

# Topic 20

## Magnetic Fields

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Revision Booklet

**This booklet covers:**

- Magnetic Fields and Field Lines
- Force on a Current-Carrying Conductor
- Force on a Moving Charge
- Hall Effect and Velocity Selector
- Magnetic Fields due to Currents
- Electromagnetic Induction

## Magnetic Fields

### Magnetic Field

A **magnetic field** is a region of space in which a moving charge, or a current-carrying conductor, experiences a force.

- Magnetic fields are produced by **moving charges** (electric currents) or by **permanent magnets**.
- The field is represented by **field lines** (flux lines); the direction is the direction of the force on a north pole.
- Field lines never cross; closer lines indicate a stronger field.
- Crosses ( $\times$ ) represent field into the page; dots ( $\bullet$ ) represent field out of the page.

## Force on a Current-Carrying Conductor

### Force on a Conductor

$$F = BIL \sin \theta$$

$F$  = force on the conductor (N)

$B$  = magnetic flux density (T)

$I$  = current in the conductor (A)

$L$  = length of conductor in the field (m)

$\theta$  = angle between the conductor and the field direction

Force is **maximum** when  $\theta = 90^\circ$  (conductor perpendicular to field):  $F = BIL$ .

Force is **zero** when  $\theta = 0^\circ$  (conductor parallel to field).

### Magnetic Flux Density $B$

**Magnetic flux density** is defined as the force per unit current per unit length acting on a wire placed **at right angles** to the field.

$$B = \frac{F}{IL} \quad \text{units: T (tesla)} \equiv \text{N A}^{-1}\text{m}^{-1}$$

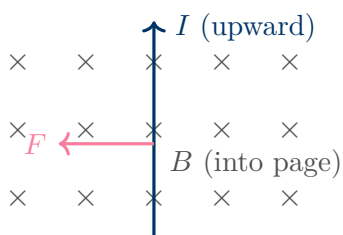
$B$  is a vector quantity. 1 T is a strong field; Earth's field is  $\approx 50 \mu\text{T}$ .

### Fleming's Left-Hand Rule

For a **conventional current** in a magnetic field:

- **Thumb:** direction of force (motion)  $F$
- **First finger:** direction of magnetic field  $B$
- **Second finger:** direction of conventional current  $I$

Remember: the rule gives the force on *positive* charges / conventional current. For electrons, reverse the direction.



### Force on a Moving Charge

#### Force on a Moving Charge

$$F = BQv \sin \theta$$

$F$  = magnetic force (N)

$B$  = magnetic flux density (T)

$Q$  = charge (C)

$v$  = speed of the charge ( $\text{m s}^{-1}$ )

$\theta$  = angle between velocity and field

For  $\theta = 90^\circ$ :  $F = BQv$ , directed perpendicular to both  $v$  and  $B$ .

#### Circular Motion in a Magnetic Field

When a charged particle moves **perpendicular** to a uniform magnetic field, the magnetic force is always perpendicular to the velocity, so no work is done and the **speed is constant**. The particle moves in a **circle**:

$$BQv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{BQ}$$

A larger momentum or smaller  $B$  gives a larger radius.

## The Hall Effect

### Hall Voltage

When a current-carrying conductor is placed in a magnetic field perpendicular to the current, charge carriers experience a sideways force. They accumulate on one face until the electric force balances the magnetic force, producing the **Hall voltage**:

$$V_H = \frac{BI}{ntq}$$

$V_H$  = Hall voltage (V)

$B$  = magnetic flux density (T)

$I$  = current through the conductor (A)

$n$  = number density of charge carriers ( $\text{m}^{-3}$ )

$t$  = thickness of the conductor in the direction of  $B$  (m)

$q$  = charge on each carrier (C)

A **Hall probe** uses this effect to measure  $B$ : since  $V_H \propto B$  at constant  $I$ .

## Velocity Selector

### Velocity Selector

A velocity selector uses **crossed** electric and magnetic fields so that only particles with a specific speed pass through undeflected:

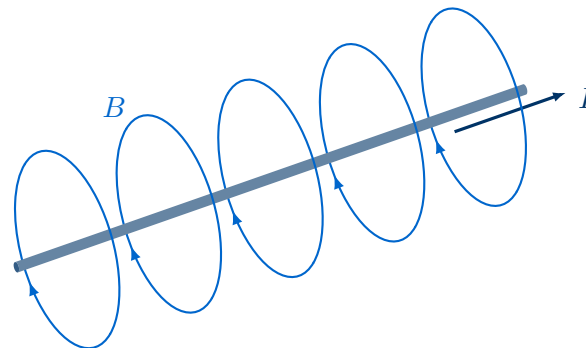
$$qE = BQv \quad \Rightarrow \quad v = \frac{E}{B}$$

- Electric force  $qE$  and magnetic force  $BQv$  act in **opposite** directions.
- Only particles where these forces balance travel in a straight line.
- Faster particles are deflected one way; slower particles the other.

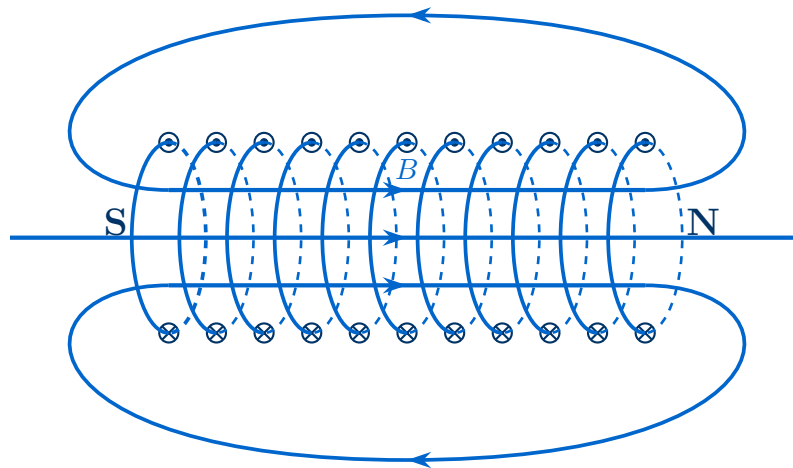
## Magnetic Fields due to Currents

### Field Patterns

- **Long straight wire:** concentric circles centred on the wire. Direction given by the **right-hand grip rule** — thumb points in direction of current, fingers curl in direction of field.
- **Flat circular coil:** field lines pass through the centre of the coil. Field at the centre is approximately uniform over a small region.
- **Long solenoid:** nearly uniform field inside, similar to a bar magnet outside. A **ferrous (iron) core** greatly increases the field strength.



Magnetic field around a straight wire



Magnetic field around a Solenoid

### Forces Between Parallel Conductors

- **Same direction currents:** conductors **attract** each other.
- **Opposite direction currents:** conductors **repel** each other.
- Each conductor sits in the magnetic field produced by the other; the force is given by  $F = BIL$ .

## Electromagnetic Induction

### Magnetic Flux

**Magnetic flux**  $\Phi$  is the product of the magnetic flux density and the cross-sectional area perpendicular to the field:

$$\Phi = BA \cos \theta \quad \text{units: Wb (weber)} \equiv \text{T m}^2$$

When  $B$  is perpendicular to the area:  $\Phi = BA$ .

**Flux linkage**  $N\Phi$  is the flux through a single turn multiplied by the number of turns  $N$  of the coil: units Wb-turns.

### Faraday's Law and Lenz's Law

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

**Faraday's Law:** the induced e.m.f. is proportional to the rate of change of flux linkage.

**Lenz's Law:** the induced e.m.f. (and hence current) acts in a direction that **opposes** the change in flux that caused it (the minus sign above).

$\mathcal{E}$  = induced e.m.f. (V)

$N$  = number of turns

$\Delta\Phi/\Delta t$  = rate of change of flux ( $\text{Wb s}^{-1} \equiv \text{V}$ )

### Factors Affecting the Induced E.M.F.

- **Rate of change** of flux: faster change  $\Rightarrow$  larger e.m.f.
- **Number of turns**  $N$ : more turns  $\Rightarrow$  larger e.m.f.
- **Strength of field**  $B$ : stronger field  $\Rightarrow$  larger flux change.
- **Area** of coil: larger area  $\Rightarrow$  more flux.

Lenz's law is a consequence of conservation of energy — the induced current creates a force that opposes the motion causing induction.

### Common Mistake

An e.m.f. is induced only when the flux is **changing**. A conductor stationary in a steady field has zero induced e.m.f., even if the field is strong.

## Formula Summary Sheet

Formula	Quantity	Units
$F = BIL \sin \theta$	Force on current-carrying conductor	N
$F = BQv \sin \theta$	Force on moving charge	N
$r = mv/(BQ)$	Radius of circular orbit in $B$ field	m
$V_H = BI/(ntq)$	Hall voltage	V
$v = E/B$	Velocity selector condition	$\text{m s}^{-1}$
$\Phi = BA \cos \theta$	Magnetic flux	Wb
$\mathcal{E} = -\Delta(N\Phi)/\Delta t$	Faraday's / Lenz's law	V

**Units:**  $1 \text{ T} = 1 \text{ N A}^{-1}\text{m}^{-1}$ ;  $1 \text{ Wb} = 1 \text{ T m}^2 = 1 \text{ V s}$

**Right-hand grip rule:** thumb  $\parallel$  current, fingers curl in direction of  $B$  field.

## Worked Examples

### Example 1 — Force on a Conductor

**Question:** A wire of length 0.15 m carries a current of 3.0 A at  $60^\circ$  to a uniform field of 0.25 T. Calculate the force on the wire.

**Solution**

$$F = BIL \sin \theta = 0.25 \times 3.0 \times 0.15 \times \sin 60^\circ$$

$$F = 0.25 \times 3.0 \times 0.15 \times 0.866 = \mathbf{9.7 \times 10^{-2} \text{ N}}$$

### Example 2 — Circular Motion of a Charged Particle

**Question:** A proton (mass  $1.67 \times 10^{-27}$  kg, charge  $1.6 \times 10^{-19}$  C) moves at  $2.0 \times 10^6$   $\text{m s}^{-1}$  perpendicular to a field of 0.15 T. Calculate the radius of its circular path.

**Solution**

$$r = \frac{mv}{BQ} = \frac{1.67 \times 10^{-27} \times 2.0 \times 10^6}{0.15 \times 1.6 \times 10^{-19}} = \frac{3.34 \times 10^{-21}}{2.40 \times 10^{-20}} = \mathbf{0.14 \text{ m}}$$

### Example 3 — Induced E.M.F.

**Question:** A coil of 200 turns and area  $50 \text{ cm}^2$  is in a field of 0.30 T perpendicular to the plane of the coil. The field drops to zero in 0.040 s. Calculate the induced e.m.f.

## Solution

$$\Delta\Phi = B \times A = 0.30 \times 50 \times 10^{-4} = 1.5 \times 10^{-3} \text{ Wb}$$

$$\mathcal{E} = N \frac{\Delta\Phi}{\Delta t} = 200 \times \frac{1.5 \times 10^{-3}}{0.040} = 7.5 \text{ V}$$

## Practice Exam Questions

## Section A — Short Answer Questions

**Q1.** Define magnetic flux density and state its SI unit.

*[2 marks]*

**Q2.** A wire of length 8.0 cm is placed perpendicular to a field of flux density 0.40 T and carries a current of 2.5 A. Calculate the force on the wire.

*[2 marks]*

**Q3.** State Faraday's Law and Lenz's Law of electromagnetic induction.

*[3 marks]*

**Q4.** Explain why the speed of a charged particle moving perpendicular to a uniform magnetic field remains constant.

*[2 marks]*

**Q5.** A Hall probe gives a voltage of 3.2 mV when a current of 50 mA passes through a slice of thickness 2.0 mm in a field  $B$ . Given  $n = 8.5 \times 10^{28} \text{ m}^{-3}$  and  $q = 1.6 \times 10^{-19} \text{ C}$ , calculate  $B$ .

*[3 marks]*

### Section B — Longer Structured Questions

**Q6.** An electron (mass  $9.11 \times 10^{-31} \text{ kg}$ , charge  $1.6 \times 10^{-19} \text{ C}$ ) enters a uniform magnetic field of  $2.0 \times 10^{-3} \text{ T}$  perpendicular to the field with speed  $5.0 \times 10^6 \text{ m s}^{-1}$ .

(a) Calculate the radius of the circular path followed by the electron.

*[2 marks]*

(b) The field strength is doubled. State and explain the effect on the radius.

*[2 marks]*

(c) An electric field is now applied perpendicular to  $B$  so that the electron travels in a straight line. Calculate the electric field strength required.

*[2 marks]*

**Q7.** A rectangular coil of 80 turns and dimensions  $4.0 \text{ cm} \times 6.0 \text{ cm}$  is placed with its plane perpendicular to a uniform field of  $0.50 \text{ T}$ .

(a) Calculate the flux linkage through the coil.

*[2 marks]*

(b) The coil is rotated so that its plane becomes parallel to the field in  $0.030 \text{ s}$ . Calculate the mean induced e.m.f.

*[2 marks]*

(c) State and explain the direction of the induced current using Lenz's law.

*[2 marks]*

## Mark Scheme and Answers

**Q1.** Magnetic flux density is the force per unit current per unit length on a wire placed at right angles to the field [1]; unit: tesla (T) or  $\text{N A}^{-1} \text{ m}^{-1}$  [1].

**Q2.**  $F = BIL \sin 90^\circ = 0.40 \times 2.5 \times 0.080 = \mathbf{0.080 \text{ N}}$  [2].

**Q3.** Faraday's Law: the induced e.m.f. is proportional to the rate of change of flux linkage [1];  $\mathcal{E} = -\Delta(N\Phi)/\Delta t$  [1]. Lenz's Law: the induced e.m.f. acts in a direction to oppose the change in flux that caused it [1].

**Q4.** The magnetic force is always perpendicular to the velocity [1]; a perpendicular force does no work, so kinetic energy and hence speed remain unchanged [1].

**Q5.**  $B = V_H n t q / I = (3.2 \times 10^{-3} \times 8.5 \times 10^{28} \times 2.0 \times 10^{-3} \times 1.6 \times 10^{-19}) / (50 \times 10^{-3})$  [2] =  $\mathbf{0.174 \text{ T}}$  [1].

**Q6(a).**  $r = mv/(BQ) = (9.11 \times 10^{-31} \times 5.0 \times 10^6)/(2.0 \times 10^{-3} \times 1.6 \times 10^{-19})$  [1]  
 $= 1.4 \times 10^{-2}$  m [1].

**Q6(b).** Radius halves [1];  $r = mv/BQ$ , so  $r \propto 1/B$ ; doubling  $B$  halves  $r$  [1].

**Q6(c).** For straight-line motion:  $qE = BQv$ ;  $E = Bv = 2.0 \times 10^{-3} \times 5.0 \times 10^6 = 1.0 \times 10^4$  V m<sup>-1</sup> [2].

**Q7(a).**  $\Phi = BA = 0.50 \times (0.040 \times 0.060) = 1.2 \times 10^{-3}$  Wb [1]; flux linkage =  $N\Phi = 80 \times 1.2 \times 10^{-3} = 9.6 \times 10^{-2}$  Wb-turns [1].

**Q7(b).**  $\Delta(N\Phi) = 9.6 \times 10^{-2}$  Wb-turns (falls to zero) [1];  $\mathcal{E} = 9.6 \times 10^{-2}/0.030 = 3.2$  V [1].

**Q7(c).** By Lenz's law the induced current opposes the decrease in flux [1]; the current flows in the direction that would create a field to oppose the rotation / maintain the flux through the coil [1].

## Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Define magnetic flux density; use $F = BIL \sin \theta$	
<input type="checkbox"/> Apply Fleming's left-hand rule to find force directions	
<input type="checkbox"/> Use $F = BQv \sin \theta$ for force on a moving charge	
<input type="checkbox"/> Derive and use $r = mv/(BQ)$ for circular orbit radius	
<input type="checkbox"/> Explain and use the Hall effect; use $V_H = BI/(ntq)$	
<input type="checkbox"/> Explain the velocity selector condition $v = E/B$	
<input type="checkbox"/> Sketch field patterns for straight wire, circular coil and solenoid	
<input type="checkbox"/> Apply the right-hand grip rule for field direction around a wire	
<input type="checkbox"/> Explain forces between parallel current-carrying conductors	
<input type="checkbox"/> Define magnetic flux $\Phi = BA \cos \theta$ and flux linkage $N\Phi$	
<input type="checkbox"/> State and apply Faraday's and Lenz's laws	
<input type="checkbox"/> Identify factors that affect the magnitude of induced e.m.f.	

*Key: 1 = Need more work    2 = Getting there    3 = Confident*

### Good luck with your revision!

Faraday's law — one equation — explains the generator, the transformer, and the electric motor. Master flux linkage and you understand how almost all electrical power is generated.