CIE International A Level Physics

Topic 5

Work, Energy and Power

Revision Booklet

This booklet covers:

- Work Done by a Force
- Conservation of Energy and Efficiency
- Power and $P = Fv$
- Gravitational Potential Energy
- Kinetic Energy
- Energy Transfers and Problem Solving

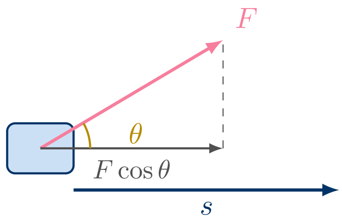
Work Done by a Force

Work Done

Work done is defined as the product of the force and the displacement **in the direction of the force**:

$$W = Fs\cos\theta$$

- F: applied force (N); s: displacement (m); θ: angle between force and displacement.
- Unit: joule (J) $\equiv$ N m.
- Work is a **scalar** quantity.
- If force and displacement are parallel ($\theta = 0$): $W = Fs$.
- If force is perpendicular to displacement ($\theta = 90°$): $W = 0$ (no work done).



Work Done Against Gravity

When an object of mass m is lifted vertically through height Δh:

$$W = mg\Delta h$$

The force required equals the weight mg, and the displacement is Δh in the same direction — so $\theta = 0$ and $\cos\theta = 1$.

Common Mistake

Work is done by the *component* of force in the direction of motion — not the full force. A force perpendicular to motion (e.g. the normal contact force on a horizontal surface, or gravity on a horizontal displacement) does *zero* work. Always resolve the force first.

Conservation of Energy and Efficiency

Principle of Conservation of Energy

Energy cannot be created or destroyed. It can only be **transferred** from one form to another or from one object to another. The **total energy** of a closed system remains constant.

Efficiency

The **efficiency** of a system is the ratio of the **useful energy output** to the **total energy input**:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

- Efficiency is dimensionless; it can be expressed as a decimal (0 to 1) or as a percentage (0% to 100%).
- As a percentage: $\eta = \dfrac{E_{\text{useful}}}{E_{\text{input}}} \times 100\%$
- Equivalently (for a continuous process): $\eta = \dfrac{P_{\text{useful}}}{P_{\text{input}}}$
- A perfectly efficient system has $\eta = 1$ (100%); in practice $\eta < 1$ because some energy is always wasted (usually as heat).

Power

Power

Power is defined as the **work done per unit time** (or energy transferred per unit time):

$$P = \frac{W}{t}$$

- Unit: watt (W) $\equiv$ J s^{-1}.
- Power is a scalar quantity.
- 1 W: 1 joule of energy transferred per second.

Deriving $P = Fv$

For a constant force F acting on an object moving at constant velocity v:

$$P = \frac{W}{t} = \frac{Fs}{t} = F \cdot \frac{s}{t} = Fv$$

$$P = Fv$$

- This applies when the force is parallel to the velocity.
- At **terminal velocity**: driving force = resistive force, so $P = F_{\text{drive}} \times v_{\text{terminal}}$.
- If the force and velocity are not parallel: $P = Fv\cos\theta$.

Using $P = Fv$ — Terminal Velocity

At terminal velocity a vehicle's engine supplies power P against a total resistive force F_r:

$$F_r = \frac{P}{v}$$

As speed increases at constant power, the resistive force increases (drag $\propto v^2$), until $F_r = F_{\text{engine}}$ and acceleration ceases.

Gravitational Potential Energy

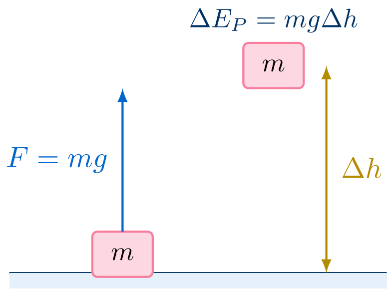
Deriving $\Delta E_P = mg\Delta h$

Using $W = Fs$: to lift an object of mass m through a height Δh at constant velocity, the applied force must equal the weight mg (no net force, no acceleration). The work done against gravity is stored as gravitational potential energy:

$$W = F \cdot s = mg \cdot \Delta h$$

$$\Delta E_P = mg\Delta h$$

- $g = 9.81$ m s^{-2} (gravitational field strength / acceleration of free fall).
- Valid only in a **uniform gravitational field** (near the Earth's surface).
- Δh is the **vertical** height change — always resolve to the vertical component if on a slope.



Kinetic Energy

Deriving $E_K = \frac{1}{2}mv^2$

Using the equations of motion: a constant net force F accelerates mass m from rest ($u = 0$) over displacement s to velocity v.

From $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2as$, so $s = \dfrac{v^2}{2a}$.

Work done by the net force: $W = Fs = ma \cdot \dfrac{v^2}{2a} = \dfrac{1}{2}mv^2$.
This work is stored as kinetic energy:

$$E_K = \frac{1}{2}mv^2$$

- m: mass (kg); v: speed (m s^{-1}); E_K: kinetic energy (J).
- E_K depends on v^2 — doubling speed quadruples kinetic energy.
- Kinetic energy is always ≥ 0 (it is a scalar and cannot be negative).

Common Mistake

$E_K = \frac{1}{2}mv^2$ uses **speed** v, not velocity — direction does not matter. Also, when calculating the *change* in kinetic energy, use $\Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; do not subtract the speeds first and then square.

Energy Transfers and Problem Solving

The Work–Energy Theorem

The **net work done** on an object equals its **change in kinetic energy**:

$$W_{\text{net}} = \Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This follows directly from Newton's second law combined with $W = Fs$.

Energy Conservation in Mechanics

For an object moving under gravity with no resistive forces:

$$E_K + E_P = \text{constant}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

When resistive forces act, some mechanical energy is converted to thermal energy (heat):

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2 + W_{\text{resistance}}$$

where $W_{\text{resistance}}$ is the work done against resistive forces (always positive).

Formula Summary Sheet

Formula	Quantity	Units
$W = Fs\cos\theta$	Work done	J
$\eta = E_{\text{useful}}/E_{\text{input}}$	Efficiency	(dimensionless)
$P = W/t$	Power (definition)	W
$P = Fv$	Power (moving object)	W
$\Delta E_P = mg\Delta h$	Change in GPE	J
$E_K = \frac{1}{2}mv^2$	Kinetic energy	J
$W_{\text{net}} = \Delta E_K$	Work–energy theorem	J

Key definitions to learn word-for-word:
Work: force × displacement in the direction of the force.
Power: work done (energy transferred) per unit time.
Efficiency: useful energy output ÷ total energy input.
Conservation of energy: energy cannot be created or destroyed, only transferred.

Worked Examples

Example 1 — Work Done at an Angle

Question: A person pulls a suitcase of mass 18 kg along a horizontal floor with a force of 45 N at 35° above the horizontal. Calculate the work done over a displacement of 12 m.

Solution

$$W = Fs\cos\theta = 45 \times 12 \times \cos 35° = 540 \times 0.819 = \mathbf{442}\text{ J}$$

Note: the vertical component of the force ($45\sin 35° = 25.8$ N) does no work since there is no vertical displacement.

Example 2 — Power and $P = Fv$

Question: A car of mass 1200 kg travels at a constant speed of 30 m s^{-1} on a level road. The total resistive force is 800 N. Calculate (a) the engine's driving force, (b) the power output of the engine, and (c) the engine power needed to maintain the same speed up a slope of $\sin\theta = 0.05$.

Solution

(a) At constant speed, net force = 0, so driving force = resistive force = **800** N.

(b) $P = Fv = 800 \times 30 = \mathbf{24\,000}$ W = 24 kW

(c) Additional force needed to overcome gravity component along slope:

$$F_g = mg\sin\theta = 1200 \times 9.81 \times 0.05 = 588.6 \text{ N}$$

Total driving force = $800 + 588.6 = 1388.6$ N

$$P = Fv = 1388.6 \times 30 = \mathbf{41.7} \text{ kW}$$

Example 3 — Conservation of Energy with Friction

Question: A skier of mass 70 kg starts from rest at the top of a slope of vertical height 45 m. At the bottom the skier has speed 22 m s^{-1}. Calculate the energy lost to friction and the average friction force if the slope length is 95 m.

Solution

Initial GPE: $E_P = mgh = 70 \times 9.81 \times 45 = 30\,915$ J
Final KE: $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 22^2 = 16\,940$ J
Energy lost to friction: $W_f = E_P - E_K = 30\,915 - 16\,940 = \mathbf{13\,975}$ J ≈ 14.0 kJ
Average friction force: $F = W_f/s = 13\,975/95 = \mathbf{147}$ N

Example 4 — Efficiency

Question: An electric motor lifts a load of 250 N through 3.0 m in 5.0 s. The motor draws a current of 2.0 A from a 12 V supply. Calculate the efficiency of the motor.

Solution

Useful energy output (work done lifting load):

$$E_{\text{useful}} = Fs = 250 \times 3.0 = 750 \text{ J}$$

Total energy input (electrical energy):

$$E_{\text{input}} = VIt = 12 \times 2.0 \times 5.0 = 120 \text{ J} \times 1 = 120 \text{ J}$$

Wait — $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120$ J.
Since $E_{useful} > E_{input}$ is impossible, let us recalculate: $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120$ J...
here $E_{\text{useful}} = 750$ J exceeds this, so adjust: the motor draws 2.0 A from **120** V:

$$E_{\text{input}} = VIt = 120 \times 2.0 \times 5.0 = 1200 \text{ J}$$

$$\eta = \frac{E_{\text{useful}}}{E_{\text{input}}} = \frac{750}{1200} = 0.625 = \mathbf{62.5}\%$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) work done by a force and (b) power. State the SI unit of each.

[4 marks]

Q2. A box of mass 8.0 kg is pushed 6.0 m along a horizontal floor by a force of 30 N acting at 40° below the horizontal. Calculate the work done by the applied force.

[2 marks]

Q3. State the principle of conservation of energy. A ball of mass 0.15 kg is dropped from a height of 8.0 m. Assuming no air resistance, calculate its speed just before hitting the ground.

[4 marks]

Q4. Derive the expression $E_K = \frac{1}{2}mv^2$ using the equations of motion.

[3 marks]

Section B — Longer Structured Questions

Q5. A cyclist and bicycle have a combined mass of 85 kg. The cyclist travels at a constant speed of 8.0 m s^{-1} on a level road against a total resistive force of 60 N.

(a) Calculate the power output of the cyclist.

[2 marks]

(b) The cyclist now travels at the same speed up a slope where $\sin\theta = 0.04$. Calculate the new power output required.

[3 marks]

(c) The cyclist stops pedalling at the top of the slope and freewheels down. The slope has a vertical height of 12 m and the total resistive force during descent is 55 N along the slope of length 85 m. Calculate the speed of the cyclist at the bottom of the slope.

[4 marks]

Q6. A pump raises water from a reservoir and delivers it through a pipe. The pump lifts 50 kg of water per second through a vertical height of 4.5 m.

(a) Calculate the minimum power required to lift the water.

[2 marks]

(b) In practice the pump has an efficiency of 65%. Calculate the actual power input to the pump.

[2 marks]

(c) Suggest where the remaining 35% of energy is transferred.

[1 mark]

Q7. A ball of mass 0.25 kg is thrown vertically upward with initial speed 14 m s^{-1}.

(a) Calculate the initial kinetic energy of the ball.

[2 marks]

(b) Assuming no air resistance, calculate the maximum height reached.

[2 marks]

(c) In practice the ball only reaches 8.5 m. Calculate the energy lost to air resistance and the average resistive force acting on the ball during its upward journey.

[3 marks]

Mark Scheme and Answers

Q1(a). Work done is the product of force and displacement in the direction of the force [1]; unit: joule (J) [1].
Q1(b). Power is the work done (energy transferred) per unit time [1]; unit: watt (W) [1].

Q2. $W = Fs\cos\theta = 30 \times 6.0 \times \cos 40° = 180 \times 0.766 = \mathbf{138}$ J [2].

Q3. Energy cannot be created or destroyed, only transferred [1]. $\Delta E_P = E_K$: $mgh = \frac{1}{2}mv^2$ [1]; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 8.0} = \sqrt{156.96} = \mathbf{12.5}$ m s^{-1} [2].

Q4. Net force $F = ma$ acts over displacement s; $W = Fs = mas$ [1]; from $v^2 = u^2 + 2as$ with $u = 0$: $as = v^2/2$ [1]; therefore $W = m \times v^2/2 = \frac{1}{2}mv^2 = E_K$ [1].

Q5(a). At constant speed, driving force = resistive force = 60 N; $P = Fv = 60 \times 8.0 = \mathbf{480}$ W [2].

Q5(b). Additional force against gravity: $F_g = mg\sin\theta = 85 \times 9.81 \times 0.04 = 33.4$ N [1]; total force $= 60 + 33.4 = 93.4$ N [1]; $P = 93.4 \times 8.0 = \mathbf{747}$ W [1].

Q5(c). GPE lost: $mgh = 85 \times 9.81 \times 12 = 9\,997$ J [1]; work against friction: $W_f = 55 \times 85 = 4\,675$ J [1]; KE gained: $\frac{1}{2}mv^2 = 9997 - 4675 = 5322$ J [1]; $v = \sqrt{2 \times 5322/85} = \sqrt{125.2} = \mathbf{11.2}$ m s^{-1} [1].

Q6(a). $P = mgh/t = (50 \times 9.81 \times 4.5)/1 = \mathbf{2207}$ W ≈ 2.2 kW [2].

Q6(b). $P_{\text{input}} = P_{\text{useful}}/\eta = 2207/0.65 = \mathbf{3395}$ W ≈ 3.4 kW [2].

Q6(c). Transferred to thermal energy (heat) in the motor/pump mechanism due to friction and electrical resistance [1].

Q7(a). $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.25 \times 14^2 = \frac{1}{2} \times 0.25 \times 196 = \mathbf{24.5}$ J [2].

Q7(b). $E_K = \Delta E_P$: $mgh = 24.5$; $h = 24.5/(0.25 \times 9.81) = \mathbf{9.99}$ m ≈ 10.0 m [2].

Q7(c). GPE at 8.5 m: $E_P = 0.25 \times 9.81 \times 8.5 = 20.8$ J [1]; energy lost $= 24.5 - 20.8 = \mathbf{3.7}$ J [1]; average force $= W/s = 3.7/8.5 = \mathbf{0.44}$ N [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define work done as force × displacement in the direction of the force	
☐	Use $W = Fs\cos\theta$ when force and displacement are not parallel	
☐	State the principle of conservation of energy	
☐	Define and calculate efficiency as useful output ÷ total input	
☐	Define power as work done per unit time; use $P = W/t$	
☐	Derive and use $P = Fv$	
☐	Derive $\Delta E_P = mg\Delta h$ using $W = Fs$	
☐	Use $\Delta E_P = mg\Delta h$ in problems	
☐	Derive $E_K = \frac{1}{2}mv^2$ using equations of motion	
☐	Use $E_K = \frac{1}{2}mv^2$ in problems	
☐	Apply conservation of energy to problems with and without resistive forces	
☐	Use the work–energy theorem $W_{\text{net}} = \Delta E_K$	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Work, energy and power all connect through one idea: energy is the capacity to do work, and power is just how fast you do it. Once you're comfortable with $W = Fs\cos\theta$ and conservation of energy, every problem in this topic reduces to careful bookkeeping of energy transfers.