CIE International A Level Physics

Topic 21

Alternating Currents

Revision Booklet

This booklet covers:

- Characteristics of Alternating Currents
- R.M.S. Values and Power
- Half-Wave Rectification
- Full-Wave (Bridge) Rectification
- Smoothing with a Capacitor

Characteristics of Alternating Currents

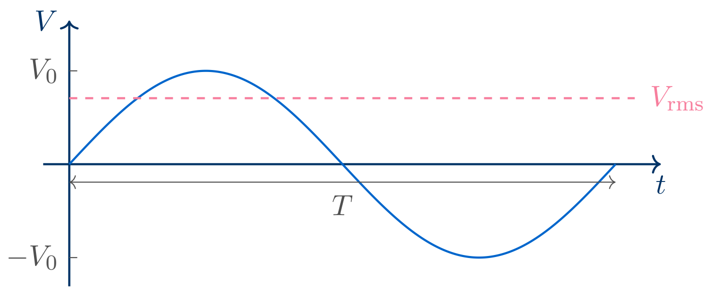
Alternating Current

An **alternating current** (a.c.) reverses direction periodically. For a sinusoidal a.c.:

$$I = I_0 \sin \omega t \qquad V = V_0 \sin \omega t$$

- I_0, V_0: **peak** (amplitude) values.
- $\omega = 2\pi f = 2\pi/T$: angular frequency (rad s^{-1}).
- T: period (s); f: frequency (Hz).
- UK mains supply: $f = 50$ Hz, $T = 20$ ms, $V_{rms} = 230$ V.

Sinusoidal a.c. voltage waveform



R.M.S. Values and Power

Root-Mean-Square (R.M.S.) Value

The **r.m.s. value** of an alternating current is defined as the value of steady direct current that would dissipate the **same power** in a purely resistive load.

R.M.S. Formulae

$$I_{rms} = \frac{I_0}{\sqrt{2}} \qquad V_{rms} = \frac{V_0}{\sqrt{2}}$$

These apply to **sinusoidal** waveforms only. For other waveforms the factor $1/\sqrt{2}$ changes.

Mean Power in a Resistive Load

$$P_{mean} = \tfrac{1}{2}P_{max} = \tfrac{1}{2}I_0^2 R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

The mean power is **half** the peak power for a sinusoidal waveform.

Why Use R.M.S.?

- The mean value of a sinusoidal current is **zero** — useless for power calculations.
- R.M.S. values allow direct use of d.c. power formulae ($P = IV$, $P = I^2R$, $P = V^2/R$).
- Meters and ratings (e.g. 230 V mains) always quote r.m.s. values.

Common Mistake

Do not use peak values in power calculations. Always convert to r.m.s. first: $V_{\text{rms}} = V_0/\sqrt{2}$. The peak mains voltage is $230\sqrt{2} \approx 325$ V, not 230 V.

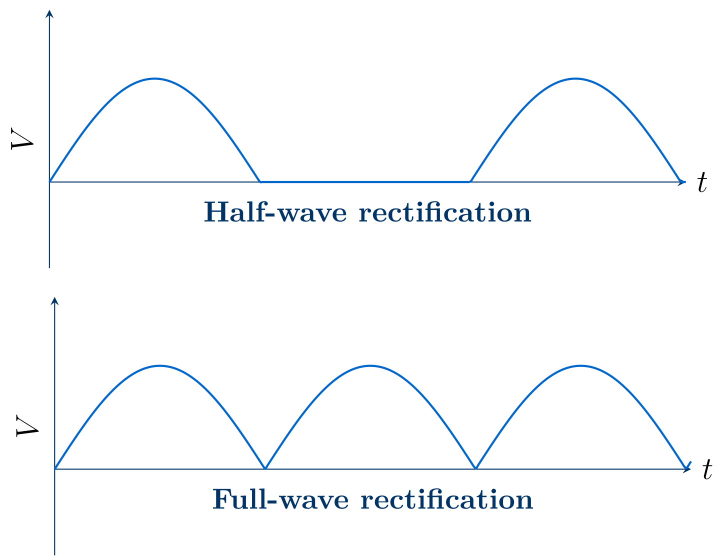
Rectification

Rectification

Rectification converts alternating current into direct current flowing in one direction only.

- **Half-wave rectification**: a single diode passes only the positive (or negative) half-cycles; output is a series of pulses with gaps.
- **Full-wave rectification**: a bridge rectifier (four diodes) inverts the negative half-cycles, producing a continuous pulsating d.c. output with no gaps.

Comparison of rectified outputs

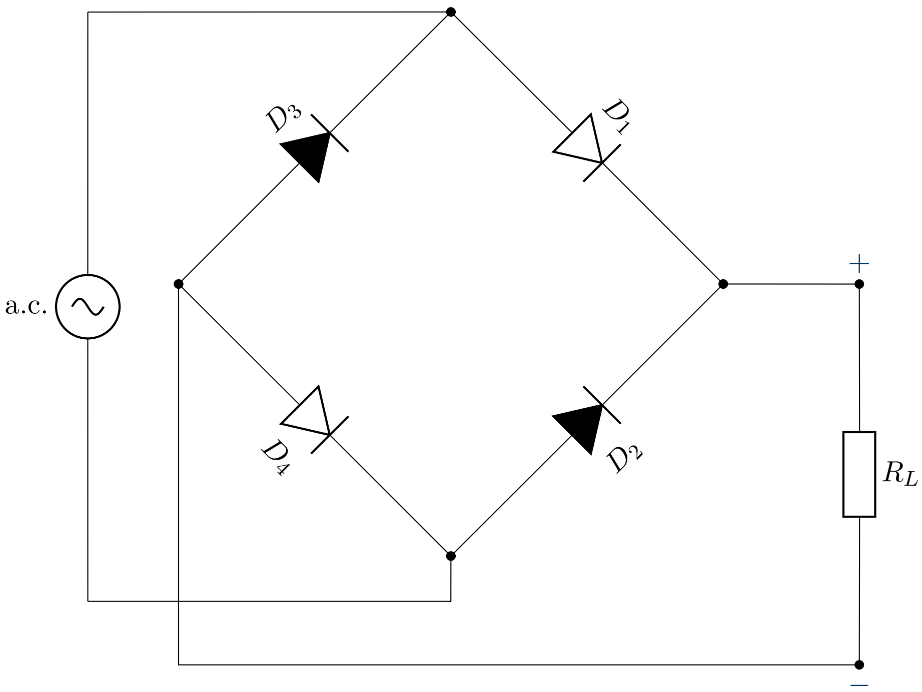


Bridge Rectifier

Bridge Rectifier

A bridge rectifier uses **four diodes** arranged in a diamond so that:

- During the **positive half-cycle**: current flows through diodes D1 and D4, through the load in the positive direction.
- During the **negative half-cycle**: current flows through diodes D2 and D3, but still through the load in the **same** direction.
- Both half-cycles contribute to the output — no wasted half-cycles.



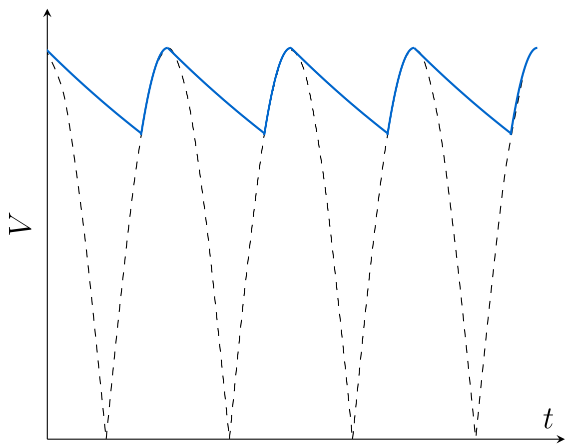
Smoothing with a Capacitor

Smoothing

A capacitor connected in **parallel with the load** reduces the ripple on a rectified output:

- The capacitor **charges up** rapidly to the peak voltage during each pulse.
- It **discharges slowly** through the load R_L between peaks, maintaining a more constant voltage.
- The output has a small residual **ripple voltage** rather than falling to zero between pulses.
- **Larger** C or **larger** R_L: longer time constant $\tau = CR_L$, less ripple.
- **Smaller** C or **smaller** R_L: faster discharge, larger ripple.

Effect of smoothing capacitor on full-wave rectified output



Common Mistake

In exam questions, always link ripple size explicitly to the time constant $\tau = CR_L$ — just saying “bigger capacitor” without explanation will not earn full marks.

Formula Summary Sheet

Formula	Quantity	Units
$I = I_0 \sin \omega t$	Sinusoidal alternating current	A
$V = V_0 \sin \omega t$	Sinusoidal alternating voltage	V
$\omega = 2\pi f = 2\pi/T$	Angular frequency	rad s^{-1}
$I_{rms} = I_0/\sqrt{2}$	R.M.S. current	A
$V_{rms} = V_0/\sqrt{2}$	R.M.S. voltage	V
$P_{mean} = \frac{1}{2}I_0^2 R$	Mean power (peak values)	W
$P_{mean} = I_{rms}^2 R = V_{rms}^2/R$	Mean power (r.m.s. values)	W

UK mains: $V_{rms} = 230$ V; $f = 50$ Hz; $V_0 = 230\sqrt{2} \approx 325$ V
Note: $1/\sqrt{2} \approx 0.707$; $P_{mean} = \frac{1}{2}P_{max}$ for sinusoidal waveform only.

Worked Examples

Example 1 — R.M.S. and Peak Values

Question: The mains supply has $V_{rms} = 230$ V at 50 Hz. Find (a) the peak voltage, (b) the angular frequency and (c) the mean power in a 1.2 kΩ resistor.

Solution

(a) $V_0 = V_{rms}\sqrt{2} = 230\sqrt{2} = \mathbf{325}$ V
(b) $\omega = 2\pi f = 2\pi \times 50 = \mathbf{314}$ rad s^{-1}
(c) $P = V_{rms}^2/R = 230^2/(1.2 \times 10^3) = 52900/1200 = \mathbf{44}$ W

Example 2 — Peak Power and Mean Power

Question: An a.c. supply has peak voltage $V_0 = 12$ V and is connected to a 60 Ω resistor. Calculate (a) the peak power and (b) the mean power dissipated.

Solution

(a) $P_{max} = V_0^2/R = 144/60 = \mathbf{2.4}$ W
(b) $P_{mean} = \frac{1}{2}P_{max} = \mathbf{1.2}$ W
Or equivalently: $V_{rms} = 12/\sqrt{2}$; $P = V_{rms}^2/R = 72/60 = 1.2$ W

Example 3 — Smoothing

Question: A full-wave rectifier feeds a 470 μF smoothing capacitor in parallel with a 2.2 kΩ load. Calculate the time constant and comment on the degree of smoothing for a 50 Hz supply.

Solution

$\tau = CR = 470 \times 10^{-6} \times 2200 = \mathbf{1.03}$ s
The period of the full-wave rectified signal is $T/2 = 1/(2 \times 50) = 10$ ms.
Since $\tau = 1.03$ s $\gg 10$ ms, the capacitor discharges very little between peaks — the output is well smoothed with very small ripple.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Explain what is meant by the r.m.s. value of an alternating current and state why it is more useful than the peak value.

[3 marks]

Q2. An a.c. supply has peak voltage 340 V. Calculate (a) the r.m.s. voltage and (b) the mean power delivered to a 680 Ω resistor.

[3 marks]

Q3. Distinguish between half-wave and full-wave rectification. State the number of diodes required for each.

[3 marks]

Q4. Explain how a capacitor connected in parallel with a load resistor smooths a rectified output. State the effect of increasing the capacitance.

[3 marks]

Section B — Longer Structured Questions

Q5. The alternating voltage from a supply is given by $V = 170 \sin(100\pi t)$, where V is in volts and t is in seconds.

(a) State the peak voltage and the frequency of the supply.

[2 marks]

(b) Calculate the r.m.s. voltage.

[1 mark]

(c) The supply is connected to a 500 Ω resistor. Calculate the mean power dissipated.

[2 marks]

(d) The supply is now passed through a bridge rectifier and a smoothing capacitor of 1000 μF is connected in parallel with the 500 Ω load. Calculate the time constant and comment on the effectiveness of smoothing.

[3 marks]

Q6. The graph below represents the output of a full-wave rectifier before smoothing. The peak voltage is 12 V and the supply frequency is 50 Hz.

(a) State the frequency of the rectified output.

[1 mark]

(b) On the same axes, sketch the output after connecting a large smoothing capacitor in parallel with the load. Indicate the approximate ripple voltage.

[2 marks]

(c) Explain what happens to the smoothing if the load resistance is reduced.

[2 marks]

Mark Scheme and Answers

Q1. The r.m.s. value is the equivalent steady d.c. that dissipates the same power in a resistive load [1]; it is more useful because power formulae ($P = I^2R$, $P = V^2/R$) can be applied directly [1]; the mean of a sinusoidal current is zero, so it gives no information about power [1].

Q2(a). $V_{\text{rms}} = V_0/\sqrt{2} = 340/\sqrt{2} = \mathbf{240}$ V [1]. **Q2(b).** $P = V_{\text{rms}}^2/R = 240^2/680 = 57600/680 = \mathbf{85}$ W [2].

Q3. Half-wave: uses **1 diode**; only one half of each cycle is passed; output has gaps of zero voltage [1]. Full-wave: uses **4 diodes** (bridge rectifier); both half-cycles are used; output is always positive with no gaps [2].

Q4. The capacitor charges to the peak voltage during each pulse [1]; between pulses it discharges slowly through the load, maintaining a more constant output voltage [1]; increasing C increases the time constant $\tau = CR$, so the capacitor discharges less between pulses and the ripple is smaller [1].

Q5(a). Peak voltage $V_0 = \mathbf{170}$ V [1]; $\omega = 100\pi$, so $f = \omega/2\pi = 100\pi/2\pi = \mathbf{50}$ Hz [1].

Q5(b). $V_{\text{rms}} = 170/\sqrt{2} = \mathbf{120}$ V [1].

Q5(c). $P = V_{\text{rms}}^2/R = (120)^2/500$ [1] $= 14400/500 = \mathbf{28.8}$ W [1].

Q5(d). $\tau = CR = 1000 \times 10^{-6} \times 500 = \mathbf{0.50}$ s [1]; period of full-wave output $= 1/(2 \times 50) = 10$ ms [1]; $\tau \gg T/2$ so capacitor barely discharges between peaks — very effective smoothing with tiny ripple [1].

Q6(a). The full-wave rectified output has frequency $2 \times 50 = \mathbf{100}$ Hz [1].

Q6(b). Sketch: smoothed output just below 12 V with small sawtooth ripple; ripple voltage is the small oscillation between the capacitor charge and discharge levels [2].

Q6(c). Reducing load resistance increases the discharge current [1]; the capacitor discharges faster between peaks ($\tau = CR$ decreases), so the ripple voltage increases and smoothing is less effective [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define period, frequency and peak value for an a.c. waveform	
☐ Use $x = x_0 \sin \omega t$ for sinusoidal current or voltage	
☐ Define r.m.s. value and explain why it is useful	
☐ Use $I_{\text{rms}} = I_0/\sqrt{2}$ and $V_{\text{rms}} = V_0/\sqrt{2}$	
☐ Calculate mean power using r.m.s. values: $P = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$	
☐ State that mean power is half peak power for sinusoidal a.c.	
☐ Distinguish half-wave and full-wave rectification graphically	
☐ Explain the action of a single diode for half-wave rectification	
☐ Explain the action of a bridge rectifier (four diodes)	
☐ Explain smoothing by a capacitor in terms of charge/discharge	
☐ Analyse the effect of C and R_L on the time constant and ripple	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

R.M.S. values are one of physics's most elegant ideas: a way to make a continuously varying quantity equivalent to a steady one. Once you see that $P_{\text{mean}} = \frac{1}{2}P_{\text{max}}$ comes directly from $\langle \sin^2 \rangle = \frac{1}{2}$, the whole topic falls into place.