

Topic 7

Waves

Revision Booklet

This booklet covers:

- Progressive Waves and Wave Properties
- The Wave Equation and Intensity
- Transverse and Longitudinal Waves
- The Doppler Effect
- The Electromagnetic Spectrum
- Polarisation and Malus's Law

Progressive Waves

Wave Motion

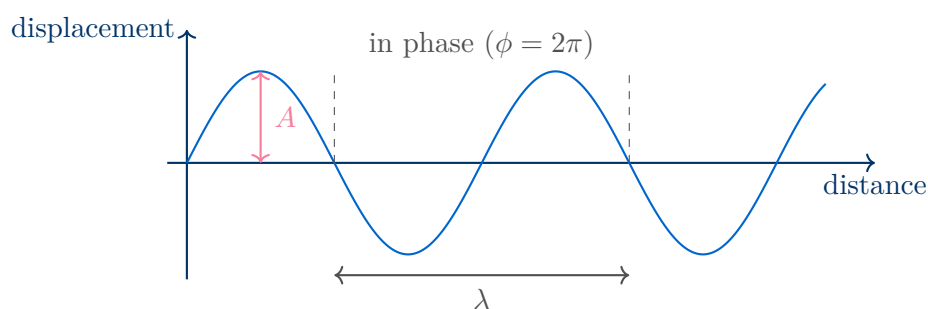
A **progressive wave** transfers energy from one place to another by means of oscillations, without any net transfer of matter. The particles of the medium vibrate about their equilibrium positions.

Examples: waves in ropes, springs, ripple tanks, sound waves, electromagnetic waves.

Key Wave Terms

Displacement (x)	Distance of a particle from its equilibrium position at a given instant; can be positive or negative. Unit: m.
Amplitude (A)	Maximum displacement from equilibrium. Unit: m.
Period (T)	Time for one complete oscillation. Unit: s.
Frequency (f)	Number of complete oscillations per unit time; $f = 1/T$. Unit: Hz.
Wavelength (λ)	Distance between two adjacent points in phase (e.g. crest to crest). Unit: m.
Wave speed (v)	Speed at which the wave profile travels through the medium. Unit: m s^{-1} .
Phase difference (ϕ)	Difference in the stage of oscillation between two points. Unit: radians or degrees.

Displacement–distance graph



Phase Difference

Two points separated by a distance d along the direction of travel of a wave of wavelength λ have a phase difference:

$$\phi = \frac{2\pi d}{\lambda} \quad (\text{in radians}) \quad \text{or} \quad \phi = \frac{360^\circ \cdot d}{\lambda}$$

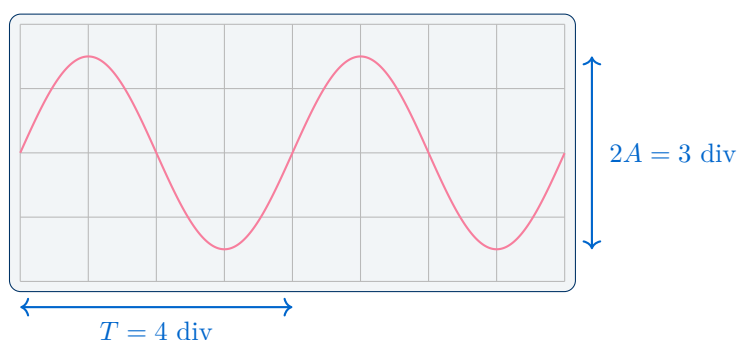
- Points separated by λ (or $n\lambda$): $\phi = 2\pi$ — **in phase**.
- Points separated by $\lambda/2$ (or $(2n - 1)\lambda/2$): $\phi = \pi$ — **in antiphase**.

CRO, Wave Equation and Intensity

Using a Cathode-Ray Oscilloscope (CRO)

A CRO displays voltage against time. Two controls are used:

- **Time-base** (s div^{-1}): sets the time represented by each horizontal division. Period $T = (\text{number of divisions per cycle}) \times (\text{time-base setting})$. Then $f = 1/T$.
- **y-gain** (V div^{-1}): sets the voltage per vertical division. Amplitude = (number of divisions from centre to peak) \times (*y*-gain setting).



The Wave Equation

Derivation: In one period T , the wave travels a distance of one wavelength λ . Speed = distance/time:

$$v = \frac{\lambda}{T} = f\lambda \quad \text{since } f = \frac{1}{T}$$

$$v = f\lambda$$

Intensity

The **intensity** of a wave is the power transmitted per unit area perpendicular to the direction of propagation:

$$I = \frac{P}{A} \quad \text{Unit: } \text{W m}^{-2}$$

For a progressive wave, intensity is proportional to the square of the amplitude:

$$I \propto (\text{amplitude})^2$$

So doubling the amplitude quadruples the intensity.

Common Mistake

$I \propto A^2$ means intensity is proportional to amplitude *squared*. Halving the amplitude reduces intensity to one quarter — not one half. This relationship holds for all types of progressive wave.

Transverse and Longitudinal Waves

Transverse Waves

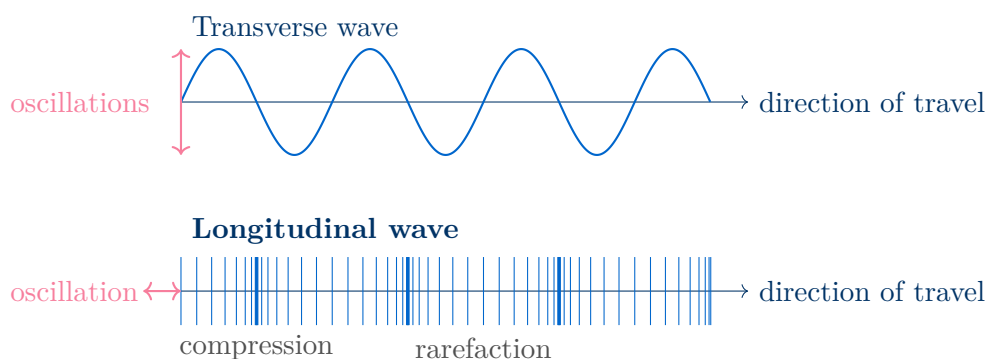
In a **transverse wave**, the oscillations of particles are **perpendicular** to the direction of energy transfer (wave propagation).

Examples: waves on a rope, water ripples, all electromagnetic waves.

Longitudinal Waves

In a **longitudinal wave**, the oscillations of particles are **parallel** to the direction of energy transfer (wave propagation). The wave consists of alternating **compressions** (regions of high pressure/density) and **rarefactions** (regions of low pressure/density).

Examples: sound waves, compression waves in a spring.



	Transverse	Longitudinal
Oscillation direction	Perpendicular to wave travel	Parallel to wave travel
Can be polarised?	Yes	No
Examples	EM waves, rope waves	Sound, spring compressions
Graphical representation	Sinusoidal displacement–distance graph	Displacement–distance graph (same shape, but displacement is along wave direction)

The Doppler Effect

Doppler Effect

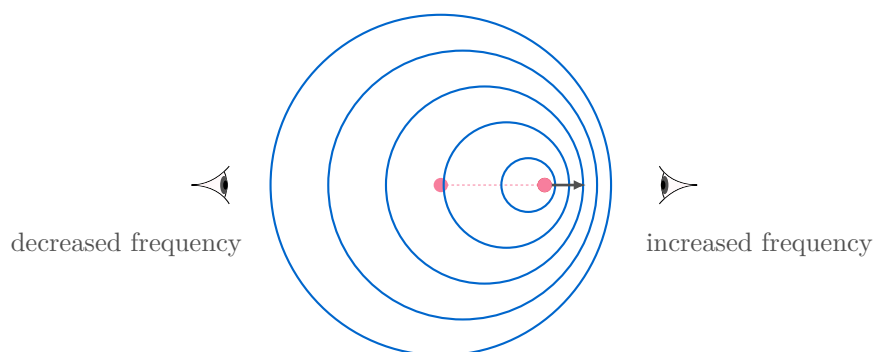
The **Doppler effect** is the change in observed frequency of a wave when the source moves relative to a stationary observer. When the source moves **towards** the observer, the observed frequency is **higher** than the source frequency; when it moves **away**, the observed frequency is **lower**.

Doppler Formula for a Moving Source

$$f_o = \frac{f_s v}{v \pm v_s}$$

- f_o : observed frequency (Hz).
- f_s : source frequency (Hz).
- v : speed of sound in the medium (m s^{-1}).
- v_s : speed of the source (m s^{-1}).
- Use $-$ when the source moves **towards** the observer ($f_o > f_s$).
- Use $+$ when the source moves **away** from the observer ($f_o < f_s$).

*Note: this formula applies to a **moving source** and stationary observer only. The Doppler effect for a stationary source and moving observer is not required.*



Sign Convention

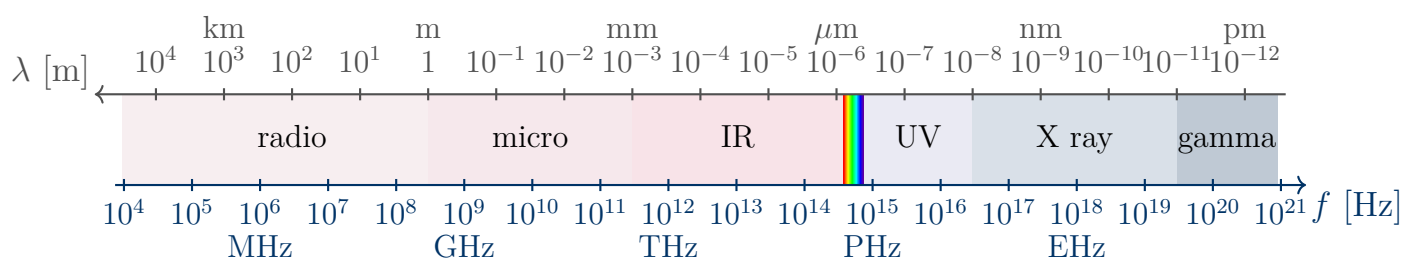
The \pm in $v \pm v_s$ is a common source of error. A useful check: if the source moves towards the observer, the denominator must be *smaller* (so use $-$), making f_o *larger*. If the source moves away, the denominator is *larger* (use $+$), making f_o *smaller*. Always sanity-check your answer against this logic.

The Electromagnetic Spectrum

Properties of Electromagnetic Waves

All electromagnetic (EM) waves:

- Are **transverse** waves.
- Travel at the **same speed** in free space (vacuum): $c = 3.0 \times 10^8 \text{ m s}^{-1}$.
- Require **no medium** — they can travel through a vacuum.
- Obey $v = f\lambda$ (with $v = c$ in free space).



Region	Approx. wavelength (m)	Approx. frequency (Hz)
Radio waves	$> 10^{-1}$	$< 3 \times 10^9$
Microwaves	10^{-3} to 10^{-1}	3×10^9 to 3×10^{11}
Infrared	7×10^{-7} to 10^{-3}	3×10^{11} to 4×10^{14}
Visible	4×10^{-7} to 7×10^{-7}	4×10^{14} to 7.5×10^{14}
Ultraviolet	10^{-8} to 4×10^{-7}	7.5×10^{14} to 3×10^{16}
X-rays	10^{-13} to 10^{-8}	3×10^{16} to 3×10^{21}
Gamma rays	$< 10^{-13}$	$> 3 \times 10^{21}$

Visible Light

The visible spectrum occupies wavelengths from approximately **400 nm** (violet) to **700 nm** (red) in free space.

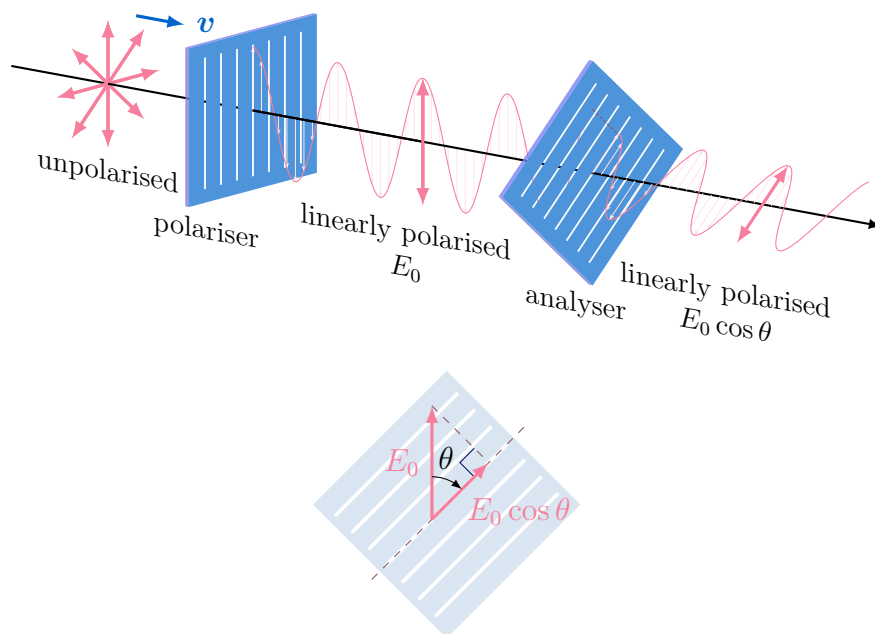
$$400 \text{ nm} = 4 \times 10^{-7} \text{ m} \quad 700 \text{ nm} = 7 \times 10^{-7} \text{ m}$$

Polarisation and Malus's Law

Polarisation

Polarisation is a phenomenon associated with **transverse waves only**. An unpolarised transverse wave has oscillations in all planes perpendicular to the direction of travel. A **plane-polarised** wave has oscillations confined to a single plane.

Longitudinal waves *cannot* be polarised — their oscillations are already along one axis (the direction of travel).



Malus's Law

When a plane-polarised electromagnetic wave of intensity I_0 passes through a polarising filter at angle θ to the plane of polarisation:

$$I = I_0 \cos^2 \theta$$

- $\theta = 0^\circ$ (filter aligned with polarisation): $I = I_0$ — maximum transmission.
- $\theta = 90^\circ$ (filter perpendicular): $I = 0$ — complete extinction.
- $\theta = 45^\circ$: $I = \frac{1}{2}I_0$ — half the intensity transmitted.
- **Note:** the effect of a polarising filter on an *unpolarised* wave is not required (but for reference, it halves the intensity).

Common Mistake

Malus's law ($I = I_0 \cos^2 \theta$) only applies to **already plane-polarised** light incident on a second filter. It does not apply to the first polarising filter acting on unpolarised light. Also remember θ is measured from the *transmission axis* of the filter to the plane of polarisation of the incoming wave — not between the two filters themselves unless the incoming wave was polarised along the first filter's axis.

Formula Summary Sheet

Formula	Quantity	Units
$v = f\lambda$	Wave equation	m s^{-1}
$f = 1/T$	Frequency–period	Hz
$\phi = 2\pi d/\lambda$	Phase difference	rad
$I = P/A$	Intensity	W m^{-2}
$I \propto (\text{amplitude})^2$	Intensity–amplitude	—
$f_o = f_s v / (v \pm v_s)$	Doppler effect	Hz
$I = I_0 \cos^2 \theta$	Malus's law	W m^{-2}

Key facts to recall:

All EM waves travel at $c = 3.0 \times 10^8 \text{ m s}^{-1}$ in free space and are transverse.

Visible light: 400 nm (violet) to 700 nm (red).

Only **transverse** waves can be polarised.

Doppler: source approaching \Rightarrow use $-$ in denominator; receding \Rightarrow use $+$.

Worked Examples

Example 1 — Wave Properties from a CRO

Question: A CRO displays a wave with a period of 2.5 divisions. The time-base is set to 4.0 ms div^{-1} and the y -gain to 0.50 V div^{-1} . The wave has a peak-to-peak height of 3.0 divisions. Find (a) the frequency and (b) the amplitude.

Solution

(a) $T = 2.5 \times 4.0 \times 10^{-3} = 1.0 \times 10^{-2} \text{ s}$; $f = 1/T = 1/(1.0 \times 10^{-2}) = \mathbf{100 \text{ Hz}}$

(b) Peak-to-peak = 3.0 div, so amplitude = 1.5 div. $A = 1.5 \times 0.50 = \mathbf{0.75 \text{ V}}$

Example 2 — Intensity and Amplitude

Question: A loudspeaker produces a sound wave of amplitude $2.4 \times 10^{-3} \text{ m}$ and intensity 1.8 W m^{-2} . The amplitude is increased to $6.0 \times 10^{-3} \text{ m}$. Calculate the new intensity.

Solution

Since $I \propto A^2$:

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{6.0 \times 10^{-3}}{2.4 \times 10^{-3}}\right)^2 = (2.5)^2 = 6.25$$

$$I_2 = 6.25 \times 1.8 = \mathbf{11.3 \text{ W m}^{-2}}$$

Example 3 — Doppler Effect

Question: An ambulance siren emits sound at $f_s = 850 \text{ Hz}$. The ambulance approaches a stationary observer at 22 m s^{-1} . The speed of sound is 340 m s^{-1} . Calculate the frequency heard by the observer (a) as the ambulance approaches and (b) as it recedes.

Solution

(a) Approaching (use $-$):

$$f_o = \frac{f_s v}{v - v_s} = \frac{850 \times 340}{340 - 22} = \frac{289\,000}{318} = \mathbf{909 \text{ Hz}}$$

(b) Receding (use $+$):

$$f_o = \frac{f_s v}{v + v_s} = \frac{850 \times 340}{340 + 22} = \frac{289\,000}{362} = \mathbf{798 \text{ Hz}}$$

Example 4 — Malus's Law

Question: Plane-polarised light of intensity 24 W m^{-2} is incident on a polarising filter. The filter's transmission axis makes an angle of 35° with the plane of polarisation. Calculate the transmitted intensity.

Solution

$$I = I_0 \cos^2 \theta = 24 \times \cos^2(35^\circ) = 24 \times (0.819)^2 = 24 \times 0.671 = \mathbf{16.1 \text{ W m}^{-2}}$$

Practice Exam Questions**Section A — Short Answer Questions**

Q1. Define the terms (a) amplitude, (b) frequency, (c) wavelength and (d) phase difference.

[4 marks]

Q2. A water wave has frequency 3.5 Hz and wavelength 0.24 m. Calculate its speed. Two points on the wave are separated by 0.18 m in the direction of travel. Calculate the phase difference between them.

[4 marks]

Q3. Distinguish between transverse and longitudinal waves. Give one example of each. Explain why polarisation is evidence that light is a transverse wave.

[4 marks]

Q4. State three properties common to all electromagnetic waves. Sketch the electromagnetic spectrum, labelling each region and giving an approximate wavelength range for visible light.

[5 marks]

Section B — Longer Structured Questions

Q5. A source of sound has frequency 640 Hz and moves towards a stationary observer at 15 m s^{-1} . The speed of sound in air is 340 m s^{-1} .

- (a) Calculate the frequency heard by the observer.

[2 marks]

- (b) The source then moves away from the observer at the same speed. Calculate the new observed frequency and the change in observed frequency between the two situations.

[3 marks]

- (c) Explain in terms of wavefronts why the observed frequency is higher when the source approaches.

[2 marks]

Q6. A plane-polarised beam of light of intensity I_0 is incident on a polarising filter.

- (a) The filter's transmission axis is at 50° to the plane of polarisation. Calculate the transmitted intensity as a fraction of I_0 .

[2 marks]

- (b) The filter is rotated to 90° . State the transmitted intensity and explain your answer.

[2 marks]

- (c) The intensity of the incident beam is now doubled to $2I_0$ and the filter is returned to 50° . Calculate the new transmitted intensity.

[1 mark]

Q7. A loudspeaker emits sound of frequency 500 Hz and produces an intensity of $3.2 \times 10^{-3} \text{ W m}^{-2}$ at a distance of 4.0 m.

- (a) The amplitude of the sound wave at this distance is A_1 . The speaker power is increased so that the amplitude doubles to $2A_1$. Calculate the new intensity.

[2 marks]

- (b) Calculate the wavelength of the sound wave. (Speed of sound = 340 m s^{-1} .)

[2 marks]

- (c) Two points in the sound wave are 0.51 m apart along the direction of travel. Calculate the phase difference between them.

[2 marks]

Mark Scheme and Answers

Q1(a). Maximum displacement of a particle from its equilibrium position [1].

Q1(b). Number of complete oscillations per unit time [1].

Q1(c). Distance between two adjacent points that are in phase [1].

Q1(d). Difference in the stage of oscillation between two points on a wave [1].

Q2. $v = f\lambda = 3.5 \times 0.24 = \mathbf{0.84} \text{ m s}^{-1}$ [2]. Phase difference: $\phi = 2\pi d/\lambda = 2\pi \times 0.18/0.24 = 2\pi \times 0.75 = \mathbf{1.5\pi} \text{ rad } (= 270^\circ)$ [2].

Q3. Transverse: oscillations perpendicular to direction of travel [1]; e.g. EM waves / rope [1]. Longitudinal: oscillations parallel to direction of travel [1]; e.g. sound [1]. Light can be polarised \Rightarrow oscillations must be perpendicular to direction of travel \Rightarrow transverse [bonus/mark scheme dependent on question wording].

Q4. Any three of: all transverse; travel at $c = 3 \times 10^8 \text{ m s}^{-1}$ in free space; require no medium; obey $v = f\lambda$ [3]. Spectrum with correct order (radio $\rightarrow \gamma$) [1]; visible 400–700 nm [1].

Q5(a). $f_o = 640 \times 340/(340 - 15) = 217\,600/325 = \mathbf{669} \text{ Hz}$ [2].

Q5(b). $f_o = 640 \times 340/(340 + 15) = 217\,600/355 = \mathbf{613} \text{ Hz}$ [2]; change = $669 - 613 = \mathbf{56} \text{ Hz}$ [1].

Q5(c). As the source approaches, successive wavefronts are emitted closer together [1]; the observer encounters more wavefronts per second, so the observed frequency is higher [1].

Q6(a). $I = I_0 \cos^2(50^\circ) = I_0 \times (0.643)^2 = \mathbf{0.413} \text{ } I_0$ [2].

Q6(b). $I = I_0 \cos^2(90^\circ) = \mathbf{0}$ [1]; the transmission axis is perpendicular to the plane of polarisation so no light passes through [1].

Q6(c). $I = 2I_0 \cos^2(50^\circ) = 2 \times 0.413 I_0 = \mathbf{0.826} \text{ } I_0$ [1].

Q7(a). $I \propto A^2$; amplitude doubles so I increases by factor 4: $I_2 = 4 \times 3.2 \times 10^{-3} = \mathbf{1.28 \times 10^{-2} \text{ W m}^{-2}}$ [2].

Q7(b). $\lambda = v/f = 340/500 = \mathbf{0.68} \text{ m}$ [2].

Q7(c). $\phi = 2\pi d/\lambda = 2\pi \times 0.51/0.68 = 2\pi \times 0.75 = \mathbf{1.5\pi} \text{ rad } (270^\circ)$ [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Define displacement, amplitude, period, frequency, wavelength, phase difference	
<input type="checkbox"/> Use the CRO to determine frequency and amplitude	
<input type="checkbox"/> Derive and use $v = f\lambda$	
<input type="checkbox"/> Use $I = P/A$ and $I \propto (\text{amplitude})^2$	
<input type="checkbox"/> Distinguish transverse and longitudinal waves with examples	
<input type="checkbox"/> Sketch and interpret displacement–distance graphs for both wave types	
<input type="checkbox"/> Understand the Doppler effect for a moving source	
<input type="checkbox"/> Use $f_o = f_s v / (v \pm v_s)$ with correct sign convention	
<input type="checkbox"/> State properties common to all EM waves	
<input type="checkbox"/> Recall the EM spectrum regions and approximate wavelength ranges	
<input type="checkbox"/> Recall visible light wavelength range (400–700 nm)	
<input type="checkbox"/> Explain polarisation and why it is evidence for transverse waves	
<input type="checkbox"/> Apply Malus's law $I = I_0 \cos^2 \theta$ to plane-polarised light	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Waves is one of those topics where a clear mental picture goes a long way. Sketch the wave, label the amplitude and wavelength, decide whether oscillations are parallel or perpendicular to travel — and most of the topic falls into place from there.