

# Topic 1

## Physical Quantities and Units

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Revision Booklet

**This booklet covers:**

- Physical Quantities and Estimation
- SI Base Units and Derived Units
- Homogeneity of Equations
- SI Prefixes
- Errors, Uncertainties, Precision and Accuracy
- Scalars and Vectors

## Physical Quantities

### Physical Quantity

Every **physical quantity** consists of two parts:

$$\text{physical quantity} = \text{numerical magnitude} \times \text{unit}$$

For example: a length of 3.5 m has magnitude 3.5 and unit metres. A quantity without a unit (or with units that cancel) is **dimensionless**.

### Reasonable Estimates

The syllabus requires you to make sensible estimates of physical quantities. Useful benchmarks to memorise:

Quantity	Estimate	Notes
Mass of a person	70 kg	
Height of a person	1.7 m	
Mass of a car	1000 kg	
Speed of a car on motorway	30 m s <sup>-1</sup>	( $\approx 110 \text{ km h}^{-1}$ )
Speed of sound in air	340 m s <sup>-1</sup>	
Speed of light	$3 \times 10^8 \text{ m s}^{-1}$	
Atmospheric pressure	$1 \times 10^5 \text{ Pa}$	
Density of water	$1000 \text{ kg m}^{-3}$	
Density of air	$1.2 \text{ kg m}^{-3}$	
Diameter of an atom	$\sim 10^{-10} \text{ m}$	
Diameter of a nucleus	$\sim 10^{-15} \text{ m}$	

## SI Base Units

### SI Base Quantities and Units

The International System of Units (SI) defines **seven base quantities**. The five required for this syllabus are:

Base quantity	SI unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K

### Derived Units

All other units are **derived** from the base units by multiplication or division. Examples:

Quantity	Derived unit	In base units
Force	newton (N)	$\text{kg m s}^{-2}$
Energy / Work	joule (J)	$\text{kg m}^2 \text{s}^{-2}$
Power	watt (W)	$\text{kg m}^2 \text{s}^{-3}$
Pressure	pascal (Pa)	$\text{kg m}^{-1} \text{s}^{-2}$
Charge	coulomb (C)	A s
Potential difference	volt (V)	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$
Resistance	ohm ( $\Omega$ )	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$
Frequency	hertz (Hz)	$\text{s}^{-1}$

## Homogeneity of Physical Equations

### Homogeneity

A physical equation is **homogeneous** if the units (dimensions) on both sides are identical. Every valid physical equation must be homogeneous.

**Method:** express every quantity in SI base units, then check both sides match.

### Using Homogeneity to Check Equations

1. Replace each symbol with its SI base units.
2. Simplify both sides independently.
3. If both sides have the same base units  $\Rightarrow$  equation is **homogeneous** (possibly correct).
4. If the sides differ  $\Rightarrow$  equation is **definitely wrong**.

*Important caveat:* homogeneity is a necessary but not sufficient condition. A homogeneous equation can still be wrong (e.g. a missing numerical factor of 2 would not be detected).

### Worked Example — Homogeneity Check

Check whether  $v^2 = u^2 + 2as$  is homogeneous.

$$\text{LHS: } [v^2] = (\text{m s}^{-1})^2 = \text{m}^2 \text{ s}^{-2}$$

$$\text{RHS: } [u^2] = \text{m}^2 \text{ s}^{-2}; \quad [2as] = (\text{m s}^{-2})(\text{m}) = \text{m}^2 \text{ s}^{-2}$$

Both terms on the RHS and the LHS have units  $\text{m}^2 \text{ s}^{-2} \Rightarrow$  equation is **homogeneous**.

✓

## SI Prefixes

Prefix	Symbol	Multiplier	Power of 10
tera	T	1 000 000 000 000	$10^{12}$
giga	G	1 000 000 000	$10^9$
mega	M	1 000 000	$10^6$
kilo	k	1 000	$10^3$
deci	d	0.1	$10^{-1}$
centi	c	0.01	$10^{-2}$
milli	m	0.001	$10^{-3}$
micro	$\mu$	0.000 001	$10^{-6}$
nano	n	0.000 000 001	$10^{-9}$
pico	p	0.000 000 000 001	$10^{-12}$

## Common Prefix Errors

- $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$ , **not**  $10^{-6} \text{ m}$  — don't confuse nano (n) with micro ( $\mu$ ).
- When squaring or cubing a unit with a prefix, the prefix is also raised to that power:  
 $1 \text{ cm}^2 = (10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2$ , not  $10^{-2} \text{ m}^2$ .
- $1 \text{ mm}^3 = (10^{-3} \text{ m})^3 = 10^{-9} \text{ m}^3$ .

## Errors, Uncertainties, Precision and Accuracy

### Types of Error

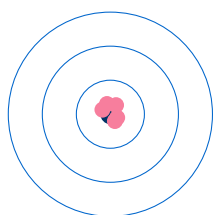
- **Systematic error:** an error that affects all readings by the same amount in the same direction. The mean of repeated readings is shifted from the true value. Cannot be reduced by repeating measurements.
  - Examples: zero error on an instrument; wrongly calibrated scale; parallax error if always reading from the same angle.
- **Random error:** an error that causes readings to scatter unpredictably above and below the true value. Can be reduced by taking more readings and averaging.
  - Examples: reaction time variation; reading a scale to the nearest division; electrical noise.
- **Zero error:** a specific systematic error where an instrument reads non-zero when it should read zero. Corrected by subtracting the zero reading from all measurements.

### Precision and Accuracy

- **Precision:** how close repeated measurements are to *each other*. A precise instrument gives small random errors (small spread). Precision is about **repeatability**.
- **Accuracy:** how close a measurement is to the *true value*. An accurate instrument has small systematic error. Accuracy is about **closeness to truth**.

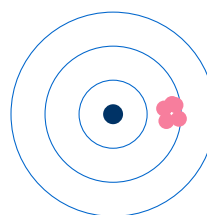
It is possible to be precise but inaccurate (systematic error shifts all readings the same way) or accurate but imprecise (readings scattered around the true value).

Precise, accurate



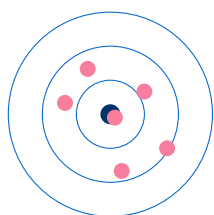
small random, small systematic

Precise, not accurate



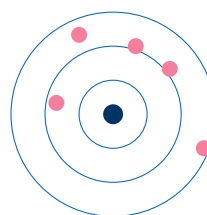
small random, large systematic

Not precise, accurate



large random, small systematic

Not precise, not accurate



large random, large systematic

## Combining Uncertainties

Operation	Rule for uncertainty
$z = x + y$ or $z = x - y$	$\Delta z = \Delta x + \Delta y$ (add <b>absolute</b> uncertainties)
$z = xy$ or $z = x/y$	$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$ (add <b>percentage</b> uncertainties)
$z = x^n$	$\frac{\Delta z}{z} =  n  \frac{\Delta x}{x}$ (multiply percentage uncertainty by $ n $ )

**Absolute uncertainty**  $\Delta x$ : has the same unit as  $x$ .

**Percentage uncertainty**:  $\frac{\Delta x}{x} \times 100\%$  — dimensionless.

## Common Uncertainty Mistakes

- For **addition/subtraction**, always add *absolute* uncertainties — never subtract.
- For **powers**:  $z = x^2$  gives  $\Delta z/z = 2 \Delta x/x$  — the power doubles the percentage uncertainty.
- A constant with no uncertainty (e.g.  $\pi$ , 2,  $g$  if treated as exact) does not contribute to the uncertainty.
- When reading an analogue scale, the uncertainty is typically  $\pm$  half the smallest division.

## Worked Example — Combining Uncertainties

A cylinder has radius  $r = (1.50 \pm 0.02)$  cm and height  $h = (4.00 \pm 0.05)$  cm. Calculate the volume and its percentage uncertainty.

$$V = \pi r^2 h = \pi(1.50)^2(4.00) = 28.3 \text{ cm}^3$$

Percentage uncertainties:

$$\frac{\Delta r}{r} = \frac{0.02}{1.50} = 1.33\%$$

$$\frac{\Delta h}{h} = \frac{0.05}{4.00} = 1.25\%$$

$$\frac{\Delta V}{V} = 2 \times 1.33\% + 1.25\% = 3.91\% \approx \mathbf{3.9\%}$$

( $r$  is squared so its percentage uncertainty is doubled;  $\pi$  contributes nothing.)

Absolute uncertainty:  $\Delta V = 0.039 \times 28.3 = 1.1 \text{ cm}^3$

$$\therefore V = (28.3 \pm 1.1) \text{ cm}^3$$

## Scalars and Vectors

### Scalars and Vectors

- A **scalar** quantity has **magnitude only**.
- A **vector** quantity has both **magnitude and direction**.

Scalars	Vectors
distance, speed	displacement, velocity
mass, density	acceleration, force, weight
energy, power, work	momentum, impulse
temperature, pressure	electric field strength
time, frequency	gravitational field strength

### Adding Vectors

Vectors must be added **tip-to-tail**. The resultant  $\vec{R} = \vec{A} + \vec{B}$  is the single vector drawn from the tail of  $\vec{A}$  to the tip of  $\vec{B}$ .

For two vectors at an angle  $\theta$  to each other, the magnitude of the resultant:

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

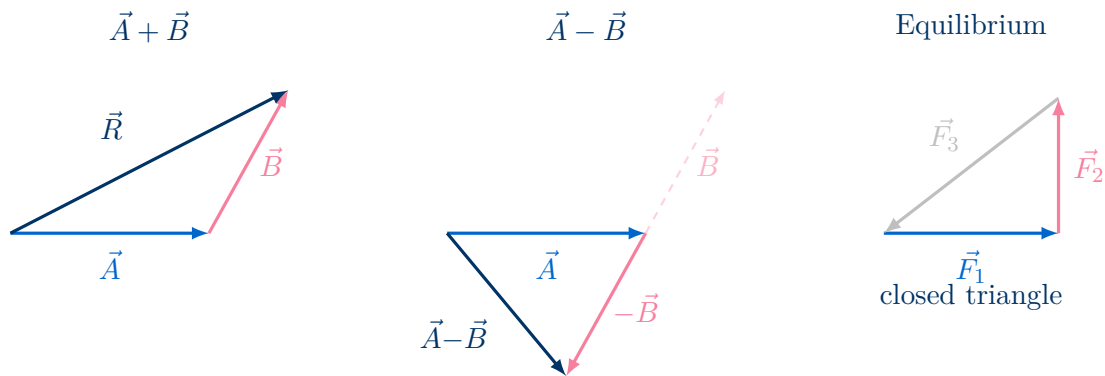
For **perpendicular** vectors ( $\theta = 90^\circ$ ):  $R = \sqrt{A^2 + B^2}$ , direction =  $\arctan(B/A)$ .

### Subtracting Vectors

To find  $\vec{A} - \vec{B}$ , reverse the direction of  $\vec{B}$  to get  $-\vec{B}$ , then add tip-to-tail:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

- $-\vec{B}$  has the **same magnitude** as  $\vec{B}$  but **opposite direction**.
- This is used whenever you need a **change in a vector quantity**, e.g. change in velocity  $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ .
- For perpendicular vectors:  $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2}$  (same magnitude as addition, different direction).



### Change in a Vector Quantity

A common application of vector subtraction is finding the **change in velocity**  $\Delta\vec{v}$ :

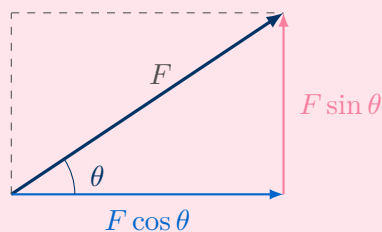
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$

Draw  $\vec{v}_1$  and  $\vec{v}_2$  from the same point.  $\Delta\vec{v}$  is the vector from the tip of  $\vec{v}_1$  to the tip of  $\vec{v}_2$ .  
*Example:* a ball bouncing off a wall at the same speed. Although  $|\vec{v}_1| = |\vec{v}_2|$ , the direction has changed so  $\Delta\vec{v} \neq 0$  — there is a non-zero acceleration (and therefore a resultant force).

### Resolving a Vector into Components

Any vector  $\vec{F}$  at angle  $\theta$  to the horizontal can be split into two perpendicular components:

$$F_x = F \cos \theta \quad F_y = F \sin \theta$$



### Coplanar Forces in Equilibrium — Vector Triangle

Three coplanar forces in equilibrium can be represented by a **closed triangle**: draw the vectors tip-to-tail; if they form a closed triangle (the last tip meets the first tail), the system is in equilibrium. This provides a graphical method for finding unknown force magnitudes or directions.

## Formula Summary Sheet

Formula / Rule	Use
$z = A + B \Rightarrow \Delta z = \Delta A + \Delta B$	Add/subtract: add absolute uncertainties
$z = AB \Rightarrow \Delta z/z = \Delta A/A + \Delta B/B$	Multiply/divide: add percentage uncertainties
$z = A^n \Rightarrow \Delta z/z =  n  \Delta A/A$	Powers: multiply % uncertainty by $ n $
$F_x = F \cos \theta$	Horizontal component of vector
$F_y = F \sin \theta$	Vertical component of vector
$R = \sqrt{A^2 + B^2}$	Resultant of two perpendicular vectors

### Key facts:

SI base units: kg, m, s, A, K.

**Systematic error:** shifts all readings same way; not reduced by repeating.

**Random error:** causes scatter; reduced by averaging more readings.

**Precision:** repeatability (small spread). **Accuracy:** closeness to true value.

**Homogeneity:** units must match on both sides of any valid equation.

## Worked Examples

### Example 1 — Derived Units in Base Units

**Question:** Show that the unit of pressure (Pa) is equivalent to  $\text{kg m}^{-1} \text{s}^{-2}$ .

#### Solution

Pressure = Force / Area. Force has units  $\text{N} = \text{kg m s}^{-2}$ . Area has units  $\text{m}^2$ .

$$[\text{Pa}] = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1} \text{s}^{-2} \quad \checkmark$$

### Example 2 — Homogeneity Check

**Question:** The period of a simple pendulum is given by  $T = 2\pi\sqrt{L/g}$ . Check this equation for homogeneity.

#### Solution

LHS:  $[T] = \text{s}$

RHS:  $2\pi$  is dimensionless.  $[L/g] = \text{m}/(\text{m s}^{-2}) = \text{s}^2$

$[\sqrt{L/g}] = \sqrt{\text{s}^2} = \text{s}$

Both sides have units of seconds  $\Rightarrow$  equation is **homogeneous**.  $\checkmark$

### Example 3 — Combining Uncertainties

**Question:** The resistance of a wire is found using  $R = V/I$ , where  $V = (6.0 \pm 0.2) \text{ V}$  and  $I = (0.30 \pm 0.01) \text{ A}$ . Calculate  $R$  and its percentage uncertainty.

#### Solution

$$R = V/I = 6.0/0.30 = \mathbf{20 \Omega}$$

Since this is division, add percentage uncertainties:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.2}{6.0} + \frac{0.01}{0.30} = 3.33\% + 3.33\% = \mathbf{6.7\%}$$

Absolute uncertainty:  $\Delta R = 0.067 \times 20 = 1.3 \Omega$

$$R = (20 \pm 1) \Omega$$

### Example 4 — Resolving and Adding Vectors

**Question:** Two forces act on a point:  $F_1 = 8.0 \text{ N}$  horizontally and  $F_2 = 6.0 \text{ N}$  vertically upward. Find the magnitude and direction of the resultant.

#### Solution

Since the forces are perpendicular:

$$R = \sqrt{F_1^2 + F_2^2} = \sqrt{8.0^2 + 6.0^2} = \sqrt{64 + 36} = \sqrt{100} = \mathbf{10 \text{ N}}$$

Direction above the horizontal:

$$\theta = \arctan\left(\frac{F_2}{F_1}\right) = \arctan\left(\frac{6.0}{8.0}\right) = \arctan(0.75) = \mathbf{36.9^\circ}$$

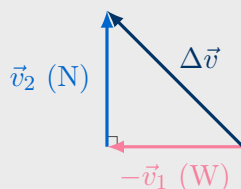
Resultant:  $10 \text{ N}$  at  $36.9^\circ$  above the horizontal.

**Example 5 — Vector Subtraction (Change in Velocity)**

**Question:** A ball of mass  $0.20 \text{ kg}$  travels at  $6.0 \text{ m s}^{-1}$  due East. It then travels at  $6.0 \text{ m s}^{-1}$  due North. Find (a) the magnitude and direction of the change in velocity  $\Delta\vec{v}$ , and (b) the magnitude of the average force if this change takes  $0.30 \text{ s}$ .

**Solution**

(a)  $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ . Reverse  $\vec{v}_1$  to get  $-\vec{v}_1 = 6.0 \text{ m s}^{-1}$  due West, then add  $\vec{v}_2 = 6.0 \text{ m s}^{-1}$  due North tip-to-tail:



$$|\Delta\vec{v}| = \sqrt{6.0^2 + 6.0^2} = \sqrt{72} = \mathbf{8.49 \text{ m s}^{-1}}$$

Direction:  $\arctan(6.0/6.0) = 45^\circ$  North of West (i.e. North-West).

(b)  $F = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{0.20 \times 8.49}{0.30} = \mathbf{5.7 \text{ N}}$  (directed North-West).

## Practice Exam Questions

### Section A — Short Answer Questions

**Q1.** State the five SI base quantities required for this syllabus and give the name and symbol of each unit.

*[5 marks]*

**Q2.** Show that the unit of the Young modulus (stress / strain) is equivalent to Pa, and express this in SI base units.

*[3 marks]*

**Q3.** Distinguish between systematic and random errors. Explain how each can be reduced or identified.

*[4 marks]*

**Q4.** Explain the difference between precision and accuracy. A thermometer consistently reads  $0.5\text{ }^{\circ}\text{C}$  too high. State whether this represents a systematic or random error, and whether the readings are precise, accurate, or both.

*[4 marks]*

## Section B — Longer Structured Questions

**Q5.** The speed  $v$  of a wave on a string is given by  $v = \sqrt{T/\mu}$ , where  $T$  is tension and  $\mu$  is mass per unit length.

- (a) Check the equation for homogeneity by expressing both sides in SI base units.

*[3 marks]*

- (b) In an experiment,  $T = (4.5 \pm 0.2)$  N and  $\mu = (8.0 \pm 0.4) \times 10^{-3}$  kg m<sup>-1</sup>. Calculate  $v$  and its percentage uncertainty.

*[4 marks]*

**Q6.** A ship travels 40 km due North, then 30 km due East.

(a) Calculate the magnitude of the resultant displacement.

*[2 marks]*

(b) Calculate the direction of the resultant displacement as a bearing.

*[2 marks]*

(c) State the difference between the total distance travelled and the magnitude of displacement. What type of quantity (scalar or vector) is each?

*[2 marks]*

**Q7.** A force of 50 N acts at  $30^\circ$  above the horizontal.

(a) Resolve the force into its horizontal and vertical components.

*[2 marks]*

(b) A second force of 20 N acts horizontally in the same direction. Find the magnitude and direction of the resultant of the two forces.

*[3 marks]*

(c) A third force is added so that the system of three forces is in equilibrium. Describe this third force.

*[2 marks]*

## Mark Scheme and Answers

**Q1.** Mass/kilogram/kg [1]; length/metre/m [1]; time/second/s [1]; electric current/ampere/A [1]; temperature/kelvin/K [1].

**Q2.** Stress = force/area  $\Rightarrow$   $\text{N m}^{-2} = \text{Pa}$  [1]. Strain = extension/length  $\Rightarrow$  dimensionless [1]. Young modulus = Pa =  $\text{kg m}^{-1} \text{s}^{-2}$  [1].

**Q3.** Systematic: affects all readings the same way [1]; identified by comparing with a known standard; reduced by recalibration or improved technique [1]. Random: causes scatter above and below true value [1]; reduced by taking more readings and averaging [1].

**Q4.** Precision: how repeatable/consistent readings are [1]; accuracy: how close readings are to the true value [1]. Constant offset of  $+0.5^\circ\text{C}$   $\Rightarrow$  **systematic error** [1]; readings are **precise** (consistent with each other) but **not accurate** (all shifted from true value) [1].

**Q5(a).** LHS:  $[v] = \text{m s}^{-1}$  [1]. RHS:  $[T/\mu] = \text{N}/(\text{kg m}^{-1}) = \text{kg m s}^{-2}/(\text{kg m}^{-1}) = \text{m}^2 \text{s}^{-2}$  [1];  $[\sqrt{T/\mu}] = \text{m s}^{-1} \Rightarrow$  homogeneous  $\checkmark$  [1].

**Q5(b).**  $v = \sqrt{4.5/(8.0 \times 10^{-3})} = \sqrt{562.5} = \mathbf{23.7} \text{ m s}^{-1}$  [1].

% unc in  $T$ :  $0.2/4.5 = 4.44\%$ ; % unc in  $\mu$ :  $0.4/8.0 = 5.00\%$  [1].

% unc in  $v = \frac{1}{2}(4.44 + 5.00)\% = \frac{1}{2}(9.44)\% = \mathbf{4.7\%}$  [2] (halved because of square root).

**Q6(a).**  $R = \sqrt{40^2 + 30^2} = \sqrt{2500} = \mathbf{50} \text{ km}$  [2].

**Q6(b).**  $\theta = \arctan(30/40) = 36.9^\circ$  East of North  $\Rightarrow$  bearing = **037°** [2].

**Q6(c).** Distance =  $40 + 30 = 70 \text{ km}$  (scalar) [1]; displacement =  $50 \text{ km}$  at  $037^\circ$  (vector) [1].

**Q7(a).**  $F_x = 50 \cos 30^\circ = \mathbf{43.3} \text{ N}$  (horizontal) [1];  $F_y = 50 \sin 30^\circ = \mathbf{25} \text{ N}$  (vertical) [1].

**Q7(b).** Total horizontal =  $43.3 + 20 = 63.3 \text{ N}$ ; total vertical =  $25 \text{ N}$  [1].  $R = \sqrt{63.3^2 + 25^2} = \sqrt{4007 + 625} = \sqrt{4632} = \mathbf{68.1} \text{ N}$  [1];  $\theta = \arctan(25/63.3) = \mathbf{21.5^\circ}$  above horizontal [1].

**Q7(c).** Third force must be equal in magnitude ( $68.1 \text{ N}$ ) [1] and opposite in direction ( $21.5^\circ$  below horizontal, pointing left) to the resultant of the first two [1].

## Revision Checklist

Learning Objective	Confidence (1–3)
<input type="checkbox"/> State that a physical quantity = magnitude $\times$ unit	
<input type="checkbox"/> Make reasonable estimates of common physical quantities	
<input type="checkbox"/> Recall the five SI base quantities and their units (kg, m, s, A, K)	
<input type="checkbox"/> Express derived units in SI base units (N, J, W, Pa, V, $\Omega$ , C)	
<input type="checkbox"/> Use base units to check homogeneity of an equation	
<input type="checkbox"/> Recall all ten SI prefixes (T, G, M, k, d, c, m, $\mu$ , n, p) with values	
<input type="checkbox"/> Convert correctly when units are raised to powers (e.g. $\text{cm}^2$ to $\text{m}^2$ )	
<input type="checkbox"/> Distinguish systematic and random errors with examples	
<input type="checkbox"/> Define and distinguish precision and accuracy	
<input type="checkbox"/> Combine absolute uncertainties for addition/subtraction	
<input type="checkbox"/> Combine percentage uncertainties for multiplication/division/powers	
<input type="checkbox"/> Distinguish scalars and vectors; give examples of each	
<input type="checkbox"/> Add and subtract coplanar vectors (tip-to-tail / component method)	
<input type="checkbox"/> Subtract vectors by reversing direction of the second vector ( $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ )	
<input type="checkbox"/> Use vector subtraction to find the change in a vector quantity (e.g. $\Delta\vec{v}$ )	
<input type="checkbox"/> Resolve a vector into two perpendicular components ( $F \cos \theta$ , $F \sin \theta$ )	
<input type="checkbox"/> Use a closed vector triangle to represent coplanar forces in equilibrium	
<hr/>	
<i>Key: 1 = Need more work    2 = Getting there    3 = Confident</i>	

**Good luck with your revision!**

Topic 1 underpins everything else in the course. Get comfortable with unit conversions, uncertainty rules and vector resolution now — you will use all three in every other topic.