

Topic 9

Electricity

Revision Booklet

This booklet covers:

- Electric Current and Charge Carriers
- Charge, Current and the Drift Velocity Equation
- Potential Difference and Power
- Resistance and Ohm's Law
- I–V Characteristics
- Resistivity, LDRs and Thermistors

Electric Current and Charge Carriers

Electric Current

An **electric current** is a flow of **charge carriers**. In metals the charge carriers are free (conduction) electrons; in electrolytes and semiconductors, other carriers (ions, holes) may contribute.

$$I = \frac{\Delta Q}{\Delta t} \quad \implies \quad Q = It$$

- I : current (A); Q : charge (C); t : time (s).
- $1 \text{ A} = 1 \text{ C s}^{-1}$.
- Conventional current flows from + to –; electron flow is from – to +.

Quantisation of Charge

The charge on any charge carrier is an **integer multiple** of the elementary charge e :

$$Q = ne \quad e = 1.60 \times 10^{-19} \text{ C}$$

Charge is **quantised** — it cannot take continuous values; it only exists in discrete packets of e .

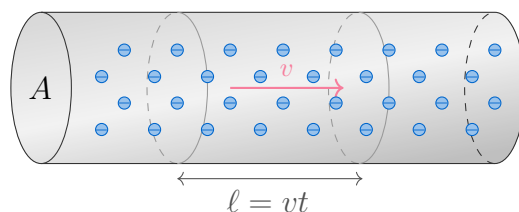
The Drift Velocity Equation

$$I = Anvq$$

For a conductor carrying current I :

$$I = Anvq$$

- A : cross-sectional area of conductor (m^2).
- n : **number density** of charge carriers (m^{-3}) — number of carriers per unit volume.
- v : **mean drift velocity** of carriers (m s^{-1}).
- q : charge on each carrier (C); for electrons $q = e = 1.60 \times 10^{-19} \text{ C}$.

Derivation of $I = Anvq$ Deriving $I = Anvq$

- In time t , carriers travel distance $\ell = vt$.
- Volume swept past the cross-section: $V = A\ell = Avt$.
- Number of carriers in this volume: $N = nAvt$.
- Total charge past the cross-section: $Q = Nq = nAvtq$.
- Therefore: $I = Q/t = nAvq$.

Using $I = Anvq$ to Compare Conductors

- **Metals** have very high n ($\sim 10^{28} \text{ m}^{-3}$) \Rightarrow very slow drift velocity ($\sim \text{mm s}^{-1}$) even for large currents.
- **Semiconductors** have lower n \Rightarrow higher v for the same current.
- At a junction where the wire narrows (A decreases), v must increase (since I , n , q are constant).
- Increasing temperature in a metal barely changes n but reduces v (increased resistance).

Potential Difference and Power

Potential Difference

The **potential difference** (p.d.) across a component is defined as the **energy transferred per unit charge** passing through it.

$$V = \frac{W}{Q}$$

- V : potential difference (V); W : energy transferred (J); Q : charge (C).
- Unit: volt (V) $\equiv \text{J C}^{-1}$.
- 1 V: 1 joule of energy transferred per coulomb of charge.

Electrical Power

$$P = VI = I^2R = \frac{V^2}{R}$$

Derivation: Power is energy per unit time. $P = W/t = (QV)/t = IV$. Substituting $V = IR$: $P = I(IR) = I^2R$; or $I = V/R$: $P = (V/R)V = V^2/R$.

- All three forms are equivalent for a resistor at any instant.
- Use $P = VI$ when both V and I are known.
- Use $P = I^2R$ when current and resistance are known.
- Use $P = V^2/R$ when voltage and resistance are known.

Common Mistake

Energy is $W = Pt = Vit$, not $W = VI$. Always multiply power by time to get energy. Also, $P = V^2/R$ and $P = I^2R$ apply to a single resistor; for a circuit with multiple components, identify V and I for each component separately.

Resistance and Ohm's Law

Resistance

The **resistance** of a component is defined as the ratio of the potential difference across it to the current flowing through it:

$$R = \frac{V}{I}$$

- Unit: ohm (Ω) \equiv V A⁻¹.
- Resistance is a property of the component — it describes how much it opposes current flow.

Ohm's Law

Ohm's law states that the current through a metallic conductor is **directly proportional** to the potential difference across it, **provided the temperature remains constant**:

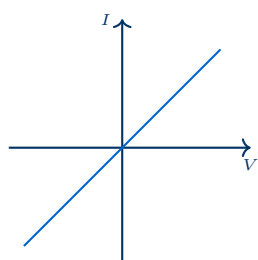
$$V \propto I \quad \Leftrightarrow \quad V = IR \quad (R \text{ constant})$$

A component that obeys Ohm's law is called **ohmic**. The I - V graph for an ohmic conductor is a straight line through the origin.

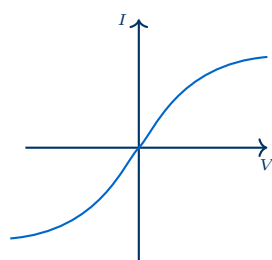
Ohm's Law is not a Definition of Resistance

$R = V/I$ defines resistance for any component. Ohm's law is a separate statement that R is *constant* (independent of V and I) for a metallic conductor at constant temperature. Not all components obey Ohm's law.

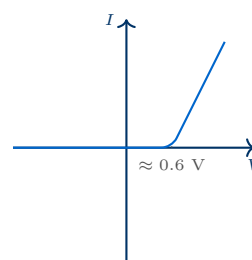
I–V Characteristics



Metallic conductor
(constant temp.)



Filament lamp
(gradient decreases)



Semiconductor diode
(threshold ≈ 0.6 V)

Interpreting I–V Graphs

- **Metallic conductor (ohmic):** straight line through origin — gradient = $1/R$ (constant). Both forward and reverse biased sections are identical straight lines.
- **Filament lamp:** S-shaped curve — gradient decreases at high V and I , meaning resistance increases. Cause: increasing current \Rightarrow increasing temperature \Rightarrow increased resistance (more collisions between electrons and vibrating lattice ions).
- **Semiconductor diode:** conducts appreciably only above the **threshold voltage** (≈ 0.6 V for silicon) in the forward direction; in reverse bias, current is essentially zero (tiny reverse leakage).

Resistivity

Resistivity

The resistance of a uniform conductor depends on its material, length and cross-sectional area:

$$R = \frac{\rho L}{A}$$

- ρ : **resistivity** of the material ($\Omega \text{ m}$).
- L : length of conductor (m).
- A : cross-sectional area (m^2).
- Resistivity is a **material property** — it does not depend on the shape of the sample.
- $R \propto L$ (double the length \Rightarrow double the resistance).
- $R \propto 1/A$ (double the area \Rightarrow halve the resistance).

Resistivity

The **resistivity** ρ of a material is defined by:

$$\rho = \frac{RA}{L}$$

It is the resistance of a 1 m cube of the material between opposite faces. Unit: $\Omega \text{ m}$.

LDRs and Thermistors

Light-Dependent Resistor (LDR)

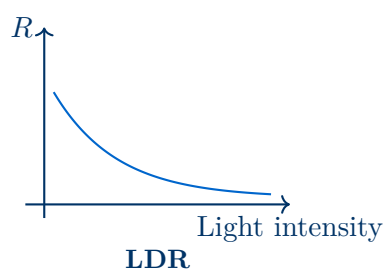
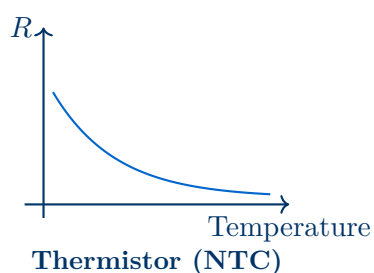
The **resistance of an LDR decreases as light intensity increases**.

- In darkness: resistance can be $\sim \text{M}\Omega$.
- In bright light: resistance falls to $\sim \text{k}\Omega$ or lower.
- Mechanism: photons give electrons enough energy to break free and become charge carriers, increasing n and thus conductivity.

Thermistor (NTC)

The **resistance of an NTC thermistor decreases as temperature increases**.

- NTC = Negative Temperature Coefficient.
- Mechanism: rising temperature gives more electrons enough energy to act as charge carriers, increasing n and reducing resistivity.
- Opposite behaviour to metals (where resistance *increases* with temperature because v is reduced).



Comparing Metals Thermistors and LDRs

Component	Effect on resistance	Reason
Metal wire	Resistance increases with T	More lattice vibration; v decreases
NTC Thermistor	Resistance decreases with T	More charge carriers; n increases
LDR	Resistance decreases with light	More charge carriers; n increases
Filament lamp	Resistance increases with I	Temperature rises (same as metal)

Formula Summary Sheet

Formula	Quantity	Units
$Q = It$	Charge	C
$Q = ne$	Quantisation of charge	C
$I = Anvq$	Current (drift velocity)	A
$V = W/Q$	Potential difference	V
$P = VI = I^2R = V^2/R$	Electrical power	W
$W = VIt$	Electrical energy	J
$R = V/I$	Resistance (definition)	Ω
$V = IR$	Ohm's law	V
$R = \rho L/A$	Resistivity	Ω

Key definitions to learn word-for-word:

P.d.: energy transferred per unit charge through a component.

Resistance: ratio of p.d. across a component to current through it ($R = V/I$).

Ohm's law: current is directly proportional to p.d. at constant temperature.

Resistivity: $\rho = RA/L$; a material property independent of sample dimensions.

Worked Examples

Example 1 — Drift Velocity

Question: A copper wire of cross-sectional area 1.5 mm^2 carries a current of 3.0 A . The number density of free electrons in copper is $8.5 \times 10^{28} \text{ m}^{-3}$. Calculate the mean drift velocity of the electrons.

Solution

Rearrange $I = Anvq$ for v :

$$v = \frac{I}{Anq} = \frac{3.0}{(1.5 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})}$$

$$v = \frac{3.0}{1.5 \times 10^{-6} \times 1.36 \times 10^{10}} = \frac{3.0}{2.04 \times 10^4} = \mathbf{1.47 \times 10^{-4} \text{ m s}^{-1}}$$

This is about 0.15 mm s^{-1} — extremely slow, yet the effect of the current is felt almost instantaneously because the electric field propagates at close to the speed of light.

Example 2 — Power and Energy

Question: A 60Ω resistor is connected to a 12 V supply for 5.0 min . Calculate (a) the current, (b) the power dissipated, and (c) the total energy transferred.

Solution

- (a) $I = V/R = 12/60 = \mathbf{0.20 \text{ A}}$
 (b) $P = V^2/R = 144/60 = \mathbf{2.4 \text{ W}}$ (or $P = I^2R = 0.04 \times 60 = 2.4 \text{ W}$ ✓)
 (c) $W = Pt = 2.4 \times (5.0 \times 60) = 2.4 \times 300 = \mathbf{720 \text{ J}}$

Example 3 — Resistivity

Question: A nichrome wire of length 0.80 m and diameter 0.50 mm has resistance 4.4Ω . Calculate the resistivity of nichrome.

Solution

Cross-sectional area: $A = \pi r^2 = \pi(0.25 \times 10^{-3})^2 = 1.96 \times 10^{-7} \text{ m}^2$

$$\rho = \frac{RA}{L} = \frac{4.4 \times 1.96 \times 10^{-7}}{0.80} = \frac{8.63 \times 10^{-7}}{0.80} = \mathbf{1.08 \times 10^{-6} \Omega \text{ m}}$$

Example 4 — I–V Characteristic

Question: The current through a filament lamp increases from 0.20 A to 0.60 A when the voltage increases from 3.0 V to 6.0 V . Show that the lamp does not obey Ohm's law.

Solution

At $V = 3.0 \text{ V}$: $R_1 = V/I = 3.0/0.20 = 15 \Omega$

At $V = 6.0 \text{ V}$: $R_2 = V/I = 6.0/0.60 = 10 \Omega$

$R_1 \neq R_2$ — resistance is not constant; it decreases as current decreases (i.e. as temperature decreases). This is characteristic of a filament lamp: at higher current the filament is

hotter and resistance is higher. The lamp **does not obey Ohm's law**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) electric current and (b) potential difference. State the SI unit of each.
[4 marks]

Q2. Explain what is meant by saying that charge is *quantised*. A current of 2.5 mA flows for 4.0 min. Calculate the charge transferred and the number of electrons passing a point.
[4 marks]

Q3. State Ohm's law and sketch the I - V characteristics of (a) a metallic conductor at constant temperature and (b) a filament lamp. Explain why the graphs have different shapes.
[5 marks]

Q4. A semiconductor diode is connected in a circuit. Sketch the I - V characteristic and mark the threshold voltage. Explain what happens in reverse bias.

[3 marks]

Section B — Longer Structured Questions

Q5. A conductor has cross-sectional area A , number density of charge carriers n , and carries a current I .

(a) Derive the expression $I = Anvq$, defining all symbols.

[3 marks]

(b) A copper wire ($n = 8.5 \times 10^{28} \text{ m}^{-3}$, diameter 1.2 mm) carries a current of 5.0 A. Calculate the mean drift velocity of the electrons.

[3 marks]

(c) The wire is replaced with one of the same material but half the diameter. The current remains 5.0 A. State and explain what happens to the drift velocity.

[2 marks]

Q6. A uniform resistance wire of length 1.20 m and cross-sectional area $3.5 \times 10^{-7} \text{ m}^2$ is made of a material of resistivity $4.9 \times 10^{-7} \Omega \text{ m}$.

(a) Calculate the resistance of the wire.

[2 marks]

(b) A potential difference of 6.0 V is applied across the wire. Calculate the current through it and the power dissipated.

[3 marks]

(c) The wire is stretched so its length doubles but its volume remains constant. Show that the resistance increases by a factor of 4.

[3 marks]

Q7. A thermistor is connected in series with a $4.7 \text{ k}\Omega$ fixed resistor across a 9.0 V supply. At 20°C the thermistor has resistance $8.2 \text{ k}\Omega$.

(a) Calculate the current in the circuit and the voltage across the thermistor at 20°C .
[3 marks]

(b) Describe and explain what happens to the current as the temperature increases.
[2 marks]

(c) Calculate the power dissipated in the fixed resistor at 20°C .
[2 marks]

Mark Scheme and Answers

Q1(a). Electric current is the rate of flow of charge [1]; unit: ampere (A) [1].

Q1(b). Potential difference is the energy transferred per unit charge through a component [1]; unit: volt (V) [1].

Q2. Charge is quantised means it only exists in discrete multiples of $e = 1.60 \times 10^{-19}$ C [1]. $Q = It = 2.5 \times 10^{-3} \times 240 = 0.60$ C [1]. Number of electrons: $N = Q/e = 0.60/(1.60 \times 10^{-19}) = 3.75 \times 10^{18}$ [2].

Q3. Ohm's law: current is proportional to p.d. at constant temperature [1]. Metallic conductor: straight line through origin [1]. Filament lamp: S-shaped curve, decreasing gradient [1]. Metal: R constant so linear [1]; lamp: higher $I \Rightarrow$ higher $T \Rightarrow$ higher R , so gradient I/V decreases [1].

Q4. Correct exponential-ish forward curve with threshold ≈ 0.6 V marked [2]; in reverse bias current is (approximately) zero — the diode blocks current [1].

Q5(a). In time t , carriers move distance vt ; volume = Avt ; number = $nAvt$; charge = $nAvtq$; current $I = Q/t = nAvq$ [3].

Q5(b). $A = \pi(0.6 \times 10^{-3})^2 = 1.131 \times 10^{-6}$ m²; $v = I/(Anq) = 5.0/(1.131 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.60 \times 10^{-19}) = 3.26 \times 10^{-4}$ m s⁻¹ [3].

Q5(c). Diameter halves \Rightarrow radius halves \Rightarrow area reduces by factor 4 [1]; since $I = Anvq$ is constant and A is quartered, v must increase by factor 4 [1].

Q6(a). $R = \rho L/A = (4.9 \times 10^{-7} \times 1.20)/(3.5 \times 10^{-7}) = 1.68 \Omega$ [2].

Q6(b). $I = V/R = 6.0/1.68 = 3.57$ A [1]; $P = VI = 6.0 \times 3.57 = 21.4$ W [2].

Q6(c). Volume constant: $A'L' = AL$; $L' = 2L$ so $A' = A/2$ [1]; new $R' = \rho L'/A' = \rho(2L)/(A/2) = 4\rho L/A = 4R$ [2].

Q7(a). $R_{\text{total}} = 8200 + 4700 = 12900 \Omega$; $I = 9.0/12900 = 6.98 \times 10^{-4}$ A [2]; $V_T = IR_T = 6.98 \times 10^{-4} \times 8200 = 5.72$ V [1].

Q7(b). As temperature increases, thermistor resistance decreases [1]; total resistance decreases so current increases (from $V = IR$) [1].

Q7(c). $P = I^2R = (6.98 \times 10^{-4})^2 \times 4700 = 4.87 \times 10^{-7} \times 4700 = 2.29 \times 10^{-3}$ W ≈ 2.3 mW [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Understand that current is a flow of charge carriers	
<input type="checkbox"/> State that charge is quantised ($Q = ne$)	
<input type="checkbox"/> Use $Q = It$ to find charge, current or time	
<input type="checkbox"/> Use and derive $I = Anvq$, defining all terms	
<input type="checkbox"/> Define p.d. as energy transferred per unit charge; use $V = W/Q$	
<input type="checkbox"/> Use all three power formulae: $P = VI$, $P = I^2R$, $P = V^2/R$	
<input type="checkbox"/> Define resistance as $R = V/I$	
<input type="checkbox"/> State Ohm's law and identify ohmic/non-ohmic behaviour	
<input type="checkbox"/> Sketch and interpret I – V graphs for metal, filament lamp and diode	
<input type="checkbox"/> Explain why filament lamp resistance increases with current	
<input type="checkbox"/> Use $R = \rho L/A$ and define resistivity	
<input type="checkbox"/> Explain how resistance of an LDR depends on light intensity	
<input type="checkbox"/> Explain how resistance of an NTC thermistor depends on temperature	
<input type="checkbox"/> Distinguish the behaviour of metals from thermistors as temperature changes	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Everything in electricity connects through two equations: $Q = It$ and $V = W/Q$. All the power formulae, resistance definitions and drift velocity expression follow from those two ideas. Once you see that, the whole topic becomes a matter of careful substitution.