CIE International A Level Physics

Topic 18

Electric Fields

Revision Booklet

This booklet covers:

- Electric Fields and Field Lines
- Uniform Electric Fields
- Coulomb's Law
- Electric Field of a Point Charge
- Electric Potential

Core Concepts and Definitions

Electric Field

An **electric field** is a region of space in which a charged object experiences a force.

- Electric field is defined as the **force per unit positive charge** acting on a small stationary test charge placed at that point.
- It is a **vector** quantity, directed along the force on a positive test charge.
- Units: N $C^{-1} \equiv$ V m^{-1}.

$$E = \frac{F}{q}$$

Radial vs Uniform Fields

- **Radial field** (around a point charge or sphere): field lines point radially inward (negative charge) or outward (positive charge); E decreases with distance.
- **Uniform field** (between parallel plates): field lines are parallel and equally spaced; E is constant throughout.

Radial Field (+) Uniform Field

Uniform Electric Fields

Field Strength Between Parallel Plates

$$E = \frac{\Delta V}{\Delta d}$$

E = electric field strength (V m^{-1})
ΔV = potential difference between the plates (V)
Δd = separation of the plates (m)

Motion of Charged Particles in a Uniform Field

- A charge q in a uniform field E experiences a constant force $F = qE$.
- The motion is analogous to projectile motion in a gravitational field:
  - Along the field: **uniform acceleration** $a = qE/m$.

  - Perpendicular to the field: **constant velocity** (if no other forces).
- The path of the charge is **parabolic**.

Common Mistake

$E = \Delta V/\Delta d$ applies **only** to uniform fields (parallel plates). Do not apply it to the field around a point charge — use $E = Q/(4\pi\varepsilon_0 r^2)$ instead.

Coulomb's Law

Coulomb's Law

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

F = electrostatic force between the charges (N)
Q_1, Q_2 = the two point charges (C)
r = separation between the charges (m)
ε_0 = permittivity of free space = 8.85×10^{-12} F m^{-1}

Key Points

- Like charges **repel**; unlike charges **attract**.
- The force obeys an **inverse-square law**: double the distance, quarter the force.
- Applies strictly to **point charges**, and to uniform spheres (treat as point charge at centre).
- Compare with gravity: same inverse-square form but gravity is **always attractive**.

Comparison: Coulomb's Law vs Newton's Law of Gravitation

Coulomb's Law	Newton's Law
$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	$F = \frac{G m_1 m_2}{r^2}$
Can be attractive or repulsive	Always attractive
Acts between charges	Acts between masses
Much stronger force	Much weaker force

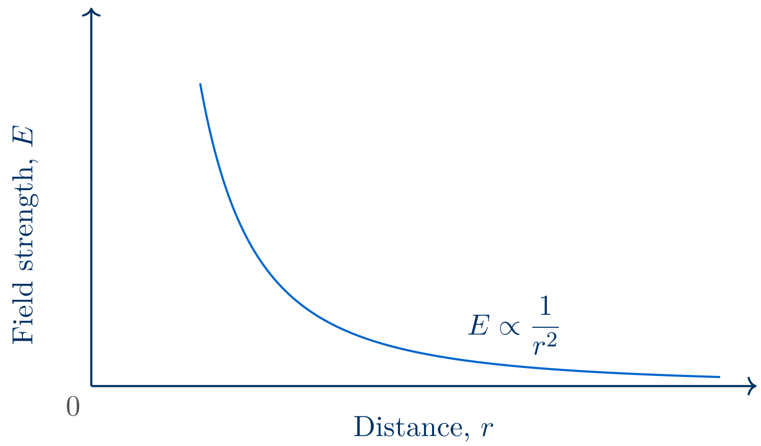
Electric Field of a Point Charge

Electric Field Strength due to a Point Charge

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

E = electric field strength at distance r (N C^{-1})
Q = point charge creating the field (C)
r = distance from the centre of Q (m)
ε_0 = 8.85×10^{-12} F m^{-1}

Variation of E with distance r



Comparing E and g fields

- Both follow an inverse-square law with distance.
- $E = Q/(4\pi\varepsilon_0 r^2)$ parallels $g = GM/r^2$.
- $F = qE$ parallels $F = mg$.
- Unlike gravitational fields, electric fields can point inward or outward depending on the sign of Q.

Electric Potential

Definition of Electric Potential V

The **electric potential** at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point.

$$V = \frac{W}{q} \qquad \text{units: J C}^{-1} \equiv \text{V}$$

- $V = 0$ at infinity (by convention).
- Around a positive charge: $V > 0$ (work must be done *against* repulsion).

- Around a negative charge: $V < 0$ (work is done *by* the field as the test charge moves in).

Electric Potential due to a Point Charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

V = electric potential at distance r (V)
Q = point charge (C)
r = distance from the charge (m)

Relationship Between E and V

Field Strength from Potential Gradient

$$E = -\frac{\Delta V}{\Delta r}$$

The electric field strength equals the **negative gradient** of the potential–distance graph. The area under an E–r graph gives the change in potential ΔV.

Electric Potential Energy

Potential Energy of Two Point Charges

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}$$

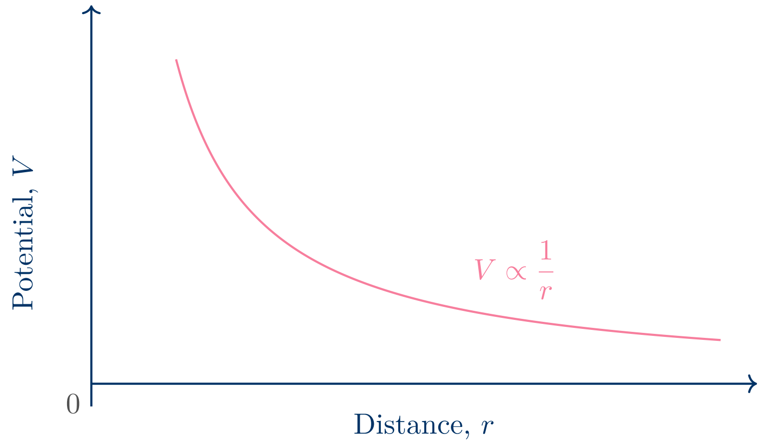
E_P = electric potential energy (J)
Q, q = the two point charges (C)
r = separation (m)

Note: $E_P = qV$, where V is the potential due to charge Q at the location of q.

Signs of Potential Energy

- Like charges ($Qq > 0$): $E_P > 0$ — energy must be supplied to bring them together.
- Unlike charges ($Qq < 0$): $E_P < 0$ — energy is released as they come together.
- $E_P = 0$ at infinite separation.

Graphs of V and E against r for a positive point charge



Common Mistake

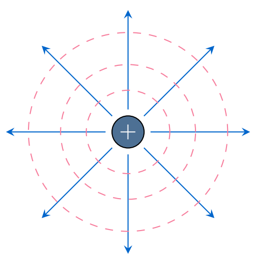
Note the difference in distance dependence: $E \propto 1/r^2$ but $V \propto 1/r$. Students often mix these up. Remember: potential falls off more *slowly* than field strength with distance.

Equipotential Surfaces

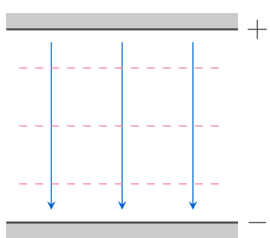
Equipotentials

An **equipotential surface** is a surface on which the electric potential is the same at every point.

- No work is done moving a charge *along* an equipotential.
- Equipotentials are always **perpendicular** to field lines.
- Around a point charge: equipotentials are concentric spheres.
- Between parallel plates: equipotentials are parallel planes equally spaced (for uniform field).



Point charge



Parallel plates

Formula Summary Sheet

Formula	Quantity	Units
$E = \dfrac{F}{q}$	Electric field strength (definition)	N C^{-1}
$F = qE$	Force on a charge in a field	N
$E = \dfrac{\Delta V}{\Delta d}$	Field between parallel plates (uniform)	V m^{-1}
$F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	Coulomb's Law	N
$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$	Field due to a point charge	N C^{-1}
$V = \dfrac{Q}{4\pi\varepsilon_0 r}$	Potential due to a point charge	V
$E = -\dfrac{\Delta V}{\Delta r}$	Field from potential gradient	V m^{-1}
$E_P = \dfrac{Qq}{4\pi\varepsilon_0 r}$	Electric potential energy	J
$E_P = qV$	Potential energy of charge in field	J

Constants: $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}, $\dfrac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}$ N m^{2}C^{-2}, $e = 1.60\times10^{-19}$ C

Note: $E \propto 1/r^2$ (inverse-square) but $V \propto 1/r$ (inverse).

Worked Examples

Example 1 — Force Between Two Charges

Question: Calculate the electrostatic force between two charges of $+3.0\ \mu$C and $-5.0\ \mu$C separated by 0.12 m.

Solution

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{3.0\times10^{-6}\times5.0\times10^{-6}}{4\pi\times8.85\times10^{-12}\times(0.12)^2}$$

$$F = \frac{1.5\times10^{-11}}{1.61\times10^{-12}} = \mathbf{9.3}\ \text{N}$$

The force is **attractive** (unlike charges).

Example 2 — Field Strength and Potential at a Distance

Question: A point charge $Q = +4.0\ \mu$C. Calculate (a) the electric field strength and (b) the electric potential at a distance of 0.30 m.

Solution

(a) $E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.30)^2}$

$E = \dfrac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} \times (0.09)^{-1}$

$$E = \frac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} = \mathbf{4.0 \times 10^5} \text{ N C}^{-1}$$

(b) $V = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.30} = \mathbf{1.2 \times 10^5} \text{ V}$

Example 3 — Uniform Field Between Plates

Question: Two parallel plates are separated by 4.0 mm with a potential difference of 240 V. Calculate (a) the field strength and (b) the force on an electron between the plates.

Solution

(a) $E = \dfrac{\Delta V}{\Delta d} = \dfrac{240}{4.0 \times 10^{-3}} = \mathbf{6.0 \times 10^4} \text{ V m}^{-1}$

(b) $F = qE = 1.6 \times 10^{-19} \times 6.0 \times 10^4 = \mathbf{9.6 \times 10^{-15}} \text{ N}$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electric field strength and state its SI units.

[2 marks]

Q2. State Coulomb's Law and identify one similarity and one difference compared with Newton's Law of Gravitation.

[3 marks]

Q3. Define electric potential at a point. Explain why the potential around an isolated positive charge is positive, but around an isolated negative charge is negative.

[3 marks]

Q4. Two parallel plates are 6.0 mm apart. The electric field strength between them is 5.0×10^4 V m^{-1}. Calculate the potential difference between the plates.

[2 marks]

Q5. A point charge produces an electric field of 3.6×10^5 N C^{-1} at a distance of 0.10 m. Calculate the magnitude of the charge.

[3 marks]

Section B — Longer Structured Questions

Q6. Two point charges $Q_1 = +6.0\ \mu$C and $Q_2 = +6.0\ \mu$C are placed 0.20 m apart.

(a) Calculate the force between the two charges.

[2 marks]

(b) Calculate the electric field strength at the midpoint between the two charges. Explain your reasoning.

[3 marks]

(c) Calculate the electric potential at the midpoint between the two charges.

[3 marks]

Q7. An electron (mass 9.11×10^{-31} kg, charge -1.6×10^{-19} C) enters horizontally between two parallel plates. The plates are 30 mm apart and have a potential difference of 150 V. The electron enters midway between the plates with horizontal speed 4.0×10^{7} m s^{-1}.

(a) Calculate the electric field strength between the plates.

[1 mark]

(b) Calculate the vertical acceleration of the electron.

[2 marks]

(c) The plates are 60 mm long. Calculate the vertical deflection of the electron as it exits the plates.

[3 marks]

(d) State and explain whether the electron hits one of the plates before exiting.

[2 marks]

Mark Scheme and Answers

Q1. Electric field strength is the force per unit positive charge acting on a small stationary test charge placed at that point [1]; units: N C^{-1} or V m^{-1} [1].

Q2. The force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation [1]. Similarity: both follow an inverse-square law [1]. Difference: gravity is always attractive; electrostatic force can be attractive or repulsive [1].

Q3. Electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point [1]. Around a positive charge: the test charge is repelled, so work must be done *on* it to bring it in from infinity — the potential is positive [1]. Around a negative charge: the test charge is attracted, so work is done *by* the field — the potential is negative [1].

Q4. $\Delta V = E \times \Delta d = 5.0 \times 10^4 \times 6.0 \times 10^{-3} = \mathbf{300}$ V [2].

Q5. $E = Q/(4\pi\varepsilon_0 r^2)$ [1]; $Q = E \times 4\pi\varepsilon_0 r^2 = 3.6 \times 10^5 \times 4\pi \times 8.85 \times 10^{-12} \times (0.10)^2$ [1] $= \mathbf{4.0 \times 10^{-7}}$ C [1].

Q6(a). $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \dfrac{(6.0 \times 10^{-6})^2}{4\pi \times 8.85 \times 10^{-12} \times (0.20)^2}$ [1] $= \mathbf{8.1}$ N [1].

Q6(b). By symmetry, the two equal charges produce equal and opposite field contributions at the midpoint [1]; the fields cancel, so $E = \mathbf{0}$ at the midpoint [2].

Q6(c). $V_1 = V_2 = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{6.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.10}$ [1] $= 5.4 \times 10^5$ V; total $V = V_1 + V_2 = \mathbf{1.08 \times 10^6}$ V [2].

Q7(a). $E = \Delta V/\Delta d = 150/(30 \times 10^{-3}) = \mathbf{5.0 \times 10^3}$ V $\mathbf{m^{-1}}$ [1].

Q7(b). $F = qE = 1.6 \times 10^{-19} \times 5.0 \times 10^3 = 8.0 \times 10^{-16}$ N [1]; $a = F/m = 8.0 \times 10^{-16}/(9.11 \times 10^{-31}) = \mathbf{8.8 \times 10^{14}}$ m $s^{\mathbf{-2}}$ [1].

Q7(c). Time in field: $t = L/v = 0.060/(4.0 \times 10^7) = 1.5 \times 10^{-9}$ s [1]; vertical deflection: $y = \frac{1}{2}at^2 = \frac{1}{2} \times 8.8 \times 10^{14} \times (1.5 \times 10^{-9})^2$ [1] $= \mathbf{9.9 \times 10^{-4}}$ m $\approx \mathbf{1.0}$ mm [1].

Q7(d). The electron enters midway so has 15 mm to the nearest plate [1]; deflection of ≈ 1 mm $\ll 15$ mm, so the electron **does not** hit a plate [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define electric field strength as force per unit positive charge; use $E = F/q$	
☐	Represent electric fields using field lines for point charges and parallel plates	
☐	Use $E = \Delta V/\Delta d$ for a uniform field between parallel plates	
☐	Describe the parabolic motion of a charge in a uniform electric field	
☐	State and apply Coulomb's Law $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$	
☐	Use $E = Q/(4\pi\varepsilon_0 r^2)$ for the field due to a point charge	
☐	Define electric potential; explain why sign depends on sign of source charge	
☐	Use $V = Q/(4\pi\varepsilon_0 r)$ for potential due to a point charge	
☐	Apply $E = -\Delta V/\Delta r$ to relate field strength and potential gradient	
☐	Use $E_P = Qq/(4\pi\varepsilon_0 r)$ for potential energy of two charges	
☐	Sketch and interpret equipotential diagrams for point charges and parallel plates	
☐	Compare electric and gravitational fields (similarities and differences)	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

The key to electric fields is seeing the deep parallel with gravitational fields: same inverse-square laws, same potential formalism — but with charge replacing mass, and the crucial difference that charge can be positive or negative.