CIE International A Level Physics

Topic 13

Gravitational Fields

Revision Booklet

This booklet covers:

- Newton's Law of Gravitation
- Gravitational Field Strength
- Gravitational Potential
- Orbital Motion & Kepler's Third Law
- Escape Velocity

Core Concepts and Definitions

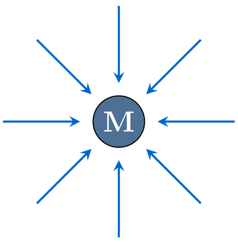
Gravitational Field

A **gravitational field** is a region of space in which a mass experiences a force due to the gravitational attraction of another mass.

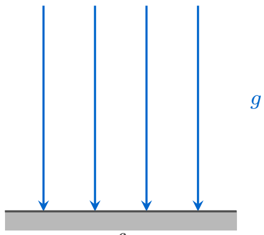
- Gravitational fields are always **attractive** — there is no gravitational repulsion.
- Any mass creates a gravitational field in the space around it.
- A **test mass** placed in the field will experience a force towards the source mass.

Radial vs Uniform Fields

- **Radial field** (e.g., around a planet): field lines point towards the centre; g decreases with distance.
- **Uniform field** (e.g., near Earth's surface over small distances): field lines are parallel and equally spaced; g is approximately constant at 9.81 N kg^{-1}.



Radial Field



Uniform Field

Newton's Law of Gravitation

Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

F = gravitational force between the two masses (N)
G = gravitational constant = 6.67×10^{-11} N m^{2}kg^{-2}
m_1, m_2 = the two point masses (or uniform spheres) (kg)
r = separation between centres of mass (m)

Key Points

- The force is always **attractive**.
- It applies strictly to **point masses**, but also works for uniform spheres if r is measured from the centre.
- r is the **centre-to-centre** distance — not surface to surface.

- The force obeys an **inverse-square law**: double the distance, quarter the force.

Gravitational Field Strength

Definition of Gravitational Field Strength g

The **gravitational field strength** at a point is the gravitational force exerted per unit mass on a small test mass placed at that point.

$$g = \frac{F}{m} \qquad \text{units: N kg}^{-1} \equiv \text{m s}^{-2}$$

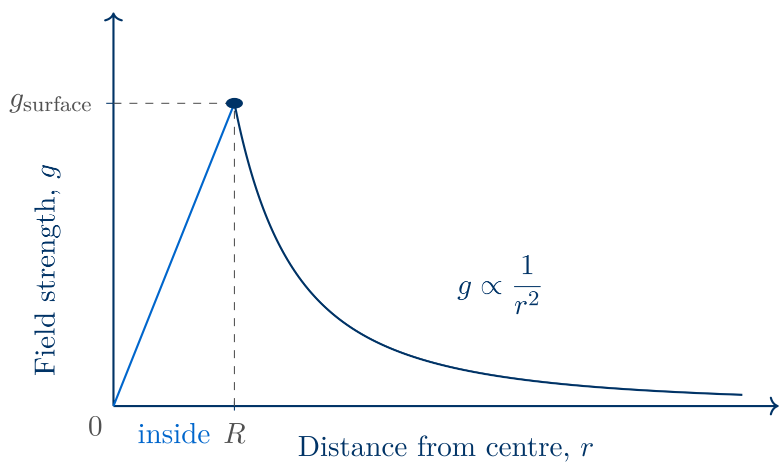
g is a **vector** quantity, directed towards the source mass.

Field Strength Formulae

$$g = \frac{GM}{r^2}$$

g = gravitational field strength at distance r (N kg^{-1})
G = gravitational constant (N m^2 kg^{-2})
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Variation of g with distance r



Common Mistake

Do not confuse g (field strength, N kg^{-1}) with G (the universal gravitational constant, N m^2 kg^{-2}). They are completely different quantities!

Gravitational Potential

Definition of Gravitational Potential ϕ

The **gravitational potential** at a point is the work done per unit mass to move a small test mass from infinity to that point.

$$\phi = \frac{W}{m} \qquad \text{units: J kg}^{-1}$$

Gravitational potential is always **negative** (zero at infinity; work is done *by* gravity as mass moves inwards, so the system loses potential energy).

Gravitational Potential Formulae

$$\phi = -\frac{GM}{r} \qquad E_p = m\phi = -\frac{GMm}{r}$$

ϕ = gravitational potential at distance r (J kg^{-1})
E_p = gravitational potential energy (J)
G = 6.67×10^{-11} N m^2 kg^{-2}
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Relationship Between g and ϕ

Field Strength from Potential Gradient

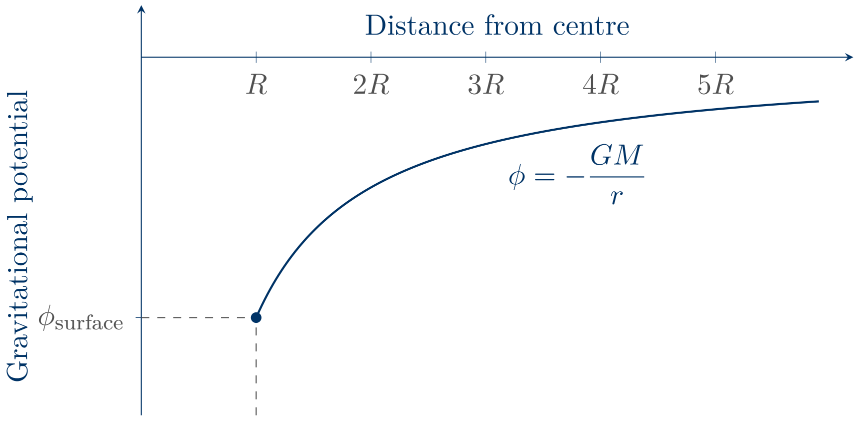
$$g = -\frac{\Delta\phi}{\Delta r}$$

Interpreting the Gradient

g is the **negative gradient** of the potential–distance graph. On a ϕ–r graph:

- A steeper gradient $\Rightarrow$ stronger field (larger g)
- The gradient becomes shallower as r increases $\Rightarrow$ g decreases
- The area under a g–r graph gives the change in potential $\Delta\phi$

Graph of ϕ against r



$\phi \to 0$ as $r \to \infty$

Equipotential Surfaces

An **equipotential surface** is a surface on which the gravitational potential is the same everywhere.

- No work is done moving a mass along an equipotential surface.
- Equipotentials are always **perpendicular** to field lines.
- For a point mass (or uniform sphere), equipotentials are concentric spheres.

Circular Orbits and Satellites

For an object of mass m in a circular orbit of radius r around a mass M, the gravitational force provides the centripetal force:

Orbital Mechanics

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\therefore \quad v = \sqrt{\frac{GM}{r}} \qquad \text{(orbital speed)}$$

$$T^2 = \frac{4\pi^2}{GM} r^3 \qquad \text{(Kepler's Third Law)}$$

Kepler's Third Law

$$T^2 \propto r^3$$

The square of the orbital period is proportional to the cube of the orbital radius. The constant of proportionality is $\frac{4\pi^2}{GM}$.

Energy in Circular Orbits

Energy of a Satellite in Circular Orbit

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r} \qquad \text{(kinetic energy — always positive)}$$
$$E_p = -\frac{GMm}{r} \qquad \text{(potential energy — always negative)}$$
$$E_{total} = E_k + E_p = -\frac{GMm}{2r} \qquad \text{(total energy — always negative)}$$

As r increases: v decreases, E_k decreases, E_p increases, E_{total} increases (becomes less negative).

Geostationary Orbits

Geostationary Satellite

A **geostationary** satellite has:

- An orbital period of exactly **24 hours**
- An orbit in the **equatorial plane**
- Movement in the **same direction** as Earth's rotation (west to east)
- An orbital radius of approximately 4.2×10^7 m ($\approx 36\,000$ km altitude)

It appears **stationary** relative to a point on Earth's surface.

Uses: satellite TV, telecommunications, weather monitoring.

Escape Velocity

Definition of Escape Velocity

The **escape velocity** is the minimum speed at which an object must be projected from the surface of a body so that it can escape to infinity against the gravitational field, without any further energy input.

Escape Velocity

Setting total energy = 0 at infinity:

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Note: $v_{esc} = \sqrt{2}\,v_{orbital}$ at the same radius.

Formula Summary Sheet

Formula	Quantity	Units
$F = \dfrac{Gm_1m_2}{r^2}$	Gravitational force	N
$g = \dfrac{F}{m}$	Field strength (definition)	N kg^{-1}
$g = \dfrac{GM}{r^2}$	Field strength (point mass)	N kg^{-1}
$\phi = -\dfrac{GM}{r}$	Gravitational potential	J kg^{-1}
$E_p = -\dfrac{GMm}{r}$	Gravitational PE	J
$g = -\dfrac{\Delta\phi}{\Delta r}$	Field from potential gradient	N kg^{-1}
$v = \sqrt{\dfrac{GM}{r}}$	Orbital speed	m s^{-1}
$T^2 = \dfrac{4\pi^2r^3}{GM}$	Kepler's Third Law	s^2, m^3
$v_{esc} = \sqrt{\dfrac{2GM}{R}}$	Escape velocity	m s^{-1}

Constants: $G = 6.67 \times 10^{-11}$ N m^{2}kg^{-2}, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Calculation Questions

1. **Identify** what you are asked to find.
2. **List** the quantities given; convert units if needed (e.g. days → seconds, km → m).
3. **Select** the appropriate formula.
4. **Substitute** values carefully, showing all working.
5. **Check** units and significant figures in your answer.

Common Errors — Avoid These!

- Using **diameter** instead of radius in $g = GM/r^2$.
- Forgetting the **negative sign** in $\phi = -GM/r$.
- Not converting **hours/days to seconds** before applying Kepler's Law.
- Confusing g (field strength, vector) with ϕ (potential, scalar).
- Saying $\phi = 0$ at the surface — it is zero only **at infinity**.

- Thinking higher orbit ⇒ faster orbital speed — it is **slower**.

Worked Examples

Example 1 — Field Strength at Altitude

Question: Calculate the gravitational field strength at a height of 400 km above Earth's surface. ($M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m)

Solution

Solution:
$r = R_E + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6$ m

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.77 \times 10^6)^2}$$

$$g = \frac{3.98 \times 10^{14}}{4.58 \times 10^{13}} = \mathbf{8.69}\ \text{N kg}^{-1}$$

Note: this is the field strength at the ISS orbit — astronauts are *not* weightless because gravity is zero!

Example 2 — Orbital Period Using Kepler's Third Law

Question: A satellite orbits Earth at a radius of 4.2×10^7 m. Calculate its orbital period.

Solution

Solution:

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 \times (4.2 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}$$

$$T^2 = \frac{4\pi^2 \times 7.41 \times 10^{22}}{3.98 \times 10^{14}} = 7.36 \times 10^9\ \text{s}^2$$

$$T = \sqrt{7.36 \times 10^9} = 8.58 \times 10^4\ \text{s} \approx \mathbf{23.8}\ \text{hours}$$

This is a geostationary orbit ($T \approx 24$ h).

Example 3 — Change in Gravitational Potential Energy

Question: A spacecraft of mass 2500 kg is moved from the Earth's surface to an altitude of 800 km. Calculate the increase in gravitational potential energy.

Solution

Solution:
$r_1 = 6.37 \times 10^6$ m, $\quad r_2 = 6.37 \times 10^6 + 8.00 \times 10^5 = 7.17 \times 10^6$ m

$$\Delta E_p = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta E_p = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 2500 \times \left(\frac{1}{6.37 \times 10^6} - \frac{1}{7.17 \times 10^6}\right)$$

$$\Delta E_p = 9.95 \times 10^{17} \times 1.75 \times 10^{-8} = \mathbf{1.74 \times 10^{10}} \text{ J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Newton's Law of Gravitation.

[2 marks]

Q2. Define gravitational field strength and state its units.

[2 marks]

Q3. Explain why gravitational potential is always a negative quantity.

[2 marks]

Q4. The gravitational field strength at Earth's surface is 9.81 N kg^{-1}. Calculate the field strength at a distance of $3R_E$ from Earth's centre, where R_E is Earth's radius.

[2 marks]

Q5. Two planets, X and Y, orbit the same star. Planet X has an orbital radius twice that of planet Y. Determine the ratio T_X/T_Y.

[3 marks]

Section B — Longer Structured Questions

Q6. A satellite of mass m orbits a planet of mass M in a circular orbit of radius r.

(a) Show that the orbital speed of the satellite is given by $v = \sqrt{GM/r}$.

[2 marks]

(b) Show that the period of the orbit satisfies $T^2 = \dfrac{4\pi^2 r^3}{GM}$.

[2 marks]

(c) The satellite is moved to a lower orbit. Explain what happens to its:

- speed
- kinetic energy
- total energy

[3 marks]

Q7. The Moon orbits the Earth with a period of 27.3 days at a mean orbital radius of 3.84×10^8 m.

(a) Use this data to calculate the mass of the Earth.

[3 marks]

(b) Calculate the gravitational potential at the Moon's orbital radius.

[2 marks]

(c) A 75 kg astronaut travels from Earth's surface to the Moon's orbital radius. Calculate the change in gravitational potential energy.

[3 marks]

Mark Scheme and Answers

Q1. Any two masses exert an attractive force on each other [1]; the force is proportional to the product of their masses and inversely proportional to the square of their separation [1].

Q2. Gravitational field strength is the gravitational force per unit mass acting on a (small test) mass placed at that point [1]; units: N kg^{-1} (or m s^{-2}) [1].

Q3. Gravitational potential is defined as zero at infinity [1]; work is done by gravity as mass moves from infinity inward, so the potential decreases below zero — it is always negative [1].

Q4. $g \propto 1/r^2$; at $3R_E$, $g = 9.81/3^2 = 9.81/9 = \mathbf{1.09}$ N kg^{-1} [2].

Q5. By Kepler's Third Law: $T^2 \propto r^3$, so $\left(\frac{T_X}{T_Y}\right)^2 = \left(\frac{2r_Y}{r_Y}\right)^3 = 8$ [2]; $\frac{T_X}{T_Y} = \sqrt{8} = \mathbf{2\sqrt{2} \approx 2.83}$ [1].

Q6(a). Gravitational force = centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]; cancel m, rearrange: $v = \sqrt{GM/r}$ [1].

Q6(b). Substitute $v = 2\pi r/T$ into result from (a) [1]; rearrange to get $T^2 = 4\pi^2 r^3/GM$ [1].

Q6(c). Speed **increases** [1]; KE **increases** [1]; total energy **decreases** (becomes more negative) [1].

Q7(a). $T = 27.3 \times 24 \times 3600 = 2.36 \times 10^6$ s; $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84 \times 10^8)^3}{6.67 \times 10^{-11}(2.36 \times 10^6)^2}$ [1] $= \mathbf{6.02 \times 10^{24}}$ kg [2].

Q7(b). $\phi = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24}}{3.84 \times 10^8} = \mathbf{-1.05 \times 10^6}$ J kg^{-1} [2].

Q7(c). $\Delta E_p = m\Delta\phi = m(\phi_{\text{Moon orbit}} - \phi_{\text{surface}})$ [1]; $\phi_{\text{surface}} = -GM/R_E = -6.25 \times 10^7$ J kg^{-1}; $\Delta\phi = -1.05 \times 10^6 - (-6.25 \times 10^7) = 6.14 \times 10^7$ J kg^{-1}; $\Delta E_p = 75 \times 6.14 \times 10^7 = \mathbf{4.6 \times 10^9}$ J [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ State and apply Newton's Law of Gravitation	
☐ Define gravitational field strength; use $g = F/m$ and $g = GM/r^2$	
☐ Sketch field line and equipotential diagrams	
☐ Define gravitational potential and explain why it is negative	
☐ Use $\phi = -GM/r$ and $E_p = -GMm/r$	
☐ Apply the relationship $g = -\Delta\phi/\Delta r$	
☐ Derive the expression for orbital speed	
☐ State and apply Kepler's Third Law ($T^2 \propto r^3$)	
☐ Analyse energy changes in satellite orbits	
☐ Describe the properties of a geostationary satellite	
☐ Derive and apply the formula for escape velocity	
☐ Interpret graphs of g vs r and ϕ vs r	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: understanding *why* formulas work is more powerful than memorising them. Practice drawing diagrams and deriving key results from first principles.