CIE International A Level Physics

Topic 17

Oscillations

Revision Booklet

This booklet covers:

- Simple Harmonic Motion
- Displacement, Velocity & Acceleration
- Energy in SHM
- Damped Oscillations
- Forced Oscillations & Resonance

Core Concepts and Definitions

Oscillatory Motion — Key Terms

- **Displacement** x: the distance from the equilibrium position, with direction (m).
- **Amplitude** x_0: the maximum displacement from equilibrium (m).
- **Period** T: the time for one complete oscillation (s).
- **Frequency** f: the number of complete oscillations per second (Hz); $f = 1/T$.
- **Angular frequency** ω: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}).
- **Phase difference** ϕ: the fraction of a cycle by which one oscillation leads or lags another, expressed in radians.

Simple Harmonic Motion (SHM)

An oscillation is **simple harmonic** if the acceleration of the object is:

- **proportional** to its displacement from a fixed equilibrium point, and
- always directed **towards** that equilibrium point (opposite to displacement).

$$a \propto -x$$

Equations of Simple Harmonic Motion

Defining Equation of SHM

$$a = -\omega^2 x$$

a = acceleration (m s^{-2})
ω = angular frequency (rad s^{-1})
x = displacement from equilibrium (m)

The negative sign confirms acceleration is always directed **opposite** to displacement.

Displacement; Velocity and Acceleration

$$x = x_0 \sin \omega t \qquad \text{(starting from equilibrium)}$$

$$v = v_0 \cos \omega t \qquad \text{where } v_0 = \omega x_0$$

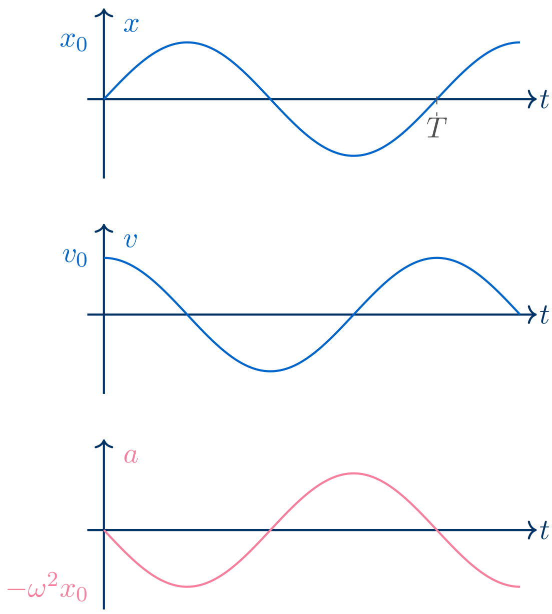
$$v = \pm\,\omega\sqrt{x_0^2 - x^2}$$

x_0 = amplitude (m)
v_0 = maximum speed = ωx_0 (m s^{-1})
t = time (s)

Maxima and Minima

- Acceleration is **maximum** at maximum displacement ($x = \pm x_0$): $|a_{\max}| = \omega^2 x_0$
- Speed is **maximum** at the equilibrium position ($x = 0$): $v_{\max} = \omega x_0$
- Speed is **zero** at the turning points ($x = \pm x_0$).

Graphs of x, v and a against time



Phase Relationships

- v leads x by $\frac{\pi}{2}$ rad (quarter period ahead).
- a leads v by $\frac{\pi}{2}$ rad, so a is **antiphase** (π rad ahead) with x.

Common Mistake

The equation $x = x_0 \sin \omega t$ assumes the object starts at the **equilibrium position** at $t = 0$. If it starts at maximum displacement, use $x = x_0 \cos \omega t$ instead. Always check the initial conditions.

Energy in Simple Harmonic Motion

Total Energy of a SHM System

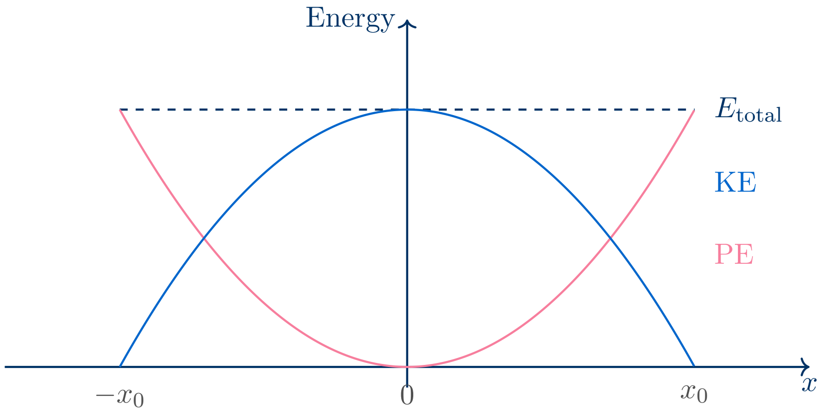
$$E = \frac{1}{2}m\omega^2 x_0^2$$

E = total mechanical energy (J)
m = mass of the oscillating object (kg)
ω = angular frequency (rad s^{-1})
x_0 = amplitude (m)

Energy Interchange During SHM

- At the **equilibrium position** ($x = 0$): KE is maximum, PE is zero.
- At the **turning points** ($x = \pm x_0$): KE is zero, PE is maximum.
- The **total energy remains constant** throughout (assuming no damping).
- $\text{KE} = \frac{1}{2}m\omega^2(x_0^2 - x^2)$ $\text{PE} = \frac{1}{2}m\omega^2 x^2$

Energy–displacement graph



Damped and Forced Oscillations

Damping

Damping

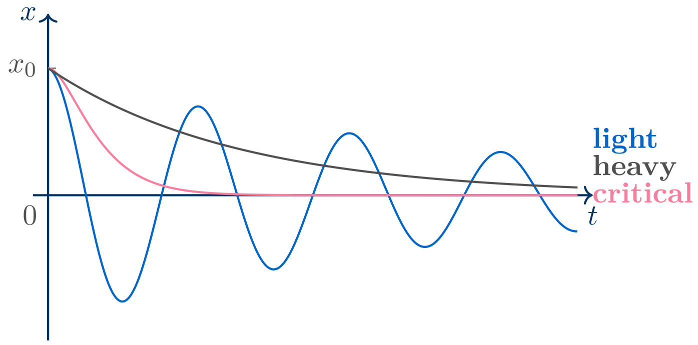
Damping occurs when a resistive force (e.g., air resistance, viscosity) acts on an oscillating system. Energy is removed from the system, causing the amplitude to **decrease over time**. The frequency is approximately unchanged for light damping.

Types of Damping

- **Light damping**: amplitude decreases gradually over many oscillations; the system oscillates for a long time before coming to rest.

- **Critical damping**: the system returns to equilibrium in the **shortest possible time** without oscillating. Used in car suspension and door closers.
- **Heavy (overdamping)**: the system returns to equilibrium **slowly** without oscillating; slower return than critical damping.

Displacement–time graphs for the three types of damping



Forced Oscillations and Resonance

Forced Oscillations and Natural Frequency

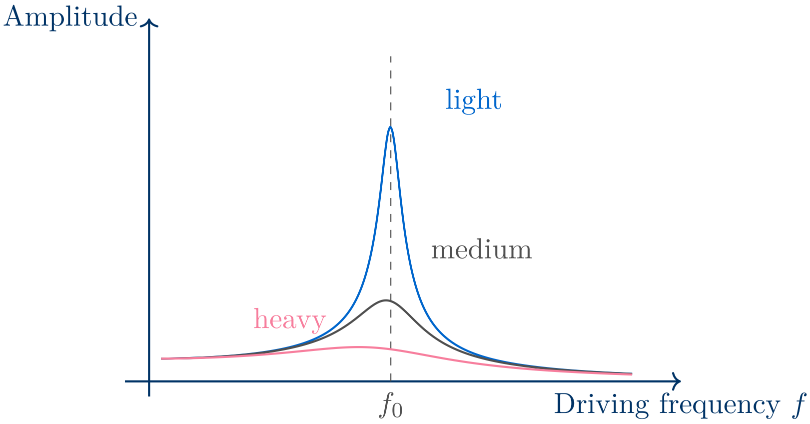
- Every oscillating system has a **natural frequency** f_0 at which it oscillates freely when displaced and released.
- A **forced oscillation** occurs when a periodic **driving force** is applied to the system at a **driving frequency** f.
- The system oscillates at the **driving frequency**, not necessarily at its natural frequency.

Resonance

Resonance occurs when the driving frequency equals the natural frequency of the system ($f = f_0$). At resonance:

- The amplitude of oscillation is a **maximum**.
- Energy transfer from the driver to the system is most efficient.
- The degree of damping determines how sharp the resonance peak is and the maximum amplitude reached.

Amplitude–frequency graph (resonance curves)



Effect of Damping on Resonance

- More damping ⇒ lower and broader resonance peak.
- More damping ⇒ peak shifts **below** f_0 (towards lower frequencies).
- Less damping ⇒ sharper, taller peak with maximum amplitude closer to f_0.
- With no damping the peak would be infinite at exactly f_0.

Resonance in Real Life

Resonance can be **useful** (e.g. MRI scanners, musical instruments, microwave ovens) or **destructive** (e.g. bridges vibrating in wind, buildings in earthquakes). Engineers use damping to control unwanted resonance.

Formula Summary Sheet

Formula	Quantity	Units
$a = -\omega^2 x$	SHM defining equation	m s^{-2}
$\omega = 2\pi f = \dfrac{2\pi}{T}$	Angular frequency	rad s^{-1}
$x = x_0 \sin \omega t$	Displacement (from equilibrium at $t = 0$)	m
$v = v_0 \cos \omega t$	Velocity	m s^{-1}
$v = \pm\omega\sqrt{x_0^2 - x^2}$	Speed at displacement x	m s^{-1}
$v_0 = \omega x_0$	Maximum speed	m s^{-1}
$a_{\max} = \omega^2 x_0$	Maximum acceleration	m s^{-2}
$E = \frac{1}{2}m\omega^2 x_0^2$	Total energy of SHM system	J
$\text{KE} = \frac{1}{2}m\omega^2(x_0^2 - x^2)$	Kinetic energy at displacement x	J
$\text{PE} = \frac{1}{2}m\omega^2 x^2$	Potential energy at displacement x	J

Key relationships: $T = 1/f$, $\omega = 2\pi f$, $v_{\max} = \omega x_0$, $a_{\max} = \omega^2 x_0$
Phase: v leads x by $\pi/2$; a is antiphase with x (leads by π)

Worked Examples

Example 1 — Finding Angular Frequency and Max Speed

Question: A mass oscillates with SHM, amplitude 4.0 cm and period 0.80 s. Calculate (a) the angular frequency and (b) the maximum speed.

Solution

(a) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = \mathbf{7.85}$ rad s^{-1}

(b) $v_{\max} = \omega x_0 = 7.85 \times 0.040 = \mathbf{0.31}$ m s^{-1}

Example 2 — Speed at a Given Displacement

Question: The same mass ($\omega = 7.85$ rad s^{-1}, $x_0 = 4.0$ cm) has displacement $x = 2.5$ cm. Find its speed.

Solution

$$v = \omega\sqrt{x_0^2 - x^2} = 7.85 \times \sqrt{(0.040)^2 - (0.025)^2}$$

$$v = 7.85 \times \sqrt{1.60 \times 10^{-3} - 6.25 \times 10^{-4}} = 7.85 \times \sqrt{9.75 \times 10^{-4}}$$

$$v = 7.85 \times 0.0312 = \mathbf{0.245}\ \text{m s}^{\mathbf{-1}}$$

Example 3 — Total Energy of SHM System

Question: A 0.20 kg mass oscillates with $\omega = 7.85$ rad s^{-1} and amplitude 4.0 cm. Calculate the total energy.

Solution

$$E = \tfrac{1}{2}m\omega^2 x_0^2 = \tfrac{1}{2} \times 0.20 \times (7.85)^2 \times (0.040)^2$$

$$E = 0.10 \times 61.6 \times 1.60 \times 10^{-3} = \mathbf{9.9 \times 10^{-3}}\ \text{J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the two conditions that define simple harmonic motion.

[2 marks]

Q2. Define the terms *amplitude*, *period* and *angular frequency* for an oscillating system.

[3 marks]

Q3. An object undergoes SHM with frequency 2.5 Hz and amplitude 6.0 cm.

(a) Calculate the angular frequency. *[1 mark]*

(b) Calculate the maximum acceleration. *[2 marks]*

(c) Calculate the speed when $x = 4.0$ cm. *[2 marks]*

Q4. Sketch displacement–time graphs for *light*, *critical* and *heavy* damping on the same axes, starting from the same initial displacement. Label each curve.

[3 marks]

Q5. Explain what is meant by *resonance* and state the condition under which it occurs.

[2 marks]

Section B — Longer Structured Questions

Q6. A 0.15 kg mass is attached to a spring and oscillates vertically with SHM. Its displacement is given by $x = 0.050\sin(12t)$, where x is in metres and t in seconds.

(a) Write down the amplitude and angular frequency of the motion.

[2 marks]

(b) Show that the maximum speed is 0.60 m s^{-1}.

[2 marks]

(c) Calculate the total energy of the oscillation.

[2 marks]

(d) Sketch graphs on the same axes showing how the kinetic energy and potential energy vary with displacement x. Label your axes clearly.

[3 marks]

Q7. A child on a swing is pushed periodically by a parent.

(a) Explain why the amplitude of the swing increases when the pushing frequency equals the natural frequency of the swing.

[2 marks]

(b) The parent now pushes at a frequency higher than the natural frequency. Describe and explain what happens to the amplitude.

[2 marks]

(c) Air resistance acts on the swing. Describe how this affects the resonance curve compared to an undamped system.

[2 marks]

Mark Scheme and Answers

Q1. Acceleration is proportional to displacement from a fixed (equilibrium) point [1]; acceleration is always directed towards that point / opposite to displacement [1].

Q2. Amplitude: maximum displacement from equilibrium [1]. Period: time for one complete oscillation [1]. Angular frequency: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}) [1].

Q3(a). $\omega = 2\pi f = 2\pi \times 2.5 = \mathbf{15.7}$ **rad s^{-1}** [1].

Q3(b). $a_{\text{max}} = \omega^2 x_0 = (15.7)^2 \times 0.060$ [1] $= \mathbf{14.8}$ **m s^{-2}** [1].

Q3(c). $v = \omega\sqrt{x_0^2 - x^2} = 15.7 \times \sqrt{(0.060)^2 - (0.040)^2}$ [1] $= 15.7 \times 0.0447 = \mathbf{0.70}$ **m s^{-1}** [1].

Q4. All three start from same x_0 [1]; light damping: decaying sinusoid, multiple oscillations; critical: returns to zero smoothly in shortest time without oscillating; heavy: slower return to zero, no oscillation — all three correctly labelled [2].

Q5. Resonance: when the driving frequency equals the natural frequency of the system [1]; the amplitude of oscillation reaches a maximum [1].

Q6(a). Amplitude $x_0 = 0.050$ m [1]; angular frequency $\omega = 12$ rad s^{-1} [1].

Q6(b). $v_{\text{max}} = \omega x_0 = 12 \times 0.050 = 0.60$ m s^{-1} [2] (must show substitution).

Q6(c). $E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.15 \times 144 \times 2.5 \times 10^{-3}$ [1] $= \mathbf{2.7 \times 10^{-2}}$ J [1].

Q6(d). PE: upward parabola, zero at $x = 0$, maximum E at $x = \pm x_0$ [1]; KE: inverted parabola, maximum E at $x = 0$, zero at $x = \pm x_0$ [1]; axes correctly labelled with E, $-x_0$, 0, x_0 [1].

Q7(a). Energy is transferred most efficiently from driver to system when $f_{\text{drive}} = f_0$ [1]; each push is in phase with the motion so energy is added each cycle, increasing amplitude [1].

Q7(b). Amplitude decreases [1]; driving frequency is not matched to natural frequency so energy transfer is less efficient / pushes are out of phase with motion [1].

Q7(c). The resonance peak is lower (smaller maximum amplitude) [1]; the peak is broader and shifts slightly to a frequency below f_0 [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define displacement, amplitude, period, frequency and angular frequency	
☐	State and apply the two conditions for SHM	
☐	Use $a = -\omega^2 x$ to identify and analyse SHM	
☐	Use $x = x_0 \sin \omega t$ and $v = v_0 \cos \omega t$	
☐	Use $v = \pm\omega\sqrt{x_0^2 - x^2}$ to find speed at any displacement	
☐	Sketch and interpret graphs of x, v and a against t	
☐	Describe the phase relationships between x, v and a	
☐	Describe the interchange between KE and PE during SHM	
☐	Use $E = \frac{1}{2}m\omega^2 x_0^2$ for total energy	
☐	Sketch and interpret KE and PE against displacement graphs	
☐	Explain and distinguish light, critical and heavy damping	
☐	Sketch displacement–time graphs for the three types of damping	
☐	Explain resonance and the condition for it to occur	
☐	Describe the effect of damping on the resonance curve	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: in SHM, acceleration and displacement are always linked by $a = -\omega^2 x$. If you can start from this equation and derive everything else, you truly understand the topic.