CIE International A Level Physics

Topic 10

D.C. Circuits

Revision Booklet

This booklet covers:

- Practical Circuits and E.M.F.
- Internal Resistance
- Kirchhoff's Laws
- Resistors in Series and Parallel
- Potential Dividers
- Thermistors and LDRs in Potential Dividers

Practical Circuits and E.M.F.

Electromotive Force (E.M.F.)

The **electromotive force** (e.m.f., symbol ε) of a source is defined as the **energy transferred per unit charge** in driving charge around a complete circuit.

$$\varepsilon = \frac{W}{Q}$$

- W: work done by the source (J); Q: charge (C).
- Unit: volt (V) $\equiv$ J C^{-1}.
- E.m.f. is a **property of the source**; it represents energy input per coulomb.

Potential Difference (P.D.)

The **potential difference** (p.d.) between two points is the **energy transferred per unit charge** by a component as charge passes between those points.

$$V = \frac{W}{Q}$$

- P.d. describes **energy released** (e.g. by a resistor); e.m.f. describes **energy supplied** (by a source).
- Both have unit: volt (V).

E.M.F. vs P.D. — Energy Perspective

- **E.m.f.**: energy is *given to* each coulomb of charge by the source (chemical $\rightarrow$ electrical).
- **P.d.**: energy is *taken from* each coulomb of charge by a component (electrical $\rightarrow$ heat, light, etc.).
- Around any complete circuit: total e.m.f. = total p.d. (conservation of energy — see Kirchhoff's Second Law).

Internal Resistance

Internal Resistance

A real source of e.m.f. has **internal resistance** r — resistance within the source itself (e.g. chemical paste in a battery). As current flows, some energy is dissipated internally.

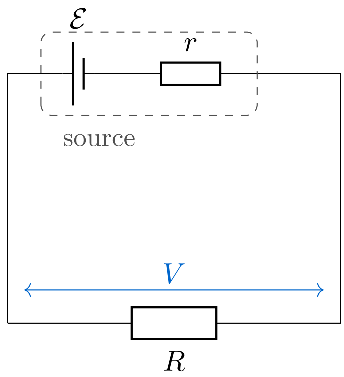
Terminal P.D. and Internal Resistance

$$\varepsilon = V + Ir \qquad \Longrightarrow \qquad V = \varepsilon - Ir$$

- $V = \varepsilon - Ir$: **terminal p.d.** (voltage across the external circuit).

- Ir: **lost volts** — p.d. across internal resistance.
- $V < \varepsilon$ whenever current flows.
- For an external resistance R: $I = \dfrac{\varepsilon}{R + r}$

Circuit diagram for a source with internal resistance

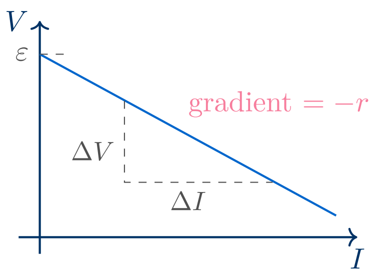


Common Mistake

The e.m.f. is *not* the same as the terminal p.d. except when $I = 0$ (open circuit). Under load, $V = \varepsilon - Ir < \varepsilon$. Always check whether the question gives e.m.f. or terminal voltage.

Graphical determination of ε and r

Rearranging: $V = \varepsilon - Ir$ has the form $y = c - mx$.



- **y-intercept**: $V = \varepsilon$ (e.m.f. when $I = 0$, i.e. open circuit).
- **Gradient**: $-r$ (magnitude of gradient gives internal resistance).
- **x-intercept**: $I = \varepsilon/r$ (short-circuit current; never reached safely).

Kirchhoff's Laws

Kirchhoff's First Law (KCL)

The sum of currents **entering** a junction equals the sum of currents **leaving** that junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

This is a consequence of **conservation of charge** — charge does not accumulate at a junction.

Kirchhoff's Second Law (KVL)

The sum of the e.m.f.s around any **closed loop** equals the sum of the potential differences (i.e. IR products) around that loop.

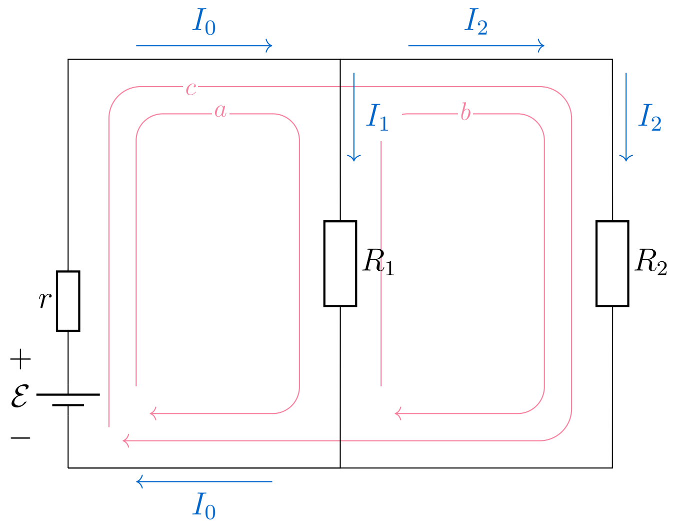
$$\sum \mathcal{E} = \sum IR$$

This is a consequence of **conservation of energy** — a charge returning to its starting point has the same potential energy.

Sign Convention for Kirchhoff's Second Law

- Choose a direction to traverse the loop.
- **E.m.f.**: positive if traversed from $-$ to $+$ terminal (source adds energy); negative if $+$ to $-$.
- IR: positive if traversed in the same direction as current; negative if opposite.
- Apply consistently — the choice of direction does not affect the final answer.

Circuit Diagram to show Kirchoff's Laws



Kirchoff's Closed Loops

There are three **closed loops** shown in the diagram above, **a,b** and **c**.
For **a**

$$\mathcal{E} = I_0 r + I_1 R_1$$

For **b**

$$0 = -I_1 R_1 + I_2 R_2$$

For **c**

$$\mathcal{E} = I_0 r + I_2 R_2$$

Also from Kirchoff's first law we know

$$I_0 = I_1 + I_2$$

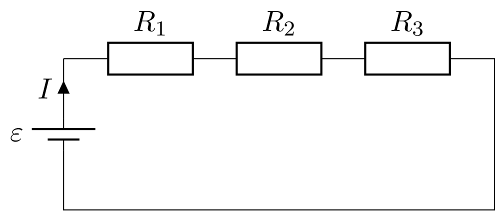
Resistors in Series and Parallel

Series Combination

For n resistors in series, the **same current** flows through each.

$$R_{\text{total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

Derivation (KVL): $V = V_1 + V_2 + \cdots$; since I is the same, $IR = IR_1 + IR_2 + \cdots$, hence $R = R_1 + R_2 + \cdots$



Series circuit — same I throughout

Parallel Combination

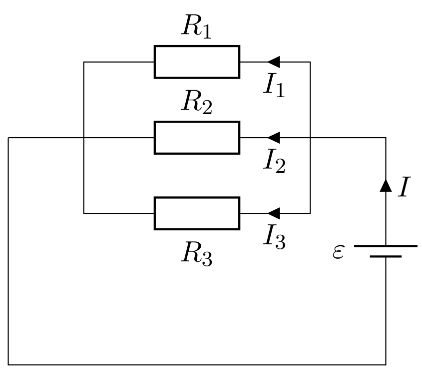
For n resistors in parallel, the **same p.d.** exists across each.

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Derivation (KCL): $I = I_1 + I_2 + \cdots$; since V is the same, $V/R = V/R_1 + V/R_2 + \cdots$, hence $1/R = 1/R_1 + 1/R_2 + \cdots$

Special case for two resistors in parallel:

$$R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



Parallel circuit — same V across each

Common Mistake

For two resistors in parallel, R_{total} is *always less* than the smaller of the two. If your answer is larger than either resistor, recheck your arithmetic — a common error is adding reciprocals incorrectly.

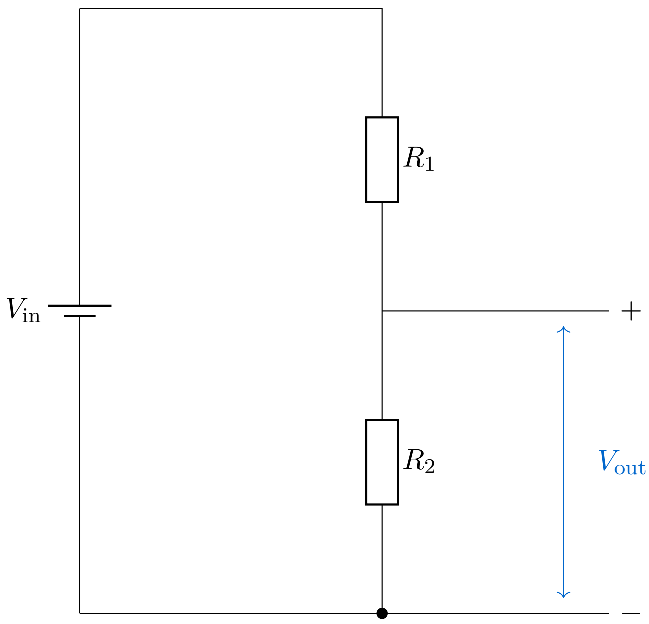
Potential Dividers

Potential Divider Principle

A **potential divider** uses two (or more) resistors in series across a supply to produce a fraction of the supply voltage as an output.

$$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1 + R_2}$$

- V_{out} is taken across R_2.
- The ratio $R_2/(R_1 + R_2)$ determines the fraction of V_{in} that appears across R_2.
- No current is drawn from the output (ideal case, or high-resistance load).



Potentiometer as a Comparison Tool

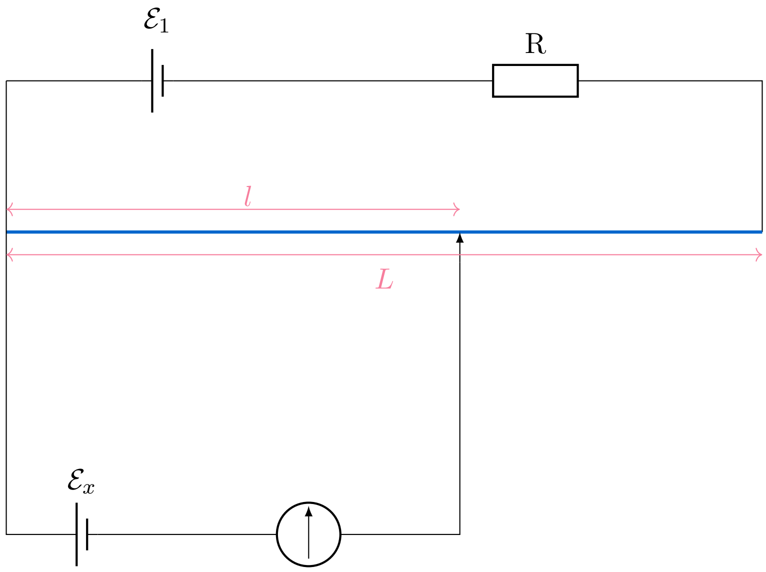
A **potentiometer** is a sliding-contact potential divider used to **compare** e.m.f.s or p.d.s precisely:

- A uniform resistance wire carries a **driver current** from a stable supply.
- The p.d. across a length l of the wire is proportional to l.
- At **balance** (null deflection on galvanometer), no current is drawn from the test source — so the measurement is unaffected by internal resistance.
- $\mathcal{E}_x = \dfrac{\mathcal{E}_1 l}{L}$ at their respective balance lengths.

Null Method

A **null method** is a measurement technique in which the quantity being measured is compared against a known standard and the detector (e.g. galvanometer) is adjusted to read **zero**. Because no current flows at balance, the result is independent of any resistance in the measuring instrument.

Circuit Diagram for a Null Method Measurement



Thermistors and LDRs in Potential Dividers

Thermistor (NTC)

A **thermistor** (negative temperature coefficient, NTC) is a resistor whose resistance **decreases** as temperature **increases**. It is used as a temperature sensor in potential divider circuits.

Light-Dependent Resistor (LDR)

An **LDR** has resistance that **decreases** as light intensity **increases**. It is used as a light sensor in potential divider circuits.

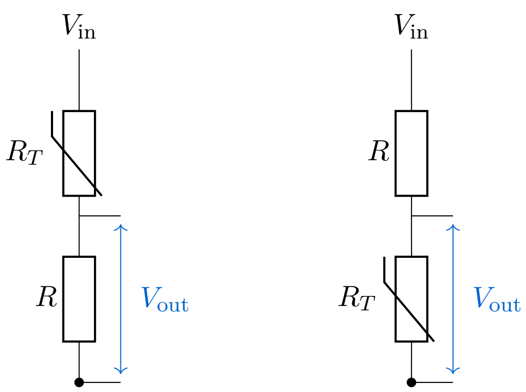
Using Sensors in Potential Dividers

For a potential divider with a thermistor (R_T) in series with a fixed resistor R, with output taken across R:

$$V_{\text{out}} = V_{\text{in}} \times \frac{R}{R_T + R}$$

- **Temperature rises** $\Rightarrow$ R_T decreases $\Rightarrow$ V_{out} **increases**.
- **Temperature falls** $\Rightarrow$ R_T increases $\Rightarrow$ V_{out} **decreases**.
- Swap the positions of R_T and R to **invert** the response (output high when cold, low when hot).

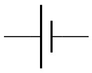
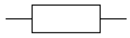
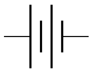
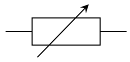
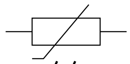
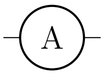
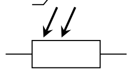
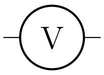
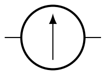
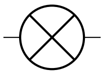
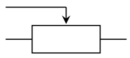
The same logic applies to an LDR replacing R_T, with light intensity replacing temperature.



V_{out} high when hot $\quad$ V_{out} high when cold

Circuit Symbols (Section 6 Reference)

The following symbols from CIE Section 6 are required for Topic 10 circuit diagrams.

Component	Symbol	Component	Symbol
Cell		Fixed resistor	
Battery of cells		Variable resistor	
Switch		Thermistor	
Ammeter		Light-dependent resistor	
Voltmeter		Galvanometer	
Lamp		Potentiometer	

Formula Summary Sheet

Formula	Quantity	Units
$\varepsilon = W/Q$	E.m.f. definition	V
$V = \varepsilon - Ir$	Terminal p.d.	V
$I = \varepsilon/(R + r)$	Current in circuit	A
$R_{\text{total}} = R_1 + R_2 + \cdots$	Series resistance	Ω
$1/R_{\text{total}} = 1/R_1 + 1/R_2 + \cdots$	Parallel resistance	Ω
$R_{\text{total}} = R_1R_2/(R_1 + R_2)$	Two resistors in parallel	Ω
$V_{\text{out}} = V_{\text{in}} \times R_2/(R_1 + R_2)$	Potential divider output	V
$\varepsilon_1/\varepsilon_2 = l_1/l_2$	Potentiometer balance	—

KCL: $\sum I_{\text{in}} = \sum I_{\text{out}}$ at any junction (conservation of charge).
KVL: $\sum \mathcal{E} = \sum IR$ around any closed loop (conservation of energy).
Note: Lost volts $= Ir$; terminal p.d. $< \mathcal{E}$ whenever current flows.

Worked Examples

Example 1 — Internal Resistance

Question: A battery of e.m.f. 12 V and internal resistance 0.5 Ω is connected to an external resistor of 3.5 Ω. Calculate (a) the current, (b) the terminal p.d., and (c) the power dissipated internally.

Solution

(a) $I = \dfrac{\varepsilon}{R+r} = \dfrac{12}{3.5+0.5} = \dfrac{12}{4.0} = \mathbf{3.0}$ A

(b) $V = \varepsilon - Ir = 12 - (3.0)(0.5) = 12 - 1.5 = \mathbf{10.5}$ V

(c) $P_{\text{internal}} = I^2 r = (3.0)^2 \times 0.5 = 9 \times 0.5 = \mathbf{4.5}$ W

Example 2 — Kirchhoff's Laws

Question: In the circuit below, $\varepsilon_1 = 10$ V, $\varepsilon_2 = 4$ V, $R_1 = 3$ Ω, $R_2 = 2$ Ω, $R_3 = 5$ Ω. Find the current flowing in the circuit (treat as a single loop, both sources driving in the same direction).

Solution

Applying KVL around the loop (taking the direction of current as positive):

$$\sum \varepsilon = \sum IR$$

$$10 - 4 = I(3 + 2 + 5)$$

$$6 = 10I$$

$$I = \mathbf{0.60} \text{ A}$$

Note: if ε_2 had opposed ε_1, we would write $10 - 4$ on the left side (one e.m.f. is negative in our sign convention).

Example 3 — Resistors in Parallel

Question: Two resistors of 6 Ω and 12 Ω are connected in parallel across a 6 V supply of negligible internal resistance. Find (a) the combined resistance, (b) the total current, and (c) the current through each resistor.

Solution

(a) $R_{\text{total}} = \dfrac{6 \times 12}{6 + 12} = \dfrac{72}{18} = \mathbf{4.0\ \Omega}$

(b) $I_{\text{total}} = V/R_{\text{total}} = 6/4.0 = \mathbf{1.5}$ A

(c) $I_1 = 6/6 = \mathbf{1.0}$ A; $I_2 = 6/12 = \mathbf{0.50}$ A

Check: $1.0 + 0.5 = 1.5$ A ✓(KCL satisfied)

Example 4 — Potential Divider with Thermistor

Question: A potential divider consists of a thermistor R_T in series with a 1.5 kΩ fixed resistor, connected across a 5.0 V supply. The output is taken across the fixed resistor. At 20 °C the thermistor has resistance 3.0 kΩ. Calculate (a) V_{out} at 20 °C, and (b) whether V_{out} increases or decreases as temperature rises.

Solution

(a) $$V_{\text{out}} = 5.0 \times \frac{1500}{3000 + 1500} = 5.0 \times \frac{1500}{4500} = 5.0 \times \frac{1}{3} = \mathbf{1.67}\text{ V}$$

(b) As temperature **rises**, R_T **decreases**. The fraction $R/(R_T + R)$ **increases**, so V_{out} **increases**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electromotive force (e.m.f.) and distinguish it from potential difference in terms of energy.

[3 marks]

Q2. A cell has e.m.f. 9.0 V and internal resistance 0.8 Ω. When connected to a resistor, a current of 3.0 A flows. Calculate (a) the terminal p.d. and (b) the power lost to internal resistance.

[3 marks]

Q3. State Kirchhoff's first and second laws and give the physical principle underlying each.

[4 marks]

Q4. Three resistors of 2 Ω, 4 Ω and 6 Ω are connected in parallel. Calculate the combined resistance.

[2 marks]

Section B — Longer Structured Questions

Q5. A battery of e.m.f. ε and internal resistance r is connected to a variable external resistor R.

(a) Show that the terminal p.d. $V = \varepsilon - Ir$ and hence that $V = \varepsilon - \dfrac{\varepsilon r}{R + r}$.

[2 marks]

(b) A graph of V against I is plotted as R is varied. Describe the graph and explain how ε and r can be determined from it.

[3 marks]

(c) Data from the graph gives $\varepsilon = 6.0$ V and $r = 0.4\ \Omega$. Calculate the current and terminal p.d. when $R = 2.6\ \Omega$.

[3 marks]

Q6. The circuit shown consists of a supply of e.m.f. 12 V and negligible internal resistance connected to resistors $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$ and $R_3 = 12\ \Omega$, where R_2 and R_3 are in parallel with each other and R_1 is in series with this parallel combination.

(a) Calculate the combined resistance of R_2 and R_3 in parallel.

[2 marks]

(b) Calculate the total resistance of the circuit and the current drawn from the supply.

[2 marks]

(c) Calculate the p.d. across the parallel combination and hence the current through each of R_2 and R_3.

[3 marks]

(d) Verify your answers using Kirchhoff's first law.

[1 mark]

Q7. A potential divider consists of a 2.0 kΩ resistor and a light-dependent resistor (LDR) connected in series across a 10 V supply. The output voltage is taken across the LDR. In bright light, the LDR has resistance 500 Ω; in darkness, its resistance is 20 kΩ.

(a) Calculate V_{out} in bright light.

[2 marks]

(b) Calculate V_{out} in darkness.

[2 marks]

(c) Explain how the circuit could be modified so that V_{out} is high in bright light rather than in darkness.

[1 mark]

Mark Scheme and Answers

Q1. E.m.f. is the energy transferred *to* each coulomb of charge by the source [1]; p.d. is the energy transferred *from* each coulomb of charge by a component [1]; e.m.f. relates to energy input (e.g. chemical to electrical), p.d. relates to energy output (e.g. electrical to heat) [1].

Q2(a). $V = \varepsilon - Ir = 9.0 - (3.0)(0.8) = 9.0 - 2.4 = \mathbf{6.6}$ V [2].
Q2(b). $P = I^2r = (3.0)^2 \times 0.8 = \mathbf{7.2}$ W [1].

Q3. KCL: sum of currents into a junction = sum of currents out [1]; consequence of conservation of charge [1]. KVL: sum of e.m.f.s around a closed loop = sum of IR products [1]; consequence of conservation of energy [1].

Q4. $\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12}$; $\quad R = \mathbf{12/11 \approx 1.1\ \Omega}$ [2].

Q5(a). By KVL: $\varepsilon = IR + Ir = I(R + r)$; rearranging gives $V = IR = \varepsilon - Ir$ [2].

Q5(b). Straight line [1]; y-intercept $= \varepsilon$ (open-circuit voltage) [1]; magnitude of gradient $= r$ [1].

Q5(c). $I = 6.0/(2.6+0.4) = 6.0/3.0 = \mathbf{2.0}$ A [1]; $V = 6.0-(2.0)(0.4) = 6.0-0.8 = \mathbf{5.2}$ V [2].

Q6(a). $R_{23} = (6 \times 12)/(6 + 12) = 72/18 = \mathbf{4.0\ \Omega}$ [2].
Q6(b). $R_{\text{total}} = 4 + 4 = \mathbf{8.0\ \Omega}$; $I = 12/8 = \mathbf{1.5}$ A [2].
Q6(c). $V_{23} = 1.5 \times 4 = \mathbf{6.0}$ V; $I_2 = 6/6 = \mathbf{1.0}$ A; $I_3 = 6/12 = \mathbf{0.50}$ A [3].
Q6(d). $I_2 + I_3 = 1.0 + 0.5 = 1.5\ \text{A} = I_{\text{total}}$ ✓[1].

Q7(a). $V_{\text{out}} = 10 \times 500/(2000 + 500) = 5000/2500 = \mathbf{2.0}$ V [2].
Q7(b). $V_{\text{out}} = 10 \times 20000/(2000 + 20000) = 200000/22000 = \mathbf{9.1}$ V [2].
Q7(c). Swap the positions of the LDR and the 2.0 kΩ resistor so that the output is taken across the fixed resistor instead [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Recall and use the circuit symbols from Section 6	
☐ Define e.m.f. as energy transferred per unit charge by a source	
☐ Distinguish between e.m.f. and p.d. in terms of energy	
☐ Use $V = \varepsilon - Ir$ and $I = \varepsilon/(R + r)$ for circuits with internal resistance	
☐ Determine ε and r from a V–I graph (intercept and gradient)	
☐ State and apply Kirchhoff's first law (conservation of charge)	
☐ State and apply Kirchhoff's second law (conservation of energy)	
☐ Derive and use $R_{\text{total}} = R_1 + R_2 + \cdots$ for series resistors	
☐ Derive and use $1/R_{\text{total}} = 1/R_1 + 1/R_2 + \cdots$ for parallel resistors	
☐ Use Kirchhoff's laws to solve multi-loop circuit problems	
☐ Understand the principle of a potential divider circuit	
☐ Use $V_{\text{out}} = V_{\text{in}} \times R_2/(R_1 + R_2)$ for a potential divider	
☐ Explain the potentiometer as a null method for comparing p.d.s	
☐ Explain how a thermistor or LDR is used in a potential divider	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Kirchhoff's laws are nothing more than conservation of charge and conservation of energy written in circuit language. Once you see that, every circuit problem becomes a matter of careful bookkeeping — choose your loops, track your signs, and the algebra does the rest.