CIE International A Level Physics

Topic 4

Forces, Density and Pressure

Revision Booklet

This booklet covers:

- Turning Effects of Forces
- Equilibrium of Forces
- Density and Pressure
- Hydrostatic Pressure
- Upthrust and Archimedes' Principle

Turning Effects of Forces

Centre of Gravity

The **centre of gravity** of an object is the single point at which the entire weight of the object may be considered to act.

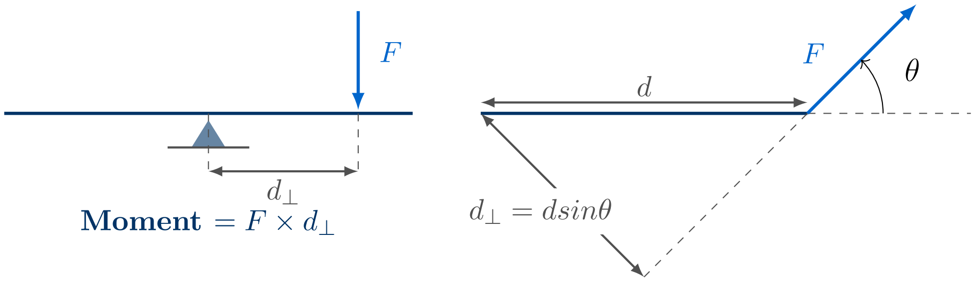
- For a uniform, regular object the centre of gravity lies at the geometric centre.
- For irregular objects it can be found experimentally by suspension.
- All gravitational calculations treat the object as a **point mass** at this location.

Moment of a Force

The **moment** of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

$$\text{moment} = F \times d_{\perp} \qquad \text{units: N m}$$

where $d_{\perp}$ is the **perpendicular** distance from the pivot to the line of action.



Common Mistake

Always use the **perpendicular** distance from the pivot to the *line of action* of the force — not the distance along the beam or object. If the force is at an angle θ, the moment is $F \sin\theta \times d$ or equivalently $F \times d \sin\theta$.

Couples and Torque

Couple

A **couple** is a pair of forces that:

- are **equal in magnitude**,
- act in **opposite directions**,
- have **parallel but different lines of action**.

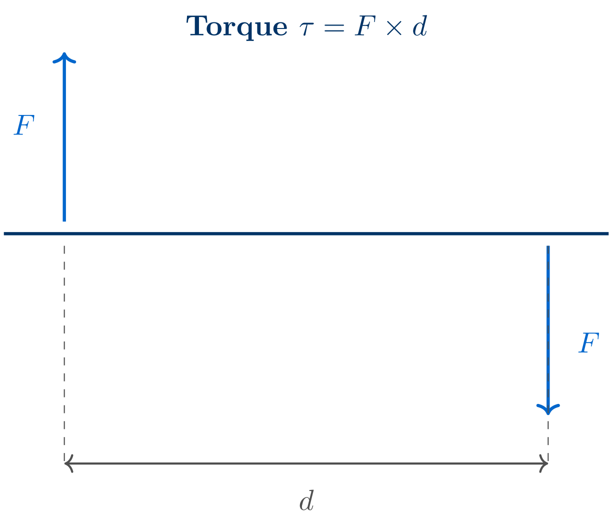
A couple produces **rotation only** — it has zero resultant force, so it produces no translational acceleration.

Torque of a Couple

$$\tau = F \times d$$

τ = torque of the couple (N m)
F = magnitude of one of the forces (N)
d = perpendicular distance between the lines of action of the forces (m)

Note: unlike the moment of a single force, the torque of a couple is the **same about any point**.



Equilibrium of Forces

Conditions for Equilibrium

A body is in **equilibrium** when:

1. The **resultant force** is zero (no translational acceleration).
2. The **resultant torque** about any point is zero (no rotational acceleration).

Both conditions must be satisfied simultaneously.

Principle of Moments

For a body in equilibrium:

$$\sum \text{clockwise moments} = \sum \text{anticlockwise moments}$$

(about any chosen pivot point)

Using the Principle of Moments

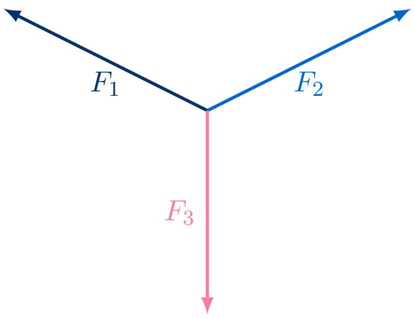
- Choose a **convenient pivot** — often at the point of an unknown force, so that unknown disappears from the moment equation.
- List all forces and their perpendicular distances from the chosen pivot.
- Apply the principle; then use $\sum F = 0$ to find any remaining unknowns.
- Check both conditions of equilibrium are satisfied.

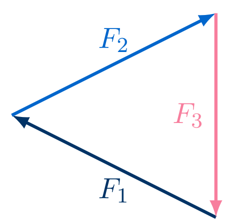
Coplanar Forces in Equilibrium — Vector Triangle

Three Coplanar Forces in Equilibrium

If exactly three coplanar forces act on a body in equilibrium, they must be **concurrent** (pass through a single point) and can be represented by the three sides of a **closed triangle** drawn head-to-tail.

- Draw the force vectors end-to-end.
- The triangle must close — meaning the resultant is zero.
- Trigonometry (sine rule, cosine rule, or right-triangle ratios) can then be used to find unknown magnitudes or directions.





Closed triangle ⇒ equilibrium

Density and Pressure

Density

Density is defined as mass per unit volume:

$$\rho = \frac{m}{V} \qquad \text{units: kg m}^{-3}$$

ρ = density (kg m^{-3})
m = mass (kg)
V = volume (m^3)

Pressure

Pressure is defined as the normal force per unit area:

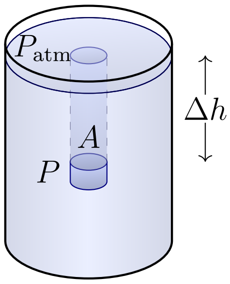
$$P = \frac{F}{A} \qquad \text{units: Pa} \equiv \text{N m}^{-2}$$

P = pressure (Pa)
F = normal force acting on the surface (N)
A = area over which the force acts (m^2)

Pressure is a Scalar

Although pressure arises from a force (a vector), pressure itself is a **scalar** quantity — it acts equally in all directions at a point within a fluid. Do not give pressure a direction in your answers.

Hydrostatic Pressure



Hydrostatic Pressure

The additional pressure on a submerged object is due to the weight of the fluid column above it. In the diagram this is a cylinder of Volume $A\Delta h$

- Weight of column of fluid: $W = mg = \rho V g = \rho \times (A\,\Delta h) \times g$
- Extra pressure on an Area A: $\Delta P = \dfrac{W}{A} = \dfrac{\rho A\,\Delta h\, g}{A}$

$$\Delta P = \rho g \Delta h$$

ΔP = increase in pressure with depth (Pa)
ρ = density of the fluid ($\mathrm{kg\,m^{-3}}$)
g = gravitational field strength ($\mathrm{N\,kg^{-1}}$)
Δh = increase in depth below the surface (m)

Key Points about Hydrostatic Pressure

- Pressure depends only on **depth** Δh, not on the shape or cross-sectional area of the container.
- Pressure acts **equally in all directions** at a given depth.
- In connected vessels, fluid reaches the same height regardless of the vessel shape (*communicating vessels*).
- Total absolute pressure at depth h: $P = P_{atm} + \rho g h$

Upthrust and Archimedes' Principle

Origin of Upthrust

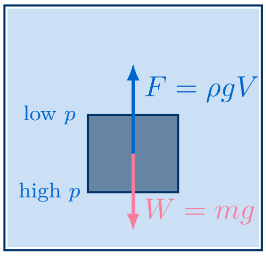
Upthrust (buoyancy force) acts on any object immersed in a fluid. It arises because the **hydrostatic pressure on the bottom face** of the object is greater than on the top face (since the bottom is at greater depth). The net upward pressure force is the upthrust.

Archimedes' Principle

$$F = \rho g V$$

F = upthrust (N)
ρ = density of the fluid ($\mathrm{kg\,m^{-3}}$)
g = gravitational field strength ($\mathrm{N\,kg^{-1}}$)
V = volume of fluid displaced by the object ($\mathrm{m^3}$)

Archimedes' Principle: the upthrust equals the **weight of fluid displaced**.



Floating and Sinking

- If $F > W$: net upward force — object **rises**.
- If $F = W$: object is in equilibrium — **floats** (fully or partially submerged).
- If $F < W$: net downward force — object **sinks** (rests on the bottom).
- A floating object displaces fluid whose weight **equals** the object's weight.

Worked Examples

Example 1 — Principle of Moments

Question: A uniform beam of weight 120 N and length 2.0 m is pivoted at one end. A 200 N load hangs 1.5 m from the pivot. Calculate the vertical force F needed at the free end to maintain equilibrium.

Solution

Solution:
Take moments about the pivot (left end):
Clockwise moments: weight of beam acts at centre (1.0 m) and load at 1.5 m:

$$\sum M_{\text{CW}} = (120 \times 1.0) + (200 \times 1.5) = 120 + 300 = 420\ \text{N m}$$

Anticlockwise moment from F at 2.0 m:

$$\sum M_{\text{ACW}} = F \times 2.0$$

Setting equal: $F \times 2.0 = 420$, so $\boxed{F = 210\ \text{N}}$ (upwards).

Example 2 — Hydrostatic Pressure

Question: A submarine is at a depth of 250 m in seawater of density 1025 kg m^{-3}. Calculate the additional pressure at this depth compared with the surface. ($g = 9.81\ \text{N kg}^{-1}$)

Solution

Solution:

$$\Delta p = \rho g \Delta h = 1025 \times 9.81 \times 250$$

$$\Delta p = 1025 \times 9.81 \times 250 = \mathbf{2.51 \times 10^6}\ \text{Pa}\ (= 2.51\ \text{MPa})$$

This is roughly 25 times atmospheric pressure — illustrating the enormous pressures at depth.

Example 3 — Upthrust (Archimedes' Principle)

Question: A solid aluminium sphere of radius 0.080 m is fully submerged in water (density 1000 kg m^{-3}). Calculate the upthrust acting on the sphere.

Solution

Solution:
Volume of sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.080)^3 = 2.14 \times 10^{-3}\ \text{m}^3$

$$F = \rho g V = 1000 \times 9.81 \times 2.14 \times 10^{-3} = \mathbf{21.0}\ \text{N}$$

Density of aluminium $\approx 2700\ \text{kg m}^{-3}$, so its weight $= 2700 \times 9.81 \times 2.14 \times 10^{-3} \approx 56.6$ N. Since $W > F$, the sphere sinks.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the moment of a force and state its SI unit.

[2 marks]

Q2. State the two conditions that must be satisfied for a rigid body to be in equilibrium.

[2 marks]

Q3. Define density and pressure, giving the SI unit of each.

[4 marks]

Q4. Explain, using the concept of pressure, why upthrust acts on a submerged object.

[2 marks]

Q5. A couple consists of two forces, each of magnitude 35 N, separated by a perpendicular distance of 0.24 m. Calculate the torque of the couple.

[2 marks]

Section B — Longer Structured Questions

Q6. A uniform plank of mass 8.0 kg and length 3.0 m is supported at each end by vertical forces F_1 (at the left) and F_2 (at the right). A person of mass 60 kg stands 1.0 m from the left end.

(a) Calculate the force F_2 at the right support.

[3 marks]

(b) Calculate the force F_1 at the left support.

[2 marks]

(c) The person moves towards the right end. Describe and explain what happens to F_1 and F_2.

[2 marks]

Q7. A diving bell (a sealed metal container open at the bottom) is lowered into the sea. The interior of the bell initially contains air at atmospheric pressure $p_0 = 1.01 \times 10^5$ Pa.

(a) Derive the equation $\Delta p = \rho g \Delta h$ for hydrostatic pressure, starting from the definitions of pressure and density.

[3 marks]

(b) The bell is lowered to a depth of 180 m in seawater of density 1025 $kg\,m^{-3}$. Calculate the total pressure at this depth.

[2 marks]

(c) A steel sphere of volume 4.5×10^{-3} m^3 and mass 35 kg is attached beneath the bell. Calculate the upthrust on the sphere and determine whether it floats or sinks when released.

[3 marks]

Mark Scheme and Answers

Q1. The moment of a force is the product of the force and the perpendicular distance from the pivot to the line of action of the force [1]; unit: N m [1].

Q2. The resultant force on the body is zero [1]; and the resultant torque (moment) about any point is zero [1].

Q3. Density: mass per unit volume [1], $kg\,m^{-3}$ [1]. Pressure: (normal) force per unit area [1], Pa (or $N\,m^{-2}$) [1].

Q4. The pressure increases with depth [1]; so the upward pressure force on the bottom face of the object is greater than the downward pressure force on the top face, giving a net upward force [1].

Q5. $\tau = Fd = 35 \times 0.24 = \mathbf{8.4}$ N m [2].

Q6(a). Take moments about left end. Clockwise: weight of plank $= 8.0 \times 9.81 = 78.5$ N at 1.5 m; person's weight $= 60 \times 9.81 = 589$ N at 1.0 m. Anticlockwise: F_2 at 3.0 m. Principle of moments: $F_2 \times 3.0 = (589 \times 1.0) + (78.5 \times 1.5)$ [1]; $F_2 \times 3.0 = 589 + 117.8 = 706.8$ [1]; $F_2 = \mathbf{236}$ N [1].

Q6(b). $\sum F = 0$: $F_1 + F_2 = 589 + 78.5 = 667.5$ N [1]; $F_1 = 667.5 - 236 = \mathbf{432}$ N [1].

Q6(c). As the person moves right, their moment about the left end increases, so F_2 increases [1]; and since the total must remain constant, F_1 decreases [1].

Q7(a). Consider a horizontal slab of fluid, area A, depth Δh, density ρ [1]. Weight of slab $= \rho A\,\Delta h\,g$ [1]; extra pressure $= W/A = \rho g\,\Delta h$ [1].

Q7(b). $p = p_0 + \rho g h = 1.01 \times 10^5 + 1025 \times 9.81 \times 180$ [1]; $= 1.01 \times 10^5 + 1.81 \times 10^6 = \mathbf{1.91 \times 10^6}$ Pa [1].

Q7(c). $F = \rho g V = 1025 \times 9.81 \times 4.5 \times 10^{-3} = \mathbf{45.2}$ N [1]; weight of sphere $= 35 \times 9.81 = 343$ N [1]; $W > F$, so the sphere **sinks** [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Explain what is meant by the centre of gravity of an object	
☐ Define and calculate the moment of a force about a point	
☐ Explain what a couple is and calculate the torque of a couple	
☐ State and apply the principle of moments	
☐ State the two conditions for equilibrium of a rigid body	
☐ Use a vector triangle to represent three coplanar forces in equilibrium	
☐ Define density and use $\rho = m/V$	
☐ Define pressure and use $p = F/A$	
☐ Derive and use the hydrostatic pressure equation $\Delta p = \rho g \Delta h$	
☐ Explain the origin of upthrust in terms of pressure difference	
☐ Calculate upthrust using Archimedes' Principle $F = \rho g V$	
☐ Determine whether an object floats or sinks by comparing weight and upthrust	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Remember: always check *both* equilibrium conditions, and use perpendicular distances when calculating moments. Drawing a clear force diagram before each problem will save you marks.