential	J, V
$\lambda = h/p = h/mv$	de Broglie wavelength	m
$\lambda = h/\sqrt{2meV}$	de Broglie $\lambda$ from accelerating pd	m
$hf = E_2 - E_1$	Photon from energy transition	J

**Constants:**  $h = 6.63 \times 10^{-34}$  J s,  $c = 3.00 \times 10^8$  m s<sup>-1</sup>,  $e = 1.60 \times 10^{-19}$  C,  $m_e = 9.11 \times 10^{-31}$  kg

## Exam Technique and Problem-Solving Strategy

### Step-by-Step Strategy for Calculation Questions

1. **Identify** the relevant formula — is it a photon, a particle, or a transition?
2. **Convert units:** eV to J ( $\times 1.60 \times 10^{-19}$ ); nm to m ( $\times 10^{-9}$ ).
3. **Substitute** values carefully, showing all working.
4. **Quote** the final answer in appropriate units with correct significant figures.

### Common Errors — Avoid These!

- Using the **wrong energy unit** — forgetting to convert eV to J before substituting.
- Confusing **threshold frequency**  $f_0$  with threshold wavelength: a higher  $f_0$  corresponds to a **shorter** threshold wavelength  $\lambda_0$ .
- Applying **intensity** changes to change photon energy — intensity only changes the **number** of photons.
- Using  $E = hf$  for the de Broglie relation — this is for photons only; for matter use  $\lambda = h/mv$ .
- Forgetting that energy levels are **negative** — the transition energy is  $|E_2 - E_1|$ ,

not just  $E_2 - E_1$  as magnitudes.

- Confusing **emission** (bright lines) and **absorption** (dark lines) spectra.

## Worked Examples

### Example 1 — Photoelectric Effect Calculation

**Question:** Light of wavelength 250 nm is incident on a metal surface with a work function of 4.5 eV. Calculate the maximum kinetic energy of emitted photoelectrons and the stopping potential.

#### Solution

##### Solution:

Convert work function:  $\phi = 4.5 \times 1.60 \times 10^{-19} = 7.20 \times 10^{-19}$  J

Photon energy:

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{250 \times 10^{-9}} = 7.96 \times 10^{-19} \text{ J}$$

Maximum KE:

$$E_{k(\max)} = hf - \phi = 7.96 \times 10^{-19} - 7.20 \times 10^{-19} = 7.6 \times 10^{-20} \text{ J}$$

Stopping potential:

$$V_s = \frac{E_{k(\max)}}{e} = \frac{7.6 \times 10^{-20}}{1.60 \times 10^{-19}} = 0.48 \text{ V}$$

### Example 2 — de Broglie Wavelength

**Question:** An electron is accelerated from rest through a potential difference of 3.0 kV. Calculate its de Broglie wavelength.

#### Solution

##### Solution:

Energy gained:  $E_k = eV = 1.60 \times 10^{-19} \times 3000 = 4.80 \times 10^{-16}$  J

Speed:  $v = \sqrt{\frac{2E_k}{m_e}} = \sqrt{\frac{2 \times 4.80 \times 10^{-16}}{9.11 \times 10^{-31}}} = 3.25 \times 10^7 \text{ m s}^{-1}$

de Broglie wavelength:

$$\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.25 \times 10^7} = 2.24 \times 10^{-11} \text{ m}$$

This is comparable to atomic spacings ( $\sim 10^{-10}$  m), confirming why electron diffraction is observed.

### Example 3 — Energy Level Transition

**Question:** An electron in a hydrogen-like atom falls from an energy level of  $-1.5$  eV to  $-3.4$  eV. Find the frequency and wavelength of the emitted photon.

**Solution****Solution:**

Energy of emitted photon:

$$hf = E_2 - E_1 = -1.5 - (-3.4) = 1.9 \text{ eV} = 1.9 \times 1.60 \times 10^{-19} = 3.04 \times 10^{-19} \text{ J}$$

Frequency:

$$f = \frac{3.04 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.59 \times 10^{14} \text{ Hz} \quad (\text{visible light — red/orange})$$

Wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{4.59 \times 10^{14}} = 654 \text{ nm}$$

**Practice Exam Questions****Section A — Short Answer Questions**

**Q1.** State what is meant by the *threshold frequency* in the photoelectric effect.

[2 marks]

**Q2.** Explain why the photoelectric effect cannot be explained by the wave model of light. Give two reasons.

[2 marks]

**Q3.** Light of frequency  $8.0 \times 10^{14}$  Hz is incident on a metal surface with work function 2.5 eV. Show whether photoelectric emission will occur and, if so, calculate the maximum kinetic energy of the emitted electrons.

[4 marks]

**Q4.** Calculate the de Broglie wavelength of a proton ( $m_p = 1.67 \times 10^{-27}$  kg) moving at  $2.0 \times 10^6$  m s<sup>-1</sup>.

*[2 marks]*

**Q5.** An electron and a proton are each accelerated from rest through the same potential difference  $V$ . Show that the de Broglie wavelength of the electron is greater than that of the proton. ( $m_e = 9.11 \times 10^{-31}$  kg,  $m_p = 1.67 \times 10^{-27}$  kg)

*[3 marks]*

### Section B — Longer Structured Questions

**Q6.** A student investigates the photoelectric effect using a metal surface. The graph of maximum kinetic energy against frequency is a straight line. The threshold frequency of the metal is  $5.5 \times 10^{14}$  Hz.

(a) Calculate the work function of the metal in joules.

*[2 marks]*

(b) Light of frequency  $9.0 \times 10^{14}$  Hz is incident on the surface. Calculate the stopping potential.

*[3 marks]*

- (c) The intensity of the incident light is doubled whilst keeping its frequency the same. State and explain what happens to (i) the rate of electron emission, and (ii) the maximum kinetic energy of emitted electrons.

*[4 marks]*

**Q7.** The energy levels of atomic hydrogen include  $E_1 = -13.6$  eV (ground state),  $E_2 = -3.4$  eV, and  $E_3 = -1.5$  eV.

- (a) Calculate the frequency of the photon emitted when an electron transitions from  $n = 3$  to  $n = 1$ .

*[3 marks]*

- (b) A photon of energy 12.1 eV is incident on a hydrogen atom in the ground state. Explain whether this photon will be absorbed.

*[2 marks]*

- (c) Describe how the line spectrum of hydrogen provides evidence for the existence of discrete energy levels in atoms.

*[3 marks]*

## Mark Scheme and Answers

**Q1.** The threshold frequency is the minimum frequency of electromagnetic radiation [1] that is capable of causing photoelectric emission from the surface [1].

**Q2.** Any two from: wave theory predicts a time delay before emission, but emission is instantaneous [1]; wave theory predicts any frequency of sufficient intensity should cause emission, but no emission occurs below  $f_0$  regardless of intensity [1]; wave theory predicts intensity should increase electron KE, but maximum KE depends only on frequency [1].

**Q3.** Photon energy:  $E = hf = 6.63 \times 10^{-34} \times 8.0 \times 10^{14} = 5.30 \times 10^{-19} \text{ J} = 3.32 \text{ eV}$  [1]. Since  $3.32 \text{ eV} > 2.5 \text{ eV}$ , emission **will** occur [1]. Maximum KE =  $3.32 - 2.5 = 0.82 \text{ eV} = 1.31 \times 10^{-19} \text{ J}$  [2].

**Q4.**  $\lambda = h/(m_p v) = 6.63 \times 10^{-34} / (1.67 \times 10^{-27} \times 2.0 \times 10^6) = \mathbf{1.98 \times 10^{-13} \text{ m}}$  [2].

**Q5.** Both particles gain the same kinetic energy  $eV$ , so  $\frac{1}{2}mv^2 = eV \Rightarrow p = mv = \sqrt{2meV}$  [1]; therefore  $\lambda = h/\sqrt{2meV}$  [1]; since  $m_e \ll m_p$ , the denominator is smaller for the electron, giving a **larger**  $\lambda$  for the electron [1].

**Q6(a).**  $\phi = hf_0 = 6.63 \times 10^{-34} \times 5.5 \times 10^{14} = \mathbf{3.65 \times 10^{-19} \text{ J}}$  [2].

**Q6(b).**  $E_{k(\text{max})} = hf - \phi = 6.63 \times 10^{-34} \times 9.0 \times 10^{14} - 3.65 \times 10^{-19} = 5.97 \times 10^{-19} - 3.65 \times 10^{-19} = 2.32 \times 10^{-19} \text{ J}$  [2];  $V_s = E_{k(\text{max})}/e = 2.32 \times 10^{-19} / 1.60 \times 10^{-19} = \mathbf{1.45 \text{ V}}$  [1].

**Q6(c).** (i) Rate of electron emission **doubles** [1]: greater intensity  $\Rightarrow$  more photons per second  $\Rightarrow$  more photoelectron emissions per second [1]. (ii) Maximum KE is **unchanged** [1]: each photon still has the same energy ( $f$  unchanged); intensity affects number of photons, not energy per photon [1].

**Q7(a).**  $\Delta E = -1.5 - (-13.6) = 12.1 \text{ eV} = 12.1 \times 1.60 \times 10^{-19} = 1.936 \times 10^{-18} \text{ J}$  [1];  $f = \Delta E/h = 1.936 \times 10^{-18} / 6.63 \times 10^{-34} = \mathbf{2.92 \times 10^{15} \text{ Hz}}$  (ultraviolet) [2].

**Q7(b).** The energy difference from  $n = 1$  to  $n = 3$  is  $-1.5 - (-13.6) = 12.1 \text{ eV}$  [1]; the photon energy exactly matches this transition, so it **will** be absorbed and the electron promoted to  $n = 3$  [1].

**Q7(c).** Each line in the emission spectrum corresponds to a specific photon frequency [1]; each frequency corresponds to a specific photon energy, given by  $hf = E_2 - E_1$  [1]; the existence of discrete lines (rather than a continuum) shows that electrons can only transition between fixed energy values — i.e. energy levels are quantised [1].

## Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> State that electromagnetic radiation is quantised into photons	
<input type="checkbox"/> Use $E = hf$ and $E = hc/\lambda$ to calculate photon energy	
<input type="checkbox"/> Convert between joules and electronvolts	
<input type="checkbox"/> Describe the photoelectric effect and state its key observations	
<input type="checkbox"/> Explain why wave theory fails to explain the photoelectric effect	
<input type="checkbox"/> Apply Einstein's equation $hf = \phi + \frac{1}{2}mv_{\max}^2$	
<input type="checkbox"/> Define work function and threshold frequency; relate via $\phi = hf_0$	
<input type="checkbox"/> Explain the significance of stopping potential and use $eV_s = E_{k(\max)}$	
<input type="checkbox"/> State and apply the de Broglie relation $\lambda = h/p$	
<input type="checkbox"/> Describe electron diffraction as evidence for wave-particle duality	
<input type="checkbox"/> Explain what is meant by atomic energy levels and ground/excited states	
<input type="checkbox"/> Use $hf = E_2 - E_1$ for emission and absorption transitions	
<input type="checkbox"/> Interpret emission and absorption line spectra	
<input type="checkbox"/> Explain how line spectra provide evidence for energy quantisation	

*Key: 1 = Need more work    2 = Getting there    3 = Confident*

### Good luck with your revision!

Quantum physics is genuinely strange — but the maths is manageable. Focus on understanding *why* classical physics fails, and let the photon model guide the rest. The equations follow naturally from the physics.