CIE International A Level Physics

Topic 8

Superposition

Revision Booklet

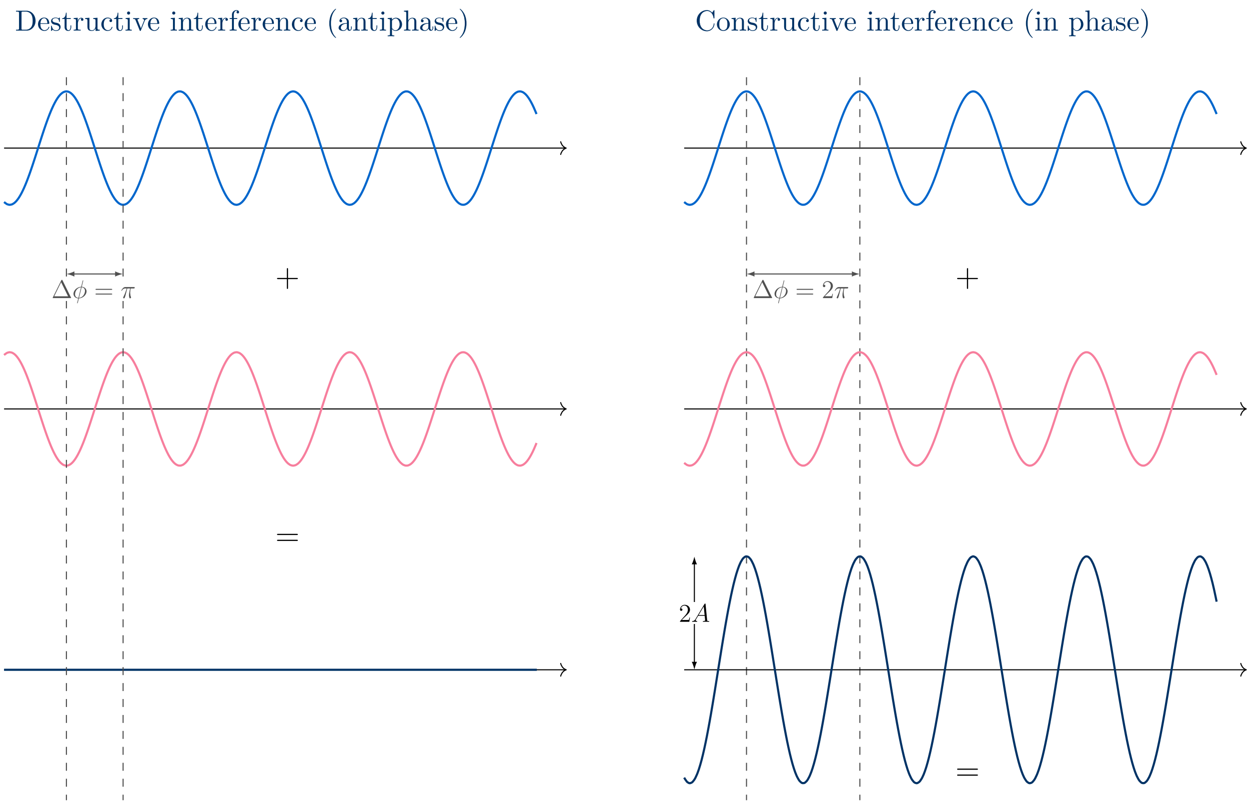
This booklet covers:

- The Principle of Superposition
- Stationary Waves: Formation and Properties
- Stationary Waves in Strings and Air Columns
- Diffraction
- Two-Source Interference and Coherence
- The Diffraction Grating

The Principle of Superposition

Principle of Superposition

When two or more waves meet at a point, the **resultant displacement** at that point is equal to the **vector sum** of the individual displacements of each wave at that point.



Stationary Waves: Formation and Properties

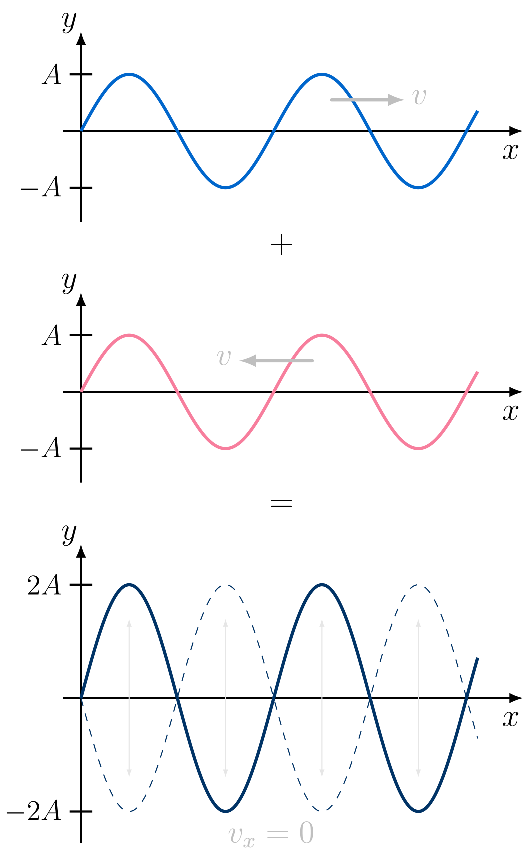
Stationary (Standing) Wave

A **stationary wave** is formed when two progressive waves of the **same frequency**, **same amplitude** and travelling in **opposite directions** superpose. The result is a wave pattern that does not travel — energy is stored rather than transmitted.

Nodes and Antinodes

- **Node**: a point of **zero amplitude** — particles are permanently at rest. Destructive superposition occurs here at all times.
- **Antinode**: a point of **maximum amplitude** — particles oscillate with the greatest displacement. Constructive superposition here.
- Adjacent nodes are separated by $\lambda/2$.
- Adjacent antinodes are also separated by $\lambda/2$.
- A node and an adjacent antinode are separated by $\lambda/4$.

Formation of a stationary wave



Differences: Stationary vs Progressive Waves

	Progressive wave	**Stationary wave**
Energy	Transferred in direction of travel	Stored; not transferred
Amplitude	Same for all particles	Varies from 0 (node) to max (antinode)
Phase	Varies continuously along wave	All particles between two nodes are in phase; adjacent segments are in antiphase
Wavelength	Distance between adjacent in-phase points	Twice the node-to-node distance

Determining Wavelength from a Stationary Wave

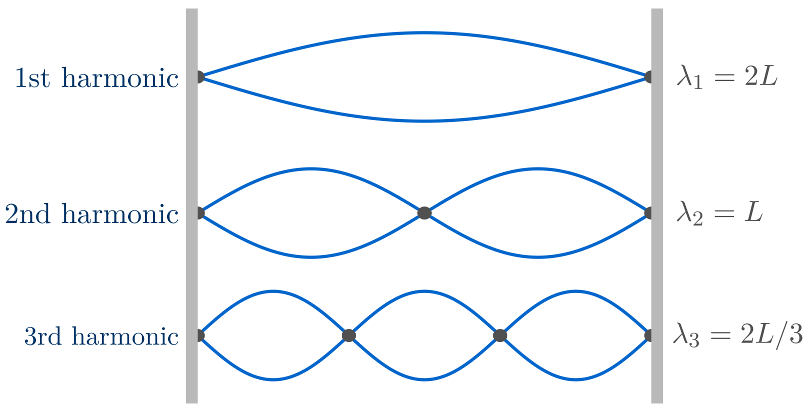
Since adjacent nodes are separated by $\lambda/2$:

$$\lambda = 2 \times (\text{node-to-node distance})$$

Measure the distance across several node spacings for greater accuracy.

Stationary Waves in Strings and Air Columns

Stretched string (both ends fixed)



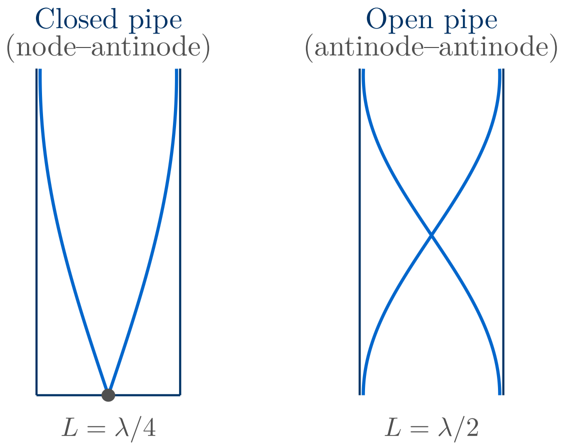
Harmonics in a String

For a string of length L fixed at both ends, the nth harmonic has:

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L} = nf_1$$

The **fundamental** (1st harmonic) has one antinode and $f_1 = v/2L$.

Air columns



Standing Waves in Air

Standing waves in air are created in a similar way. Waves are reflected from the ends of a tube and the waves travelling in opposite directions superpose to form a standing wave as long as the boundary conditions are satisfied.

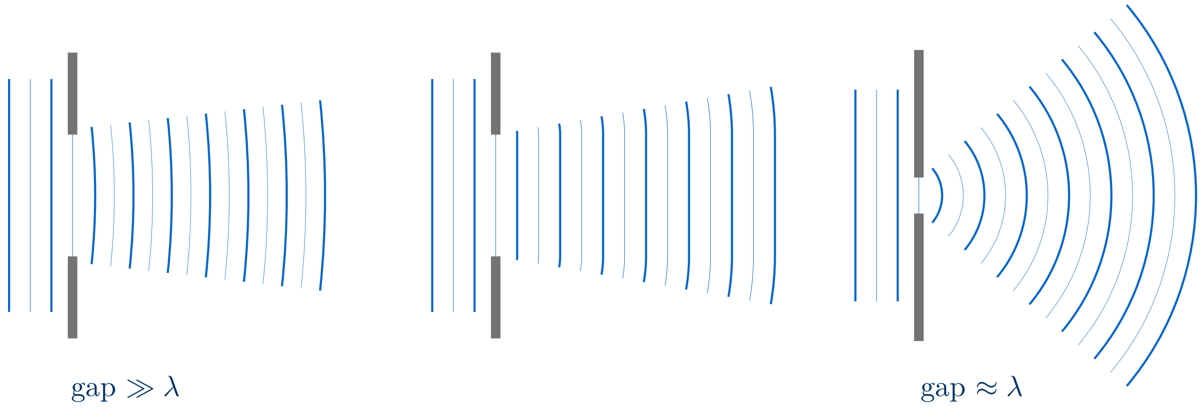
- For a closed pipe: A **Node** at the closed end and an **Antinode** at the open end.
- For an open pipe: An **Antinode** at both ends.

Diffraction

Diffraction

Diffraction is the spreading of a wave as it passes through a gap or around an obstacle. It is a property of all waves.

- Diffraction is most pronounced when the **gap width ≈ wavelength**.
- When gap width $\gg \lambda$: very little spreading — the wave passes through with little diffraction.
- When gap width $\approx \lambda$: maximum spreading — the wave spreads out as a near-semicircle.
- When gap width $< \lambda$: the gap acts almost like a point source.



Two-Source Interference and Coherence

Coherence

Two sources are **coherent** if they have:

- The **same frequency** (and hence wavelength).
- A **constant phase difference** (not necessarily zero).

Coherence is essential for a stable interference pattern to be observed.

Interference

Interference is the superposition of waves from two coherent sources, producing a pattern of **maxima** (constructive) and **minima** (destructive).

- **Constructive interference**: path difference $= n\lambda$ ($n = 0, 1, 2, \ldots$); waves arrive in phase.
- **Destructive interference**: path difference $= (n + \frac{1}{2})\lambda$; waves arrive in antiphase.

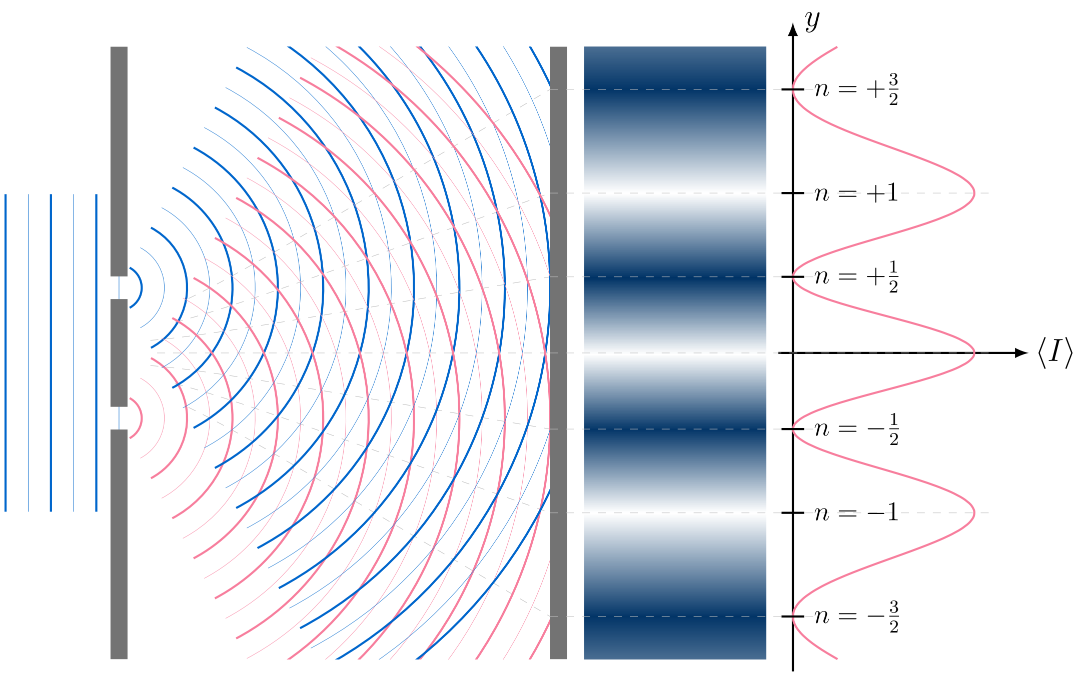
Conditions for Observable Two-Source Fringes

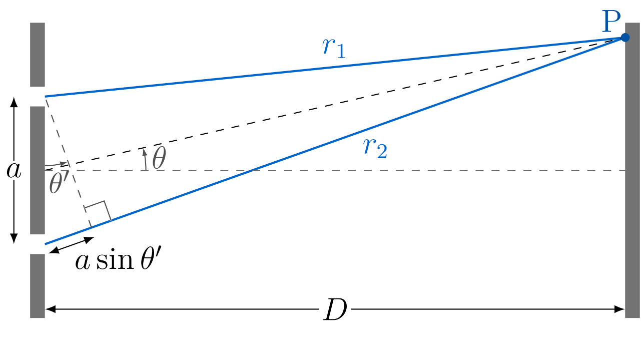
1. Sources must be **coherent** (same frequency, constant phase difference).
2. Sources must have **similar amplitudes** (for clear minima — zero intensity at destructive points).
3. For light: sources must be **monochromatic** or fringes from different wavelengths overlap and blur.
4. The **slit separation** a must be comparable to λ (much larger $\Rightarrow$ fringes too close together to resolve).

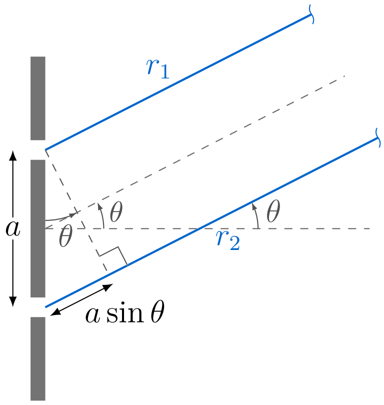
Double-Slit Fringe Spacing: $\lambda = ax/D$

$$\lambda = \frac{ax}{D}$$

- a: slit separation (m) — centre to centre.
- x: fringe spacing (m) — distance between adjacent bright fringes.
- D: distance from slits to screen (m).
- Valid when $D \gg a$ (small-angle approximation).







Derivation of Double Slit formula

- For small angles $r_1 \approx r_2$; $\theta' \approx \theta$
- Path difference $= a\sin\theta$
- For the first maximum: Path difference $= \lambda = a\sin\theta$
- By similar triangles and $\tan\theta \approx \sin\theta$: $\frac{x}{D} = \frac{a\sin\theta}{a}$

$$\lambda = \frac{ax}{D}$$

Common Mistakes with $\lambda = ax/D$

- a is the **slit separation**, not the slit width.
- x is the fringe **spacing** (centre of one fringe to the centre of the next), not the total width of the pattern.
- Increasing D or decreasing a *increases* fringe spacing x — fringes spread out.
- The formula requires $D \gg a$; if this is not satisfied, the small-angle approximation breaks down.

The Diffraction Grating

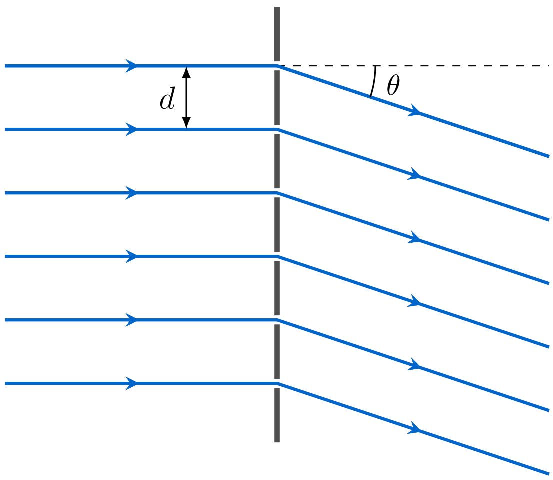
Diffraction Grating

A **diffraction grating** consists of many equally-spaced parallel slits. When light is incident normally, each slit diffracts the light, and the diffracted beams from all slits interfere. Sharp, bright maxima (orders) are produced at specific angles.

Grating Equation

$$d \sin\theta = n\lambda$$

- d: grating spacing — distance between adjacent slits (m). If a grating has N lines per metre, then $d = 1/N$.
- θ: angle of the nth order maximum to the straight-through (zero-order) direction.
- n: order number ($n = 0, \pm1, \pm2, \ldots$).
- λ: wavelength of light (m).
- Maximum possible order: $n_{\text{max}} = \lfloor d/\lambda \rfloor$ (since $\sin\theta \leq 1$).



Using a Diffraction Grating to Measure Wavelength

1. Direct light normally at the grating.
2. Measure the angle θ_n to the nth order maximum using a protractor or spectrometer scale.
3. Apply $d \sin\theta_n = n\lambda$ to find λ.
4. Repeat for several orders and take a mean for greater accuracy.
5. The grating spacing d is found from $d = 1/N$ where N is the number of lines per metre (usually stated as lines per mm on the grating — convert carefully).

Common Mistakes with the Grating Equation

- Lines per mm must be converted: e.g. 300 lines mm^{-1} = 3×10^5 lines m^{-1}, so $d = 1/(3 \times 10^5)$ m.
- $\sin\theta$ cannot exceed 1, so check that your calculated n_{max} is physically possible.
- The grating produces sharper, more widely spaced maxima than double slits — do not confuse the two formulae.

Formula Summary Sheet

Formula	Quantity	Units
$\lambda = 2\times$ (node spacing)	Wavelength from stationary wave	m
$f_n = nv/2L$	nth harmonic in a string	Hz
$\lambda = ax/D$	Double-slit fringe spacing	m
$d \sin\theta = n\lambda$	Diffraction grating	m
$d = 1/N$	Grating spacing from line density	m

Key facts:
Superposition: resultant displacement = vector sum of individual displacements.
Nodes separated by $\lambda/2$; **antinodes** separated by $\lambda/2$.
Constructive: path difference = $n\lambda$; **Destructive**: path difference = $(n + \frac{1}{2})\lambda$.
Coherence: same frequency + constant phase difference.
Diffraction is maximum when gap width $\approx \lambda$.

Worked Examples

Example 1 — Wavelength from a Stationary Wave

Question: A stretched string vibrates in its third harmonic. The distance between the first and last nodes is 72 cm. Find the wavelength.

Solution

The third harmonic has 4 nodes and 3 loops. Distance from first to last node $= 3 \times (\lambda/2)$:

$$72 \text{ cm} = 3 \times \frac{\lambda}{2} \implies \lambda = \frac{2 \times 72}{3} = \mathbf{48} \text{ cm} = 0.48 \text{ m}$$

Example 2 — Double-Slit Interference

Question: In a Young's double-slit experiment, the slit separation is 0.45 mm, the screen is 1.8 m from the slits, and the fringe spacing is 2.4 mm. Calculate the wavelength of light used.

Solution

$$\lambda = \frac{ax}{D} = \frac{(0.45 \times 10^{-3})(2.4 \times 10^{-3})}{1.8} = \frac{1.08 \times 10^{-6}}{1.8} = \mathbf{6.0 \times 10^{-7}} \text{ m} = 600 \text{ nm}$$

Example 3 — Diffraction Grating

Question: A diffraction grating has 400 lines per mm. Light of wavelength 589 nm is incident normally. Find (a) the grating spacing, (b) the angle of the second-order maximum, and (c) the highest order observable.

Solution

(a) $d = \dfrac{1}{400 \times 10^3 \text{ m}^{-1}} = 2.5 \times 10^{-6}$ m

(b) $d \sin\theta = n\lambda \implies \sin\theta = \dfrac{2 \times 589 \times 10^{-9}}{2.5 \times 10^{-6}} = \dfrac{1.178 \times 10^{-6}}{2.5 \times 10^{-6}} = 0.4712$

$\theta = \arcsin(0.4712) = \mathbf{28.1}^\circ$

(c) $n_{\max} = \dfrac{d}{\lambda} = \dfrac{2.5 \times 10^{-6}}{589 \times 10^{-9}} = 4.24 \implies n_{\max} = \mathbf{4}$ (must be a whole number ≤ 4.24)

Example 4 — Path Difference and Interference

Question: Two coherent sources of sound ($\lambda = 0.40$ m) are placed 1.5 m apart. A detector at a point P has path differences of 1.00 m from the two sources. State whether P is a maximum or minimum and explain why.

Solution

Path difference = 1.00 m.
$1.00/\lambda = 1.00/0.40 = 2.5$
Path difference $= 2.5\lambda = (2 + \frac{1}{2})\lambda$ — this is a half-integer multiple of λ.
Therefore P is a **minimum** (destructive interference): the waves arrive in antiphase.

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the principle of superposition. Distinguish between constructive and destructive interference in terms of path difference.

[4 marks]

Q2. Explain what is meant by (a) a node and (b) an antinode in a stationary wave. State the distance between adjacent nodes in terms of wavelength.

[3 marks]

Q3. Explain what is meant by diffraction. State the condition on gap width relative to wavelength for maximum diffraction to occur.

[3 marks]

Q4. Define coherence. State three conditions required for a stable two-source interference pattern to be observed using light.

[4 marks]

Section B — Longer Structured Questions

Q5. A stationary wave is set up on a stretched string of length 0.90 m fixed at both ends. The string vibrates in its second harmonic. The speed of waves on the string is 120 m s^{-1}.

(a) Sketch the stationary wave pattern, labelling all nodes and antinodes.

[2 marks]

(b) Calculate the wavelength and frequency of this harmonic.

[3 marks]

(c) Describe how the stationary wave is formed, referring to the two progressive waves involved.

[3 marks]

Q6. In a Young's double-slit experiment using light of wavelength 540 nm, the slit separation is 0.30 mm.

(a) Calculate the fringe spacing when the screen is 2.0 m from the slits.

[2 marks]

(b) The screen is moved further from the slits. State and explain the effect on the fringe spacing.

[2 marks]

(c) The experiment is repeated with white light. Describe and explain the appearance of the fringe pattern.

[3 marks]

Q7. A diffraction grating with 600 lines per mm is illuminated normally by monochromatic light.

(a) Calculate the grating spacing d.

[1 mark]

(b) The second-order maximum is observed at $\theta = 43.2°$. Calculate the wavelength of the light.

[2 marks]

(c) Show that the third-order maximum cannot be observed for this wavelength and grating.

[2 marks]

(d) Explain why the maxima from a diffraction grating are much sharper than the fringes from a double slit.

[2 marks]

Mark Scheme and Answers

Q1. Resultant displacement at any point equals the vector sum of the individual displacements [1]. Constructive: path difference $= n\lambda$; waves arrive in phase; amplitude/intensity is maximum [2]. Destructive: path difference $= (n + \frac{1}{2})\lambda$; waves arrive in antiphase; amplitude/intensity is minimum (zero if amplitudes equal) [1].

Q2(a). Node: point of zero (minimum) amplitude in a stationary wave; particles permanently at rest [1].
Q2(b). Antinode: point of maximum amplitude; particles oscillate with greatest displacement [1].
Distance between adjacent nodes $= \lambda/2$ [1].

Q3. Diffraction: spreading of a wave as it passes through a gap or around an obstacle [2]. Maximum diffraction when gap width $\approx \lambda$ [1].

Q4. Coherence: two sources with same frequency and constant phase difference [1]. Conditions for light fringes: sources coherent [1]; sources monochromatic [1]; slit separation comparable to λ [1].

Q5(a). Second harmonic: 3 nodes (at both ends + centre), 2 antinodes (at quarter-points) [2].

Q5(b). $\lambda_2 = L = 0.90$ m [1] (for 2nd harmonic, $L = \lambda$); $f = v/\lambda = 120/0.90 = \mathbf{133}$ Hz [2].

Q5(c). Two progressive waves of same frequency and amplitude travel in opposite directions [1]; they superpose according to the principle of superposition [1]; the resultant produces fixed nodes and antinodes — the pattern does not travel [1].

Q6(a). $x = \lambda D/a = (540 \times 10^{-9} \times 2.0)/(0.30 \times 10^{-3}) = 1.08 \times 10^{-6}/3 \times 10^{-4} = \mathbf{3.6 \times 10^{-3}}$ m $= 3.6$ mm [2].

Q6(b). Fringe spacing **increases** [1]; from $x = \lambda D/a$, increasing D increases x directly [1].

Q6(c). Central fringe is white [1]; coloured fringes either side — violet (shortest λ, smallest x) closest to centre, red (longest λ) furthest [1]; fringes overlap at large distances giving white/blurred pattern [1].

Q7(a). $d = 1/(600 \times 10^3) = \mathbf{1.67 \times 10^{-6}}$ m [1].

Q7(b). $\lambda = d\sin\theta/n = (1.67 \times 10^{-6} \times \sin 43.2°)/2 = (1.67 \times 10^{-6} \times 0.684)/2 = \mathbf{5.71 \times 10^{-7}}$ m ≈ 571 nm [2].

Q7(c). For $n = 3$: $\sin\theta = 3\lambda/d = 3 \times 5.71 \times 10^{-7}/1.67 \times 10^{-6} = 1.026 > 1$ [1]; since $\sin\theta$ cannot exceed 1, the third order does not exist [1].

Q7(d). The grating has many slits [1]; for a maximum to form, all slits must constructively interfere simultaneously — any slight deviation from the exact angle causes destructive interference from the large number of slits, producing very sharp peaks [1].

Revision Checklist

	Learning Objective	Confidence (1–3)
☐	State and apply the principle of superposition	
☐	Explain formation of a stationary wave graphically	
☐	Identify nodes and antinodes; state their separation ($\lambda/2$)	
☐	Describe stationary wave experiments: strings, air columns, microwaves	
☐	Determine wavelength from node/antinode positions	
☐	Explain the differences between stationary and progressive waves	
☐	Define diffraction and explain the effect of gap width relative to λ	
☐	Define coherence (same frequency + constant phase difference)	
☐	State conditions required to observe two-source interference fringes	
☐	Use $\lambda = ax/D$ for double-slit interference	
☐	Distinguish constructive and destructive interference by path difference	
☐	Use $d \sin\theta = n\lambda$ for the diffraction grating	
☐	Calculate grating spacing from lines per mm	
☐	Find the maximum order observable ($\sin\theta \leq 1$)	
☐	Describe how a grating is used to measure wavelength	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Superposition is the single idea that unifies this whole topic. Stationary waves, interference fringes and grating maxima are all just superposition playing out in different geometries. Get the path difference conditions clear in your mind and the rest follows naturally.