CIE International A Level Physics

Topic 19

Capacitance

Revision Booklet

This booklet covers:

- Capacitors and Capacitance
- Capacitors in Series and Parallel
- Energy Stored in a Capacitor
- Capacitor Discharge
- The Time Constant

Core Concepts and Definitions

Capacitance

The **capacitance** of a conductor is the charge stored per unit potential difference across it.

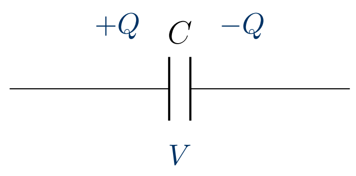
$$C = \frac{Q}{V} \qquad \text{units: F (farads)} \equiv \text{C V}^{-1}$$

- 1 F is a very large unit; practical capacitors are typically μF, nF or pF.
- Capacitance depends on the **geometry** of the conductor and the medium between the plates, not on Q or V individually.
- For an **isolated spherical conductor** of radius R: $C = 4\pi\varepsilon_0 R$.

The Parallel Plate Capacitor

Two conducting plates of area A separated by distance d:

- Charging the capacitor stores charge $+Q$ on one plate and $-Q$ on the other.
- The electric field between the plates is uniform: $E = V/d$.
- Capacitance increases with larger plate area A and smaller separation d.



Capacitors in Series and Parallel

Capacitors in Parallel

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$$

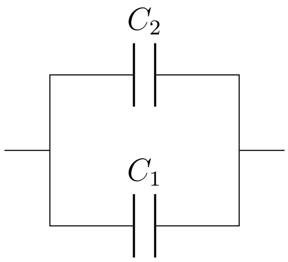
Each capacitor has the **same voltage**; charges add: $Q_{\text{total}} = Q_1 + Q_2 + \cdots$

Capacitors in Series

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Each capacitor has the **same charge**; voltages add: $V_{\text{total}} = V_1 + V_2 + \cdots$

Circuit diagrams



Parallel: $C_T = C_1 + C_2$

Series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

Derivation of Series Formula

For capacitors in series, each carries the same charge Q. The total voltage is:

$$V_{\text{total}} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Since $C_{\text{total}} = Q/V_{\text{total}}$, dividing through by Q gives $\dfrac{1}{C_{\text{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Common Mistake

Capacitors in series combine like **resistors in parallel** (reciprocal rule), and capacitors in parallel combine like **resistors in series** (direct sum). This is the opposite to resistors — don't mix them up.

Energy Stored in a Capacitor

Energy Stored

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

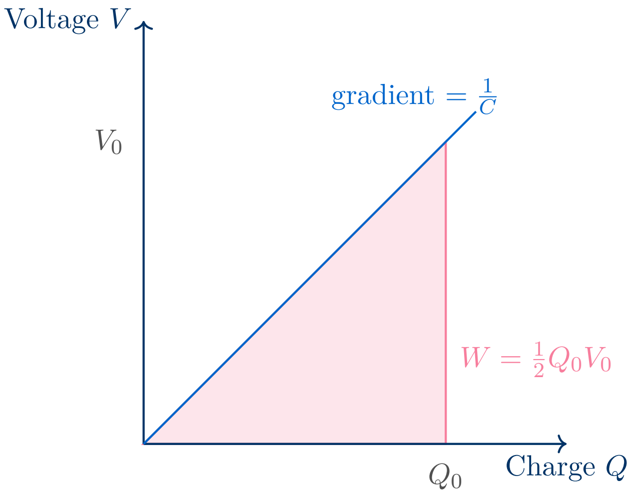
W = energy stored (J)
Q = charge stored (C)
V = potential difference across capacitor (V)
C = capacitance (F)

Energy from the V–Q Graph

The energy stored equals the **area under the V–Q graph** (a straight line through the origin with gradient $1/C$):

$$W = \text{area of triangle} = \tfrac{1}{2} \times Q \times V = \tfrac{1}{2}QV$$

V–Q graph for a capacitor



Discharging a Capacitor

Capacitor Discharge Through a Resistor

When a charged capacitor discharges through a resistor R, the charge, voltage and current all decay **exponentially** with time:

$$Q = Q_0\, e^{-t/RC} \qquad V = V_0\, e^{-t/RC} \qquad I = I_0\, e^{-t/RC}$$

where $I_0 = V_0/R = Q_0/RC$ is the initial current.

Time Constant

$$\tau = RC$$

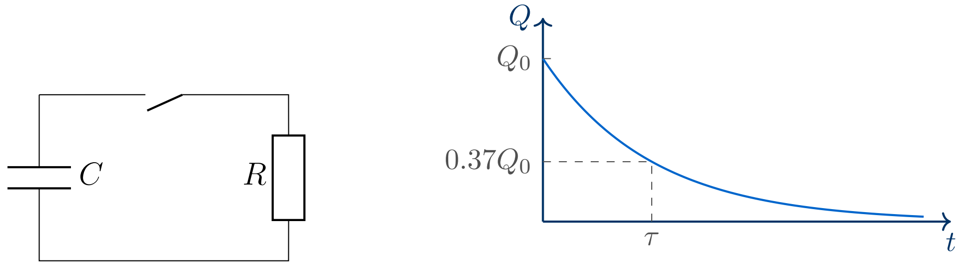
τ = time constant (s)
R = resistance of discharge path (Ω)
C = capacitance (F)

After one time constant ($t = \tau$), the charge/voltage/current has fallen to $e^{-1} \approx$ **37%** of its initial value.

Key Values During Discharge

- $t = \tau$: $Q = 0.37\, Q_0$ (37%)
- $t = 2\tau$: $Q = 0.135\, Q_0$ (13.5%)
- $t = 5\tau$: $Q \approx 0.007\, Q_0$ — capacitor considered fully discharged.
- A larger τ means slower discharge; a smaller τ means faster discharge.

Discharge circuit and Q–t graph



Analysing Discharge Graphs

- A **linear** $\ln Q$ vs t graph confirms exponential decay; gradient $= -1/RC$.
- The time constant τ can be read directly as the time for Q to fall to $0.37\,Q_0$.
- Doubling R or C doubles τ and halves the rate of discharge.

Linearising the discharge: $\ln Q$ against t

Taking logarithms of $Q = Q_0\,e^{-t/RC}$:

$$\ln Q = \ln Q_0 - \frac{1}{RC}\,t$$

This is of the form $y = mx + c$, so a graph of $\ln Q$ against t gives:

- gradient $= -\dfrac{1}{RC} = -\dfrac{1}{\tau}$
- y-intercept $= \ln Q_0$

Formula Summary Sheet

Formula	Quantity	Units
$C = Q/V$	Capacitance (definition)	F
$C_T = C_1 + C_2 + \cdots$	Capacitors in parallel	F
$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$	Capacitors in series	F
$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$	Energy stored	J
$\tau = RC$	Time constant	s
$Q = Q_0\, e^{-t/RC}$	Charge during discharge	C
$V = V_0\, e^{-t/RC}$	Voltage during discharge	V
$I = I_0\, e^{-t/RC}$	Current during discharge	A
$\ln Q = \ln Q_0 - \frac{t}{RC}$	Linearised discharge	—

Constants and values: $e^{-1} \approx 0.368$; after 1τ: 37% remains; after 5τ: $< 1\%$ remains

Units check: $[\tau] = [\Omega][\text{F}] = [\text{V A}^{-1}][\text{C V}^{-1}] = [\text{C A}^{-1}] = \text{s}$

Worked Examples

Example 1 — Capacitors in Series and Parallel

Question: Three capacitors of 2.0 μF, 3.0 μF and 6.0 μF are connected (a) in parallel and (b) in series. Find the total capacitance in each case.

Solution

(a) Parallel:

$$C_T = 2.0 + 3.0 + 6.0 = \mathbf{11.0}\ \mu\text{F}$$

(b) Series:

$$\frac{1}{C_T} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3+2+1}{6.0} = \frac{6}{6.0} = 1.0\ \mu\text{F}^{-1}$$

$$C_T = \mathbf{1.0}\ \mu\text{F}$$

Example 2 — Energy Stored

Question: A 470 μF capacitor is charged to 12 V. Calculate (a) the charge stored and (b) the energy stored.

Solution

(a) $Q = CV = 470 \times 10^{-6} \times 12 = \mathbf{5.64 \times 10^{-3}}$ C
(b) $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 470 \times 10^{-6} \times 12^2 = \mathbf{3.38 \times 10^{-2}}$ J

Example 3 — Capacitor Discharge

Question: A 220 μF capacitor charged to 9.0 V discharges through a 47 kΩ resistor. Calculate (a) the time constant and (b) the voltage after 5.0 s.

Solution

(a) $\tau = RC = 47 \times 10^3 \times 220 \times 10^{-6} = \mathbf{10.3}$ s
(b) $V = V_0\, e^{-t/RC} = 9.0 \times e^{-5.0/10.3} = 9.0 \times e^{-0.485} = 9.0 \times 0.616 = \mathbf{5.5}$ V

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define capacitance and state its SI unit.

[2 marks]

Q2. A capacitor stores 360 μC of charge when connected to a 12 V supply. Calculate its capacitance.

[2 marks]

Q3. Two capacitors of 4.0 μF and 12 μF are connected in series across a 6.0 V supply.

(a) Calculate the total capacitance. *[2 marks]*

(b) Calculate the total charge stored. *[1 mark]*

(c) Calculate the voltage across the 4.0 μF capacitor. *[2 marks]*

Q4. Explain, with reference to the V–Q graph, why the energy stored in a capacitor is $W = \frac{1}{2}QV$ and not $W = QV$.

[2 marks]

Q5. Define the time constant for a capacitor–resistor discharge circuit and state what fraction of the initial charge remains after two time constants.

[3 marks]

Section B — Longer Structured Questions

Q6. A 100 μF capacitor is charged to 20 V and then discharged through a 25 kΩ resistor.

(a) Calculate the initial charge stored on the capacitor.

[1 mark]

(b) Calculate the initial discharge current.

[2 marks]

(c) Calculate the time constant for the discharge.

[1 mark]

(d) Calculate the charge remaining after 4.0 s.

[2 marks]

(e) The student plots a graph of $\ln(Q/\text{C})$ against t/s. State the gradient and y-intercept of this graph.

[2 marks]

Q7. A 50 μF and a 200 μF capacitor are connected in series and charged from a 15 V supply.

(a) Calculate the combined capacitance.

[2 marks]

(b) Calculate the energy stored in the combination.

[2 marks]

(c) The two capacitors are now reconnected in parallel across the same supply. Calculate the new total energy stored and explain why it differs from part (b).

[3 marks]

Mark Scheme and Answers

Q1. Capacitance is the charge stored per unit potential difference [1]; unit: farad (F) or C V^{-1} [1].

Q2. $C = Q/V = 360 \times 10^{-6}/12 = \mathbf{30}$ μF [2].

Q3(a). $\frac{1}{C_T} = \frac{1}{4.0} + \frac{1}{12} = \frac{3+1}{12} = \frac{4}{12}$ [1]; $C_T = \mathbf{3.0}$ μF [1].

Q3(b). $Q = C_T V = 3.0 \times 10^{-6} \times 6.0 = \mathbf{1.8 \times 10^{-5}}$ C [1].

Q3(c). Same charge on each capacitor in series; $V_1 = Q/C_1 = 1.8 \times 10^{-5}/(4.0 \times 10^{-6}) =$ **4.5** V [2].

Q4. The V–Q graph is a straight line through the origin [1]; the energy is the area under this graph (a triangle), which is $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}QV$ — not QV because the voltage builds up gradually from 0 to V as charge is stored [1].

Q5. The time constant is the product RC [1]; it is the time taken for the charge (or voltage or current) to fall to $1/e \approx 37\%$ of its initial value [1]; after 2τ: $e^{-2} \approx \mathbf{13.5}\%$ remains [1].

Q6(a). $Q_0 = CV_0 = 100 \times 10^{-6} \times 20 = \mathbf{2.0 \times 10^{-3}}$ C [1].

Q6(b). $I_0 = V_0/R = 20/(25 \times 10^3)$ [1] $= \mathbf{8.0 \times 10^{-4}}$ A [1].

Q6(c). $\tau = RC = 25 \times 10^3 \times 100 \times 10^{-6} = \mathbf{2.5}$ s [1].

Q6(d). $Q = Q_0\, e^{-t/RC} = 2.0 \times 10^{-3} \times e^{-4.0/2.5}$ [1] $= 2.0 \times 10^{-3} \times e^{-1.6} = 2.0 \times 10^{-3} \times 0.202 = \mathbf{4.0 \times 10^{-4}}$ C [1].

Q6(e). Gradient $= -1/RC = -1/2.5 = \mathbf{-0.40}$ $\mathbf{s^{-1}}$ [1]; y-intercept $= \ln Q_0 = \ln(2.0 \times 10^{-3}) = \mathbf{-6.2}$ [1].

Q7(a). $\dfrac{1}{C_T} = \dfrac{1}{50} + \dfrac{1}{200} = \dfrac{4+1}{200} = \dfrac{5}{200}$ [1]; $C_T = \mathbf{40}$ μF [1].

Q7(b). $W = \frac{1}{2}C_T V^2 = \frac{1}{2} \times 40 \times 10^{-6} \times 15^2$ [1] $= \mathbf{4.5 \times 10^{-3}}$ J [1].

Q7(c). $C_T = 50 + 200 = 250$ μF; $W = \frac{1}{2} \times 250 \times 10^{-6} \times 15^2$ [1] $= \mathbf{2.8 \times 10^{-2}}$ J [1]; more energy is stored in parallel because the total capacitance is greater — more charge is drawn from the supply [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	**Confidence (1–3)**
☐	Define capacitance; use $C = Q/V$	
☐	Calculate combined capacitance for series and parallel combinations	
☐	Derive the series and parallel formulae from $C = Q/V$	
☐	Use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$ for energy stored	
☐	Explain why energy stored is the area under a V–Q graph	
☐	Describe the exponential decay of Q, V and I during discharge	
☐	Use $x = x_0\, e^{-t/RC}$ for discharge of charge, voltage or current	
☐	Define the time constant $\tau = RC$ and state its physical significance	
☐	Determine τ from a discharge graph (directly or via $\ln Q$ vs t)	
☐	Sketch discharge curves for Q, V and I against time	
☐	Linearise the discharge equation and interpret gradient and intercept	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

The exponential decay of a capacitor is one of the most important mathematical forms in physics. Once you can recognise it, linearise it, and extract τ from a graph, you have a powerful tool that reappears throughout the course.