

CIE AS-Level Physics

Revision Booklet

This booklet covers:

- Topic 1 — Physical Quantities and Units
- Topic 2 — Kinematics
- Topic 3 — Dynamics
- Topic 4 — Forces, Density and Pressure
- Topic 5 — Work, Energy and Power
- Topic 6 — Deformation of Solids
- Topic 7 — Waves
- Topic 8 — Superposition
- Topic 9 — Electricity
- Topic 10 — D.C. Circuits
- Topic 11 — Particle Physics

Topic 1

Physical Quantities and Units

Revision Booklet

This booklet covers:

- Physical Quantities and Estimation
- SI Base Units and Derived Units
- Homogeneity of Equations
- SI Prefixes
- Errors, Uncertainties, Precision and Accuracy
- Scalars and Vectors

Physical Quantities

Physical Quantity

Every **physical quantity** consists of two parts:

$$\text{physical quantity} = \text{numerical magnitude} \times \text{unit}$$

For example: a length of 3.5 m has magnitude 3.5 and unit metres. A quantity without a unit (or with units that cancel) is **dimensionless**.

Reasonable Estimates

The syllabus requires you to make sensible estimates of physical quantities. Useful benchmarks to memorise:

Quantity	Estimate	Notes
Mass of a person	70 kg	
Height of a person	1.7 m	
Mass of a car	1000 kg	
Speed of a car on motorway	30 m s ⁻¹	($\approx 110 \text{ km h}^{-1}$)
Speed of sound in air	340 m s ⁻¹	
Speed of light	$3 \times 10^8 \text{ m s}^{-1}$	
Atmospheric pressure	$1 \times 10^5 \text{ Pa}$	
Density of water	1000 kg m^{-3}	
Density of air	1.2 kg m^{-3}	
Diameter of an atom	$\sim 10^{-10} \text{ m}$	
Diameter of a nucleus	$\sim 10^{-15} \text{ m}$	

SI Base Units

SI Base Quantities and Units

The International System of Units (SI) defines **seven base quantities**. The five required for this syllabus are:

Base quantity	SI unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K

Derived Units

All other units are **derived** from the base units by multiplication or division. Examples:

Quantity	Derived unit	In base units
Force	newton (N)	kg m s^{-2}
Energy / Work	joule (J)	$\text{kg m}^2 \text{s}^{-2}$
Power	watt (W)	$\text{kg m}^2 \text{s}^{-3}$
Pressure	pascal (Pa)	$\text{kg m}^{-1} \text{s}^{-2}$
Charge	coulomb (C)	A s
Potential difference	volt (V)	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$
Resistance	ohm (Ω)	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$
Frequency	hertz (Hz)	s^{-1}

Homogeneity of Physical Equations

Homogeneity

A physical equation is **homogeneous** if the units (dimensions) on both sides are identical. Every valid physical equation must be homogeneous.

Method: express every quantity in SI base units, then check both sides match.

Using Homogeneity to Check Equations

1. Replace each symbol with its SI base units.
2. Simplify both sides independently.
3. If both sides have the same base units \Rightarrow equation is **homogeneous** (possibly correct).
4. If the sides differ \Rightarrow equation is **definitely wrong**.

Important caveat: homogeneity is a necessary but not sufficient condition. A homogeneous equation can still be wrong (e.g. a missing numerical factor of 2 would not be detected).

Worked Example — Homogeneity Check

Check whether $v^2 = u^2 + 2as$ is homogeneous.

$$\text{LHS: } [v^2] = (\text{m s}^{-1})^2 = \text{m}^2 \text{ s}^{-2}$$

$$\text{RHS: } [u^2] = \text{m}^2 \text{ s}^{-2}; \quad [2as] = (\text{m s}^{-2})(\text{m}) = \text{m}^2 \text{ s}^{-2}$$

Both terms on the RHS and the LHS have units $\text{m}^2 \text{ s}^{-2} \Rightarrow$ equation is **homogeneous**.

✓

SI Prefixes

Prefix	Symbol	Multiplier	Power of 10
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^9
mega	M	1 000 000	10^6
kilo	k	1 000	10^3
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}

Common Prefix Errors

- $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$, **not** 10^{-6} m — don't confuse nano (n) with micro (μ).
- When squaring or cubing a unit with a prefix, the prefix is also raised to that power:
 $1 \text{ cm}^2 = (10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2$, not 10^{-2} m^2 .
- $1 \text{ mm}^3 = (10^{-3} \text{ m})^3 = 10^{-9} \text{ m}^3$.

Errors, Uncertainties, Precision and Accuracy

Types of Error

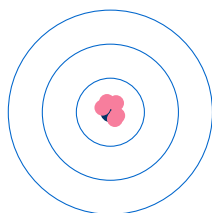
- **Systematic error:** an error that affects all readings by the same amount in the same direction. The mean of repeated readings is shifted from the true value. Cannot be reduced by repeating measurements.
 - Examples: zero error on an instrument; wrongly calibrated scale; parallax error if always reading from the same angle.
- **Random error:** an error that causes readings to scatter unpredictably above and below the true value. Can be reduced by taking more readings and averaging.
 - Examples: reaction time variation; reading a scale to the nearest division; electrical noise.
- **Zero error:** a specific systematic error where an instrument reads non-zero when it should read zero. Corrected by subtracting the zero reading from all measurements.

Precision and Accuracy

- **Precision:** how close repeated measurements are to *each other*. A precise instrument gives small random errors (small spread). Precision is about **repeatability**.
- **Accuracy:** how close a measurement is to the *true value*. An accurate instrument has small systematic error. Accuracy is about **closeness to truth**.

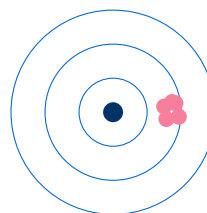
It is possible to be precise but inaccurate (systematic error shifts all readings the same way) or accurate but imprecise (readings scattered around the true value).

Precise, accurate



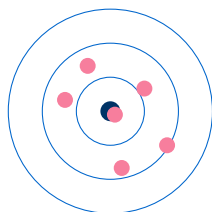
small random, small systematic

Precise, not accurate



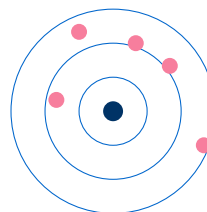
small random, large systematic

Not precise, accurate



large random, small systematic

Not precise, not accurate



large random, large systematic

Combining Uncertainties

Operation	Rule for uncertainty
$z = x + y$ or $z = x - y$	$\Delta z = \Delta x + \Delta y$ (add absolute uncertainties)
$z = xy$ or $z = x/y$	$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$ (add percentage uncertainties)
$z = x^n$	$\frac{\Delta z}{z} = n \frac{\Delta x}{x}$ (multiply percentage uncertainty by $ n $)

Absolute uncertainty Δx : has the same unit as x .

Percentage uncertainty: $\frac{\Delta x}{x} \times 100\%$ — dimensionless.

Common Uncertainty Mistakes

- For **addition/subtraction**, always add *absolute* uncertainties — never subtract.
- For **powers**: $z = x^2$ gives $\Delta z/z = 2 \Delta x/x$ — the power doubles the percentage uncertainty.
- A constant with no uncertainty (e.g. π , 2, g if treated as exact) does not contribute to the uncertainty.
- When reading an analogue scale, the uncertainty is typically \pm half the smallest division.

Worked Example — Combining Uncertainties

A cylinder has radius $r = (1.50 \pm 0.02)$ cm and height $h = (4.00 \pm 0.05)$ cm. Calculate the volume and its percentage uncertainty.

$$V = \pi r^2 h = \pi(1.50)^2(4.00) = 28.3 \text{ cm}^3$$

Percentage uncertainties:

$$\frac{\Delta r}{r} = \frac{0.02}{1.50} = 1.33\%$$

$$\frac{\Delta h}{h} = \frac{0.05}{4.00} = 1.25\%$$

$$\frac{\Delta V}{V} = 2 \times 1.33\% + 1.25\% = 3.91\% \approx \mathbf{3.9\%}$$

(r is squared so its percentage uncertainty is doubled; π contributes nothing.)

Absolute uncertainty: $\Delta V = 0.039 \times 28.3 = 1.1 \text{ cm}^3$

$$\therefore V = (28.3 \pm 1.1) \text{ cm}^3$$

Scalars and Vectors

Scalars and Vectors

- A **scalar** quantity has **magnitude only**.
- A **vector** quantity has both **magnitude and direction**.

Scalars	Vectors
distance, speed	displacement, velocity
mass, density	acceleration, force, weight
energy, power, work	momentum, impulse
temperature, pressure	electric field strength
time, frequency	gravitational field strength

Adding Vectors

Vectors must be added **tip-to-tail**. The resultant $\vec{R} = \vec{A} + \vec{B}$ is the single vector drawn from the tail of \vec{A} to the tip of \vec{B} .

For two vectors at an angle θ to each other, the magnitude of the resultant:

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

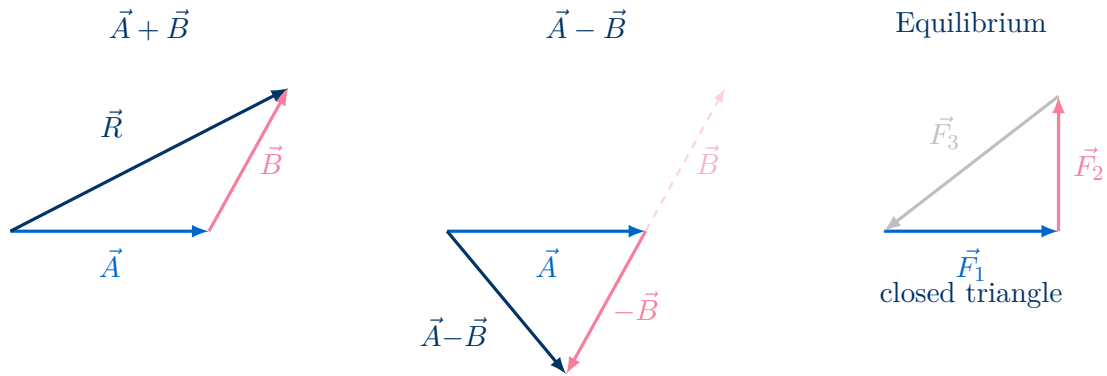
For **perpendicular** vectors ($\theta = 90^\circ$): $R = \sqrt{A^2 + B^2}$, direction = $\arctan(B/A)$.

Subtracting Vectors

To find $\vec{A} - \vec{B}$, reverse the direction of \vec{B} to get $-\vec{B}$, then add tip-to-tail:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

- $-\vec{B}$ has the **same magnitude** as \vec{B} but **opposite direction**.
- This is used whenever you need a **change in a vector quantity**, e.g. change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.
- For perpendicular vectors: $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2}$ (same magnitude as addition, different direction).



Change in a Vector Quantity

A common application of vector subtraction is finding the **change in velocity** $\Delta\vec{v}$:

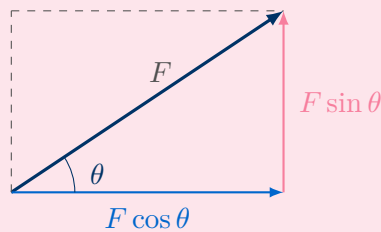
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$

Draw \vec{v}_1 and \vec{v}_2 from the same point. $\Delta\vec{v}$ is the vector from the tip of \vec{v}_1 to the tip of \vec{v}_2 . *Example:* a ball bouncing off a wall at the same speed. Although $|\vec{v}_1| = |\vec{v}_2|$, the direction has changed so $\Delta\vec{v} \neq 0$ — there is a non-zero acceleration (and therefore a resultant force).

Resolving a Vector into Components

Any vector \vec{F} at angle θ to the horizontal can be split into two perpendicular components:

$$F_x = F \cos \theta \quad F_y = F \sin \theta$$



Coplanar Forces in Equilibrium — Vector Triangle

Three coplanar forces in equilibrium can be represented by a **closed triangle**: draw the vectors tip-to-tail; if they form a closed triangle (the last tip meets the first tail), the system is in equilibrium. This provides a graphical method for finding unknown force magnitudes or directions.

Formula Summary Sheet

Formula / Rule	Use
$z = A + B \Rightarrow \Delta z = \Delta A + \Delta B$	Add/subtract: add absolute uncertainties
$z = AB \Rightarrow \Delta z/z = \Delta A/A + \Delta B/B$	Multiply/divide: add percentage uncertainties
$z = A^n \Rightarrow \Delta z/z = n \Delta A/A$	Powers: multiply % uncertainty by $ n $
$F_x = F \cos \theta$	Horizontal component of vector
$F_y = F \sin \theta$	Vertical component of vector
$R = \sqrt{A^2 + B^2}$	Resultant of two perpendicular vectors

Key facts:

SI base units: kg, m, s, A, K.

Systematic error: shifts all readings same way; not reduced by repeating.

Random error: causes scatter; reduced by averaging more readings.

Precision: repeatability (small spread). **Accuracy:** closeness to true value.

Homogeneity: units must match on both sides of any valid equation.

Worked Examples

Example 1 — Derived Units in Base Units

Question: Show that the unit of pressure (Pa) is equivalent to $\text{kg m}^{-1} \text{s}^{-2}$.

Solution

Pressure = Force / Area. Force has units $\text{N} = \text{kg m s}^{-2}$. Area has units m^2 .

$$[\text{Pa}] = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1} \text{s}^{-2} \quad \checkmark$$

Example 2 — Homogeneity Check

Question: The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$. Check this equation for homogeneity.

Solution

LHS: $[T] = \text{s}$

RHS: 2π is dimensionless. $[L/g] = \text{m}/(\text{m s}^{-2}) = \text{s}^2$

$[\sqrt{L/g}] = \sqrt{\text{s}^2} = \text{s}$

Both sides have units of seconds \Rightarrow equation is **homogeneous**. \checkmark

Example 3 — Combining Uncertainties

Question: The resistance of a wire is found using $R = V/I$, where $V = (6.0 \pm 0.2) \text{ V}$ and $I = (0.30 \pm 0.01) \text{ A}$. Calculate R and its percentage uncertainty.

Solution

$$R = V/I = 6.0/0.30 = \mathbf{20 \Omega}$$

Since this is division, add percentage uncertainties:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.2}{6.0} + \frac{0.01}{0.30} = 3.33\% + 3.33\% = \mathbf{6.7\%}$$

Absolute uncertainty: $\Delta R = 0.067 \times 20 = 1.3 \Omega$

$$R = (20 \pm 1) \Omega$$

Example 4 — Resolving and Adding Vectors

Question: Two forces act on a point: $F_1 = 8.0 \text{ N}$ horizontally and $F_2 = 6.0 \text{ N}$ vertically upward. Find the magnitude and direction of the resultant.

Solution

Since the forces are perpendicular:

$$R = \sqrt{F_1^2 + F_2^2} = \sqrt{8.0^2 + 6.0^2} = \sqrt{64 + 36} = \sqrt{100} = \mathbf{10 \text{ N}}$$

Direction above the horizontal:

$$\theta = \arctan\left(\frac{F_2}{F_1}\right) = \arctan\left(\frac{6.0}{8.0}\right) = \arctan(0.75) = \mathbf{36.9^\circ}$$

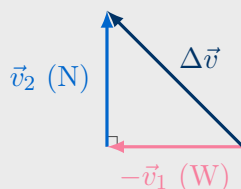
Resultant: 10 N at 36.9° above the horizontal.

Example 5 — Vector Subtraction (Change in Velocity)

Question: A ball of mass 0.20 kg travels at 6.0 m s^{-1} due East. It then travels at 6.0 m s^{-1} due North. Find (a) the magnitude and direction of the change in velocity $\Delta\vec{v}$, and (b) the magnitude of the average force if this change takes 0.30 s.

Solution

(a) $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. Reverse \vec{v}_1 to get $-\vec{v}_1 = 6.0 \text{ m s}^{-1}$ due West, then add $\vec{v}_2 = 6.0 \text{ m s}^{-1}$ due North tip-to-tail:



$$|\Delta\vec{v}| = \sqrt{6.0^2 + 6.0^2} = \sqrt{72} = \mathbf{8.49 \text{ m s}^{-1}}$$

Direction: $\arctan(6.0/6.0) = 45^\circ$ North of West (i.e. North-West).

(b) $F = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{0.20 \times 8.49}{0.30} = \mathbf{5.7 \text{ N}}$ (directed North-West).

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the five SI base quantities required for this syllabus and give the name and symbol of each unit.

[5 marks]

Q2. Show that the unit of the Young modulus (stress / strain) is equivalent to Pa, and express this in SI base units.

[3 marks]

Q3. Distinguish between systematic and random errors. Explain how each can be reduced or identified.

[4 marks]

Q4. Explain the difference between precision and accuracy. A thermometer consistently reads $0.5\text{ }^{\circ}\text{C}$ too high. State whether this represents a systematic or random error, and whether the readings are precise, accurate, or both.

[4 marks]

Section B — Longer Structured Questions

Q5. The speed v of a wave on a string is given by $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.

- (a) Check the equation for homogeneity by expressing both sides in SI base units.

[3 marks]

- (b) In an experiment, $T = (4.5 \pm 0.2)$ N and $\mu = (8.0 \pm 0.4) \times 10^{-3}$ kg m⁻¹. Calculate v and its percentage uncertainty.

[4 marks]

Q6. A ship travels 40 km due North, then 30 km due East.

(a) Calculate the magnitude of the resultant displacement.

[2 marks]

(b) Calculate the direction of the resultant displacement as a bearing.

[2 marks]

(c) State the difference between the total distance travelled and the magnitude of displacement. What type of quantity (scalar or vector) is each?

[2 marks]

Q7. A force of 50 N acts at 30° above the horizontal.

(a) Resolve the force into its horizontal and vertical components.

[2 marks]

(b) A second force of 20 N acts horizontally in the same direction. Find the magnitude and direction of the resultant of the two forces.

[3 marks]

(c) A third force is added so that the system of three forces is in equilibrium. Describe this third force.

[2 marks]

Mark Scheme and Answers

Q1. Mass/kilogram/kg [1]; length/metre/m [1]; time/second/s [1]; electric current/ampere/A [1]; temperature/kelvin/K [1].

Q2. Stress = force/area \Rightarrow N m⁻² = Pa [1]. Strain = extension/length \Rightarrow dimensionless [1]. Young modulus = Pa = kg m⁻¹ s⁻² [1].

Q3. Systematic: affects all readings the same way [1]; identified by comparing with a known standard; reduced by recalibration or improved technique [1]. Random: causes scatter above and below true value [1]; reduced by taking more readings and averaging [1].

Q4. Precision: how repeatable/consistent readings are [1]; accuracy: how close readings are to the true value [1]. Constant offset of +0.5°C \Rightarrow **systematic error** [1]; readings are **precise** (consistent with each other) but **not accurate** (all shifted from true value) [1].

Q5(a). LHS: $[v] = \text{m s}^{-1}$ [1]. RHS: $[T/\mu] = \text{N}/(\text{kg m}^{-1}) = \text{kg m s}^{-2}/(\text{kg m}^{-1}) = \text{m}^2 \text{s}^{-2}$ [1]; $[\sqrt{T/\mu}] = \text{m s}^{-1} \Rightarrow$ homogeneous \checkmark [1].

Q5(b). $v = \sqrt{4.5/(8.0 \times 10^{-3})} = \sqrt{562.5} = \mathbf{23.7} \text{ m s}^{-1}$ [1].

% unc in T : $0.2/4.5 = 4.44\%$; % unc in μ : $0.4/8.0 = 5.00\%$ [1].

% unc in $v = \frac{1}{2}(4.44 + 5.00)\% = \frac{1}{2}(9.44)\% = \mathbf{4.7\%}$ [2] (halved because of square root).

Q6(a). $R = \sqrt{40^2 + 30^2} = \sqrt{2500} = \mathbf{50} \text{ km}$ [2].

Q6(b). $\theta = \arctan(30/40) = 36.9^\circ$ East of North \Rightarrow bearing = **037°** [2].

Q6(c). Distance = $40 + 30 = 70 \text{ km}$ (scalar) [1]; displacement = 50 km at 037° (vector) [1].

Q7(a). $F_x = 50 \cos 30^\circ = \mathbf{43.3} \text{ N}$ (horizontal) [1]; $F_y = 50 \sin 30^\circ = \mathbf{25} \text{ N}$ (vertical) [1].

Q7(b). Total horizontal = $43.3 + 20 = 63.3 \text{ N}$; total vertical = 25 N [1]. $R = \sqrt{63.3^2 + 25^2} = \sqrt{4007 + 625} = \sqrt{4632} = \mathbf{68.1} \text{ N}$ [1]; $\theta = \arctan(25/63.3) = \mathbf{21.5^\circ}$ above horizontal [1].

Q7(c). Third force must be equal in magnitude (68.1 N) [1] and opposite in direction (21.5° below horizontal, pointing left) to the resultant of the first two [1].

Revision Checklist

Learning Objective	Confidence (1–3)
<input type="checkbox"/> State that a physical quantity = magnitude \times unit	
<input type="checkbox"/> Make reasonable estimates of common physical quantities	
<input type="checkbox"/> Recall the five SI base quantities and their units (kg, m, s, A, K)	
<input type="checkbox"/> Express derived units in SI base units (N, J, W, Pa, V, Ω , C)	
<input type="checkbox"/> Use base units to check homogeneity of an equation	
<input type="checkbox"/> Recall all ten SI prefixes (T, G, M, k, d, c, m, μ , n, p) with values	
<input type="checkbox"/> Convert correctly when units are raised to powers (e.g. cm^2 to m^2)	
<input type="checkbox"/> Distinguish systematic and random errors with examples	
<input type="checkbox"/> Define and distinguish precision and accuracy	
<input type="checkbox"/> Combine absolute uncertainties for addition/subtraction	
<input type="checkbox"/> Combine percentage uncertainties for multiplication/division/powers	
<input type="checkbox"/> Distinguish scalars and vectors; give examples of each	
<input type="checkbox"/> Add and subtract coplanar vectors (tip-to-tail / component method)	
<input type="checkbox"/> Subtract vectors by reversing direction of the second vector ($\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$)	
<input type="checkbox"/> Use vector subtraction to find the change in a vector quantity (e.g. $\Delta\vec{v}$)	
<input type="checkbox"/> Resolve a vector into two perpendicular components ($F \cos \theta$, $F \sin \theta$)	
<input type="checkbox"/> Use a closed vector triangle to represent coplanar forces in equilibrium	
<hr/>	
<i>Key: 1 = Need more work 2 = Getting there 3 = Confident</i>	

Good luck with your revision!

Topic 1 underpins everything else in the course. Get comfortable with unit conversions, uncertainty rules and vector resolution now — you will use all three in every other topic.

Topic 2

Kinematics

Revision Booklet

This booklet covers:

- Distance, Displacement, Speed, Velocity and Acceleration
- Graphs of Motion
- Equations of Uniform Acceleration (SUVAT)
- Free Fall and Measuring g
- Projectile Motion

Core Definitions

Scalar and Vector Quantities

- **Distance** (scalar): the total length of path travelled, regardless of direction. Units: m.
- **Displacement** s (vector): the straight-line distance from start to finish *in a specified direction*. Units: m.
- **Speed** (scalar): distance travelled per unit time. Units: m s^{-1} .
- **Velocity** v (vector): rate of change of displacement. Units: m s^{-1} .
- **Acceleration** a (vector): rate of change of velocity. Units: m s^{-2} .

Defining Equations

$$v = \frac{\Delta s}{\Delta t} \qquad a = \frac{\Delta v}{\Delta t}$$

Δs = change in displacement (m)

Δv = change in velocity (m s^{-1})

Δt = time interval (s)

Distance vs Displacement

A car that travels 400 m north then 400 m south has a total **distance** of 800 m but a **displacement** of zero. Always identify whether the question asks for a scalar or vector quantity before calculating.

Graphs of Motion

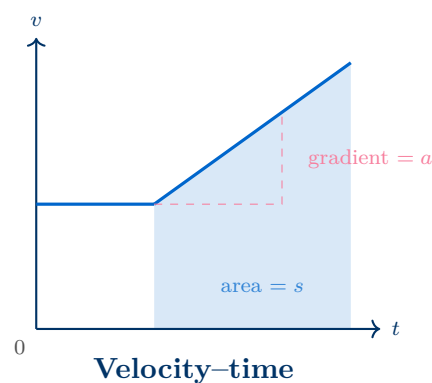
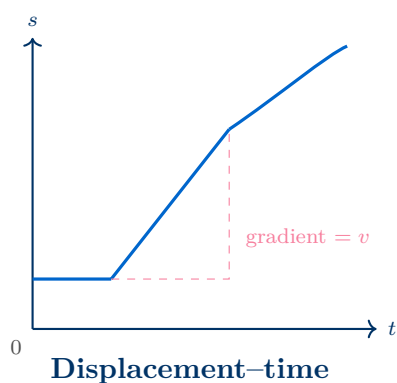
Reading Motion Graphs

Displacement–time ($s-t$) graph:

- Gradient = instantaneous velocity.
- Horizontal line \Rightarrow stationary.
- Straight line \Rightarrow constant (uniform) velocity.
- Curve \Rightarrow changing velocity (acceleration present).

Velocity–time ($v-t$) graph:

- Gradient = instantaneous acceleration.
- Area under the graph = displacement.
- Horizontal line \Rightarrow constant velocity (zero acceleration).
- Straight line \Rightarrow uniform acceleration.



Equations of Uniform Acceleration (SUVAT)

Derivation

For **constant** acceleration, starting from the definitions of velocity and acceleration:

$$1. \text{ From } a = \Delta v / \Delta t: \quad a = (v - u) / t \quad \Rightarrow \quad \boxed{v = u + at}$$

$$2. \text{ Mean velocity} \times \text{time}: \quad s = \frac{u + v}{2} \times t \quad \Rightarrow \quad \boxed{s = \frac{1}{2}(u + v)t}$$

$$3. \text{ Substitute equation (1) into (2): } s = \frac{u + u + at}{2} t \quad \Rightarrow \quad \boxed{s = ut + \frac{1}{2}at^2}$$

$$4. \text{ Eliminate } t: \text{ from (1), } t = (v - u) / a; \text{ substitute into (2): } \quad \Rightarrow \quad \boxed{v^2 = u^2 + 2as}$$

The SUVAT Equations

Equation	Variable not present
$v = u + at$	s
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t
$s = \frac{1}{2}(u + v)t$	a

s = displacement (m) u = initial velocity (m s^{-1})
 v = final velocity (m s^{-1}) a = acceleration (m s^{-2}) t = time (s)

Valid only for **uniform** (constant) acceleration in a straight line.

SUVAT Problem Strategy

1. Write down the five variables: s , u , v , a , t .
2. Fill in the known values; identify what you are finding.
3. Choose the equation that contains those four variables (three known + one unknown).
4. Solve; check units and sign conventions.

Positive direction must be defined clearly at the start — be consistent throughout.

Free Fall and Measuring g

Free Fall

Free fall is the motion of an object under gravity alone, with no air resistance. The object accelerates uniformly downwards at the **acceleration of free fall**:

$$g = 9.81 \text{ m s}^{-2} \quad (\text{near Earth's surface})$$

Free fall is uniform acceleration, so all four SUVAT equations apply with $a = g$.

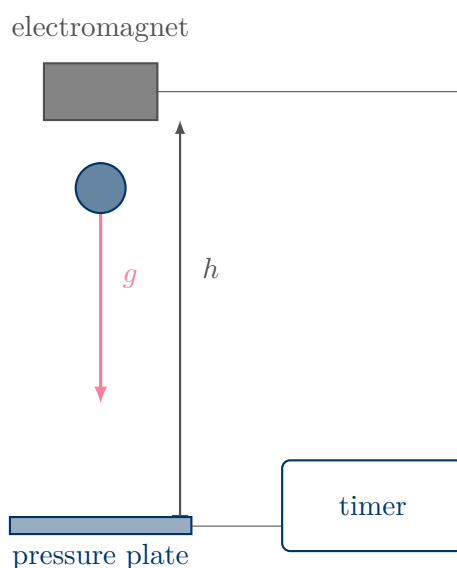
Experiment: Determining g Using a Falling Object

Method — Electromagnetic Release and Trapdoor Timer

1. A steel ball is held by an electromagnet at a measured height h above a trapdoor switch.
2. The circuit is broken simultaneously: the ball is released and a millisecond timer starts.
3. When the ball hits the pressure plate the timer stops, giving the time of fall t .
4. Use $h = \frac{1}{2}gt^2$ (with $u = 0$) to find g :

$$g = \frac{2h}{t^2}$$

5. Repeat for several values of h ; plot h against t^2 — the gradient equals $g/2$.



Sources of Uncertainty and Improvements

- **Reaction time:** eliminated by electronic (not manual) timing.
- **Residual magnetism:** the ball may not release instantly — add a thin bit of tape between the ball and the magnet.
- **Air resistance:** use a dense, compact sphere to minimise drag.
- **Height measurement:** measure from the *bottom* of the ball to the pressure plate; use a ruler held vertically alongside.
- **Graph method:** plotting h vs t^2 and finding the gradient = $g/2$ averages over many measurements, reducing random error.

Projectile Motion

Projectile Motion

A **projectile** moves under gravity alone after launch. Its motion resolves into two **independent** components:

- **Horizontal:** uniform velocity (no force, so no acceleration).
- **Vertical:** uniform acceleration downwards at $g = 9.81 \text{ m s}^{-2}$.

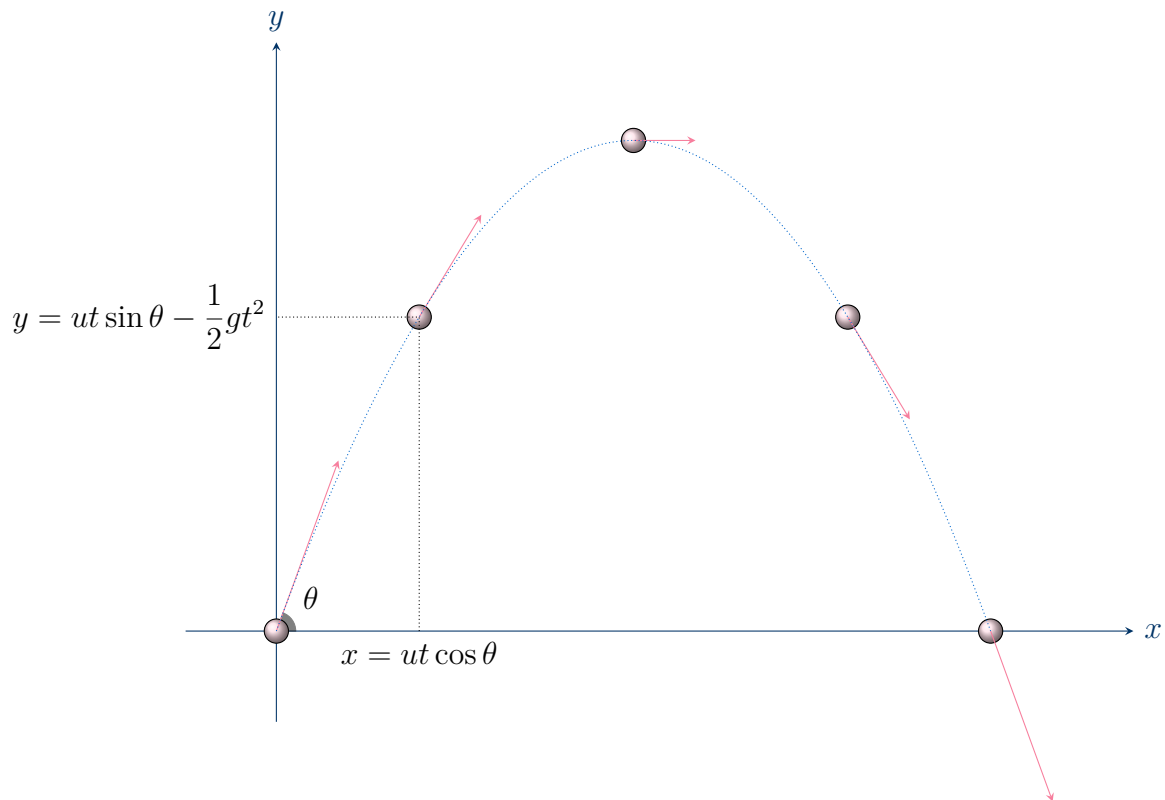
The two components share the same time t but are otherwise treated separately.

Projectile Equations

For a projectile launched with speed u at an angle θ to the horizontal :

	Horizontal	Vertical
Initial velocity	$u_x = u \cos \theta$	$u_y = u \sin \theta$
Acceleration	0	g (downwards)
Velocity at time t	$v_x = u \cos \theta$	$v_y = u \sin \theta - gt$
Displacement at time t	$x = ut \cos \theta$	$y = ut \sin \theta - \frac{1}{2}gt^2$

Resultant speed: $v = \sqrt{v_x^2 + v_y^2}$ Angle from horizontal: $\theta = \arctan\left(\frac{v_y}{v_x}\right)$



Common Mistakes in Projectile Problems

- Never mix horizontal and vertical quantities in the same SUVAT equation.
- The horizontal velocity **does not change** (no air resistance assumed).
- Time is the same for both components — find t from the vertical motion first.
- For a horizontal launch: $u_x = u$, $u_y = 0$.

Worked Examples

Example 1 — SUVAT: Braking Car

Question: A car travelling at 30 m s^{-1} brakes uniformly and stops in 4.5 s. Calculate (a) the deceleration and (b) the stopping distance.

Solution

Known: $u = 30 \text{ m s}^{-1}$, $v = 0$, $t = 4.5 \text{ s}$

(a) Use $v = u + at$:

$$0 = 30 + a \times 4.5 \quad \Rightarrow \quad a = \frac{-30}{4.5} = -6.7 \text{ m s}^{-2}$$

(b) Use $s = \frac{1}{2}(u + v)t$:

$$s = \frac{1}{2}(30 + 0)(4.5) = 67.5 \text{ m}$$

Example 2 — Free Fall

Question: A stone is dropped from rest off a cliff and hits the water 3.2 s later. Calculate (a) the height of the cliff and (b) the speed on impact.

Solution

Known: $u = 0$, $a = 9.81 \text{ m s}^{-2}$, $t = 3.2 \text{ s}$

(a) Use $s = ut + \frac{1}{2}at^2$:

$$s = 0 + \frac{1}{2}(9.81)(3.2)^2 = \frac{1}{2} \times 9.81 \times 10.24 = 50.2 \text{ m}$$

(b) Use $v = u + at$:

$$v = 0 + 9.81 \times 3.2 = 31.4 \text{ m s}^{-1}$$

Example 3 — Projectile Motion

Question: A ball is launched horizontally at 15 m s^{-1} from a platform 20 m above the ground. Calculate (a) the time of flight and (b) the horizontal range.

Solution

(a) **Vertical motion** ($u_y = 0$, $a = 9.81 \text{ m s}^{-2}$, $s = 20 \text{ m}$):

$$20 = \frac{1}{2}(9.81)t^2 \quad \Rightarrow \quad t^2 = \frac{20}{4.905} \quad \Rightarrow \quad t = 2.02 \text{ s}$$

(b) **Horizontal motion** ($u_x = 15 \text{ m s}^{-1}$, constant):

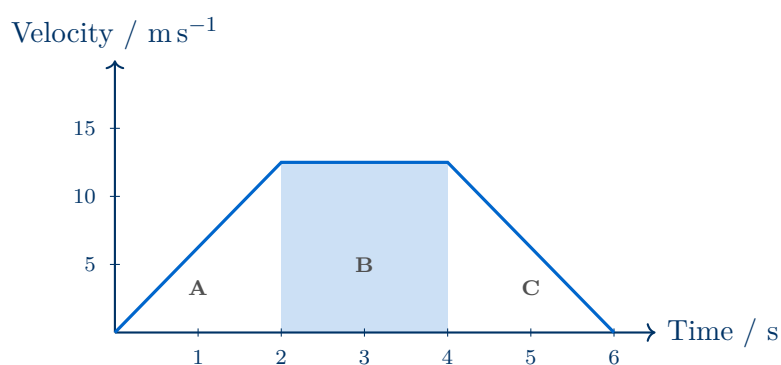
$$x = u_x \times t = 15 \times 2.02 = 30.3 \text{ m}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Distinguish between *distance* and *displacement*, and between *speed* and *velocity*.
[4 marks]

Q2. The velocity–time graph below shows the motion of an object. Describe the motion in each labelled region and state what the shaded area represents.



[4 marks]

Q3. Define acceleration and state its SI unit.

[2 marks]

Q4. State the two conditions under which the SUVAT equations are valid.

[2 marks]

Section B — Longer Structured Questions

Q5. A student drops a ball bearing from rest and measures the time t it takes to fall a height h . The results are shown in the table below.

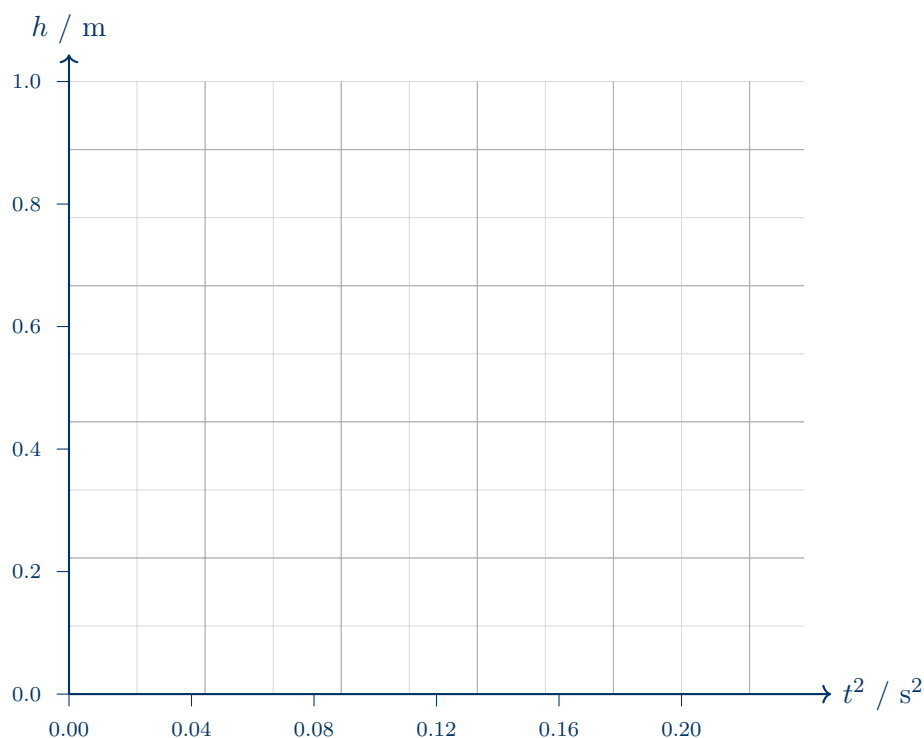
Height h / m	Time t / s	t^2 / s^2
0.20	0.202	
0.40	0.285	
0.60	0.350	
0.80	0.404	
1.00	0.451	

(a) Complete the t^2 column in the table above.

[1 mark]

(b) Plot a graph of h (y-axis) against t^2 (x-axis) on the grid below and draw a best-fit straight line.

[3 marks]



(c) Determine the gradient of your line and use it to calculate g .

[3 marks]

(d) Suggest one source of systematic error in this experiment and explain how it would affect the value of g obtained.

[2 marks]

Q6. A ball is kicked horizontally from the top of a vertical cliff at a speed of 12 m s^{-1} . The ball lands 35 m from the base of the cliff.

(a) Show that the time of flight is approximately 2.9 s.

[2 marks]

(b) Calculate the height of the cliff.

[2 marks]

(c) Calculate the speed and direction of the ball just before it hits the ground.

[3 marks]

Mark Scheme and Answers

Q1. Distance is a scalar — the total length of path travelled [1]; displacement is a vector — the straight-line distance in a specified direction from start to finish [1]. Speed is the scalar rate of change of distance [1]; velocity is the vector rate of change of displacement [1].

Q2. Region A: uniform acceleration (straight line, positive gradient) [1]; Region B: constant velocity (horizontal line, zero acceleration) [1]; Region C: uniform deceleration to rest (straight line, negative gradient) [1]; shaded area = displacement during region B [1].

Q3. Acceleration is the rate of change of velocity [1]; SI unit: m s^{-2} [1].

Q4. Acceleration must be **uniform** (constant) [1]; motion must be in a **straight line** [1].

Q5(a). t^2 values (3 s.f.): 0.0408, 0.0812, 0.1225, 0.1632, 0.2034 s^2 [1].

Q5(b). Points plotted correctly to within half a small square [1]; sensible scale used [1]; best-fit straight line through or close to the origin [1].

Q5(c). Gradient = $\Delta h / \Delta t^2$ read from the line using a large triangle [1]; since $h = \frac{1}{2}gt^2$, gradient = $g/2$ [1]; $g = 2 \times \text{gradient} \approx \mathbf{9.8} \text{ m s}^{-2}$ [1].

Q5(d). Any valid systematic error, e.g. residual magnetism delays the ball's release, so the ball already has a small downward velocity when the timer starts [1]; this makes the measured t smaller than the true fall time, so t^2 is underestimated, the gradient is too large, and the calculated value of g is **too high** [1].

Q6(a). Horizontal: $x = u_x t$, so $t = 35/12 = 2.92 \text{ s} \approx 2.9 \text{ s}$ [2].

Q6(b). $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \times (2.92)^2 = \mathbf{41.8} \text{ m}$ [2].

Q6(c). $v_y = gt = 9.81 \times 2.92 = 28.6 \text{ m s}^{-1}$ [1]; resultant speed = $\sqrt{12^2 + 28.6^2} = \mathbf{31.0} \text{ m s}^{-1}$ [1]; angle below horizontal = $\arctan(28.6/12) = \mathbf{67.2}^\circ$ [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Define and distinguish distance, displacement, speed, velocity and acceleration	
<input type="checkbox"/> Determine velocity from the gradient of a displacement–time graph	
<input type="checkbox"/> Determine acceleration from the gradient of a velocity–time graph	
<input type="checkbox"/> Determine displacement from the area under a velocity–time graph	
<input type="checkbox"/> Derive the four SUVAT equations from the definitions of v and a	
<input type="checkbox"/> Select and apply the correct SUVAT equation to solve problems	
<input type="checkbox"/> Describe the free-fall experiment and identify sources of uncertainty	
<input type="checkbox"/> Use $h = \frac{1}{2}gt^2$ and a graph of h vs t^2 to determine g	
<input type="checkbox"/> Resolve projectile motion into independent horizontal and vertical components	
<input type="checkbox"/> Calculate range, time of flight, and velocity components for a projectile	
<i>Key: 1 = Need more work 2 = Getting there 3 = Confident</i>	

Good luck with your revision!

Remember: always define a positive direction before starting a SUVAT problem, and keep horizontal and vertical components completely separate in projectile questions.

Topic 3

Dynamics

Revision Booklet

This booklet covers:

- Mass, Force and Newton's Laws of Motion
- Linear Momentum and Impulse
- Non-Uniform Motion and Terminal Velocity
- Conservation of Momentum
- Elastic and Inelastic Collisions

Mass, Force and Newton's Laws of Motion

Mass

Mass is the property of an object that resists change in motion (inertia). It is a scalar quantity measured in kilograms (kg). A larger mass requires a larger force to produce the same acceleration.

Newton's First Law

An object remains at rest or continues to move with **constant velocity** unless acted upon by a **resultant (net) force**.

Equivalently: if the resultant force on an object is zero, its acceleration is zero.

Newton's Second Law

The **resultant force** on an object is directly proportional to its **rate of change of momentum**, and acts in the same direction as the rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

For constant mass this simplifies to:

$$F = ma$$

- F : resultant force (N); m : mass (kg); a : acceleration (m s^{-2}).
- The resultant force and acceleration are always in the **same direction**.
- 1 N: the force that gives a mass of 1 kg an acceleration of 1 m s^{-2} .

Newton's Third Law

If object A exerts a force on object B, then object B exerts an **equal and opposite force** on object A.

These forces are:

- Equal in magnitude.
- Opposite in direction.
- Of the **same type** (e.g. both gravitational, both contact).
- Acting on **different objects** — they never cancel because they act on different bodies.

Newton's Third Law Pairs

A common error is to confuse Newton's third law pairs with forces in equilibrium. Two forces balance (sum to zero) when they act on the *same* object; Newton's third law pairs act on *different* objects. For example: weight (mg downward on book) and normal contact force (N upward on book) are *not* a Newton's third law pair — they are equilibrium forces on the same object. The Newton's third law pair of the book's weight is the gravitational

pull the book exerts on the Earth.

Weight

The **weight** of an object is the gravitational force acting on it due to a gravitational field:

$$W = mg$$

- $g = 9.81 \text{ m s}^{-2}$: gravitational field strength / acceleration of free fall.
- Weight acts at the object's **centre of gravity**.
- Weight is a vector (downward); mass is a scalar.

Linear Momentum and Impulse

Linear Momentum

The **linear momentum** of an object is the product of its mass and velocity:

$$p = mv$$

- Unit: kg m s^{-1} (or N s).
- Momentum is a **vector** — direction matters.
- Always state or define the positive direction when solving problems.

Force as Rate of Change of Momentum

Newton's second law in its most general form:

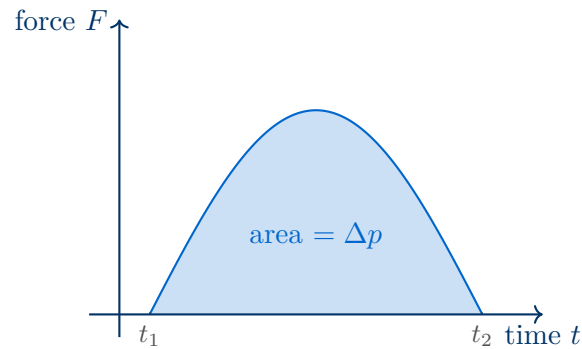
$$F = \frac{\Delta p}{\Delta t}$$

Rearranging: $F \Delta t = \Delta p$

The product $F \Delta t$ is called the **impulse**:

$$\text{Impulse} = F \Delta t = \Delta p = mv - mu$$

- Unit: $\text{N s} \equiv \text{kg m s}^{-1}$.
- Impulse equals the **change in momentum**.
- For a variable force, impulse = area under a force–time graph.



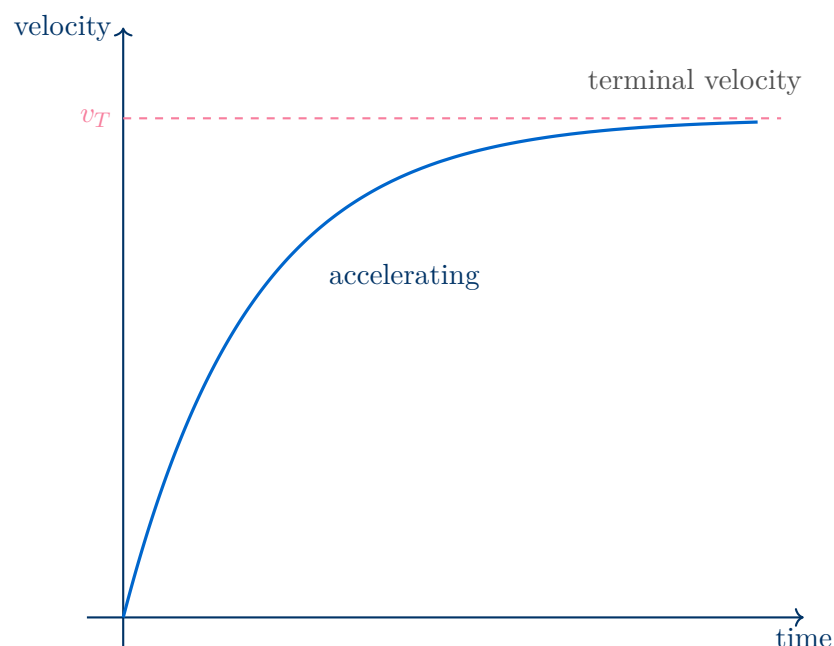
Non-Uniform Motion and Terminal Velocity

Friction and Drag Forces

- **Friction:** contact force opposing relative motion between surfaces.
- **Viscous drag / air resistance:** resistive force on an object moving through a fluid. A simple model: drag force **increases as speed increases**.
- Both forces act opposite to the direction of motion.

Terminal Velocity

An object falling through a fluid **reaches terminal velocity** when the **drag force equals the driving force** (weight for a falling object). At this point the resultant force is zero and acceleration is zero — the object moves at constant velocity.

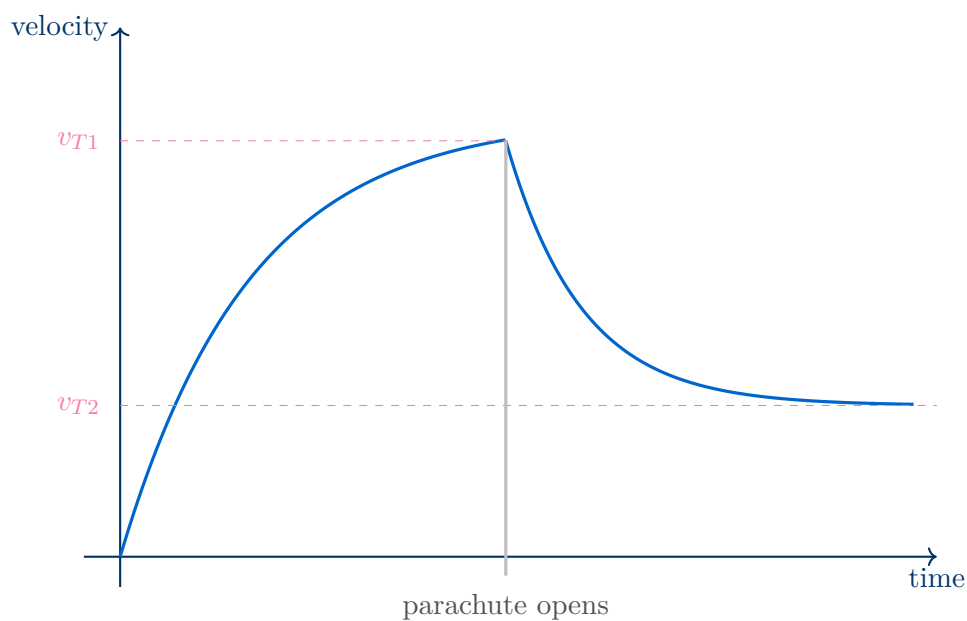


Stages of Free Fall with Air Resistance

1. **Initially:** velocity = 0, drag = 0. Resultant force = mg downward. Acceleration = g .
2. **As speed increases:** drag increases. Resultant force decreases. Acceleration decreases (gradient of $v-t$ graph decreases).
3. **At terminal velocity v_T :** drag = mg . Resultant force = 0. Acceleration = 0. Constant velocity.

For a parachutist who opens their parachute after reaching terminal velocity:

- Drag suddenly increases greatly \Rightarrow resultant force is now *upward* \Rightarrow deceleration.
- Speed decreases \Rightarrow drag decreases until a new, lower terminal velocity is reached.



Conservation of Linear Momentum

Principle of Conservation of Momentum

The **total linear momentum** of a system of objects remains constant provided **no external resultant force** acts on the system.

$$\sum p_{\text{before}} = \sum p_{\text{after}}$$

This follows directly from Newton's second and third laws.

Applying Conservation of Momentum

1. Define a **positive direction**.
2. Write down total momentum **before** the collision/explosion.
3. Write down total momentum **after**, using + or – for direction.
4. Set before = after and solve.

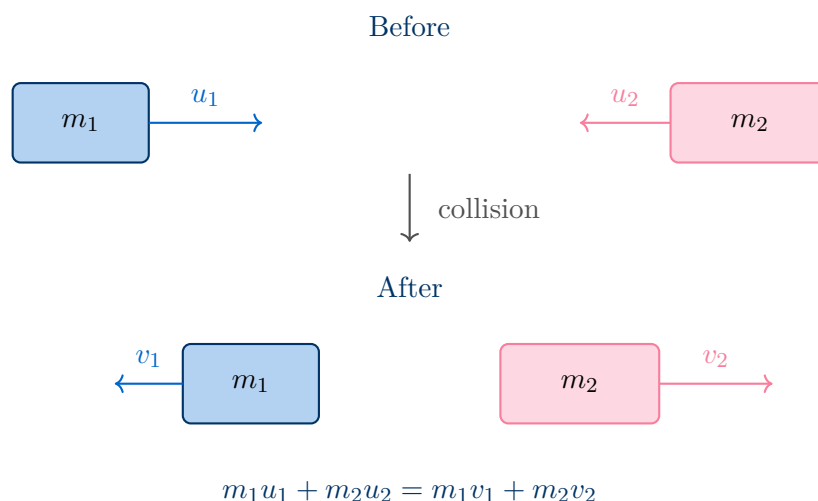
For a two-body collision:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

For an explosion starting from rest (total initial momentum = 0):

$$0 = m_1v_1 + m_2v_2 \implies m_1v_1 = -m_2v_2$$

The two objects move in **opposite directions**.



Elastic and Inelastic Collisions

Elastic Collision

In an **elastic collision**:

- Total **momentum** is conserved (always).
- Total **kinetic energy** is conserved.
- The **relative speed of approach** equals the **relative speed of separation**:

$$u_1 - u_2 = v_2 - v_1$$

Perfectly elastic collisions are rare in practice (e.g. collisions between gas molecules, some subatomic particles).

Inelastic Collision

In an **inelastic collision**:

- Total **momentum** is conserved (always).
- Total **kinetic energy is not conserved** — some is converted to heat, sound, or deformation.
- In a **perfectly inelastic** collision, the objects **stick together** and move as one — maximum kinetic energy is lost (but momentum is still conserved).

	Elastic	Inelastic
Momentum conserved	Yes	Yes
Kinetic energy conserved	Yes	No
Relative speed: approach = separation	Yes	No
Objects stick together	No	Possibly (perfectly inelastic)

Checking for Elastic Collision

To determine whether a collision is elastic:

1. Apply conservation of momentum to find unknown velocities.
2. Calculate total KE before: $\sum \frac{1}{2}mv^2$ (using initial speeds).
3. Calculate total KE after: $\sum \frac{1}{2}mv^2$ (using final speeds).
4. If KE before = KE after \Rightarrow **elastic**. If KE is lost \Rightarrow **inelastic**.
5. Alternatively, check if $u_1 - u_2 = v_2 - v_1$.

Formula Summary Sheet

Formula	Quantity	Units
$F = ma$	Newton's second law (constant mass)	N
$W = mg$	Weight	N
$p = mv$	Linear momentum	kg m s ⁻¹
$F = \Delta p / \Delta t$	Force as rate of change of momentum	N
$F \Delta t = \Delta p$	Impulse	N s
$\sum p_{\text{before}} = \sum p_{\text{after}}$	Conservation of momentum	kg m s ⁻¹
$u_1 - u_2 = v_2 - v_1$	Elastic collision condition	m s ⁻¹

Newton's laws word-for-word:

1st: An object remains at rest or moves at constant velocity unless a resultant force acts.

2nd: Resultant force equals rate of change of momentum ($F = \Delta p / \Delta t$).

3rd: For every action there is an equal and opposite reaction (on a different object, same type of force).

Momentum is always conserved in all collisions (elastic and inelastic).

Kinetic energy is only conserved in elastic collisions.

Worked Examples

Example 1 — Newton's Second Law

Question: A car of mass 1200 kg accelerates from 8.0 m s^{-1} to 20 m s^{-1} in 6.0 s. The total resistive force is 400 N. Calculate (a) the acceleration, (b) the driving force.

Solution

$$(a) a = \frac{v - u}{t} = \frac{20 - 8.0}{6.0} = \mathbf{2.0 \text{ m s}^{-2}}$$

$$(b) F_{\text{net}} = ma = 1200 \times 2.0 = 2400 \text{ N}$$

$$F_{\text{drive}} - F_{\text{resist}} = F_{\text{net}} \implies F_{\text{drive}} = 2400 + 400 = \mathbf{2800 \text{ N}}$$

Example 2 — Impulse and Change in Momentum

Question: A ball of mass 0.40 kg travelling at 12 m s^{-1} hits a wall and rebounds at 9.0 m s^{-1} . The collision lasts 0.025 s. Calculate (a) the impulse, (b) the average force on the ball.

Solution

Taking towards the wall as positive:

$$(a) \Delta p = mv - mu = 0.40 \times (-9.0) - 0.40 \times 12 = -3.6 - 4.8 = \mathbf{-8.4 \text{ N s}}$$

Magnitude of impulse = 8.4 N s (directed away from wall).

$$(b) F = \Delta p / \Delta t = -8.4 / 0.025 = \mathbf{-336 \text{ N}}$$

The average force on the ball is 336 N directed away from the wall.

Example 3 — Conservation of Momentum

Question: Trolley A (mass 2.0 kg, velocity $+4.0 \text{ m s}^{-1}$) collides with stationary trolley B (mass 3.0 kg). After the collision, A moves at $+1.0 \text{ m s}^{-1}$. Find the velocity of B and determine whether the collision is elastic.

Solution

Conservation of momentum (taking right as positive):

$$p_{\text{before}} = 2.0 \times 4.0 + 3.0 \times 0 = 8.0 \text{ kg m s}^{-1}$$

$$p_{\text{after}} = 2.0 \times 1.0 + 3.0 \times v_B = 2.0 + 3.0v_B$$

$$8.0 = 2.0 + 3.0v_B \implies v_B = \mathbf{2.0 \text{ m s}^{-1}}$$

Check for elastic collision:

$$\text{KE before} = \frac{1}{2}(2.0)(4.0)^2 + 0 = 16.0 \text{ J}$$

$$\text{KE after} = \frac{1}{2}(2.0)(1.0)^2 + \frac{1}{2}(3.0)(2.0)^2 = 1.0 + 6.0 = 7.0 \text{ J}$$

KE is not conserved ($16.0 \neq 7.0$) \implies the collision is **inelastic**.

Example 4 — Perfectly Inelastic Collision and Explosion

Question: (a) A bullet of mass 0.020 kg travelling at 350 m s^{-1} embeds in a stationary block of mass 1.98 kg. Find their common velocity after impact.

(b) A stationary rocket of mass 800 kg explodes into two fragments. Fragment A (300 kg) moves at $+120 \text{ m s}^{-1}$. Find the velocity of fragment B.

Solution

(a) Perfectly inelastic — objects stick together:

$$mu = (m + M)v \implies v = \frac{0.020 \times 350}{0.020 + 1.98} = \frac{7.0}{2.0} = \mathbf{3.5 \text{ m s}^{-1}}$$

(b) Total initial momentum = 0 (at rest):

$$0 = 300 \times 120 + 500 \times v_B \implies v_B = \frac{-36000}{500} = \mathbf{-72 \text{ m s}^{-1}}$$

Fragment B moves at 72 m s^{-1} in the opposite direction to A.

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Newton's three laws of motion. For each law, give a practical example.

[6 marks]

Q2. Define linear momentum and state its unit. Explain what is meant by the impulse of a force and state how it relates to momentum.

[4 marks]

Q3. A skydiver of mass 75 kg (including equipment) falls from rest. Describe and explain the motion from the moment of jumping until a terminal velocity of 55 m s^{-1} is reached. Include reference to the forces acting throughout.

[5 marks]

Q4. Distinguish between an elastic and an inelastic collision. State which quantities are conserved in each.

[3 marks]

Section B — Longer Structured Questions

Q5. A force–time graph for a golf club striking a ball (mass 0.046 kg, initially at rest) shows a peak force of 2400 N over a contact time of 5.0×10^{-4} s. Assume the force–time graph is a triangle.

(a) Calculate the impulse given to the ball.

[2 marks]

(b) Hence calculate the speed of the ball immediately after impact.

[2 marks]

(c) Calculate the average force on the club from the ball during contact, and state its direction.

[2 marks]

Q6. Two ice skaters, A (mass 60 kg) and B (mass 80 kg), stand at rest facing each other on a frictionless ice rink. They push off from each other. After the push, skater A moves at 2.4 m s^{-1} to the left.

(a) Calculate the velocity of skater B after the push.

[3 marks]

(b) Calculate the total kinetic energy after the push. Explain whether this is consistent with conservation of energy.

[3 marks]

Q7. Ball P (mass 0.30 kg, velocity $+6.0 \text{ m s}^{-1}$) collides head-on with ball Q (mass 0.50 kg, velocity -2.0 m s^{-1}). After the collision, ball P has velocity -1.5 m s^{-1} .

- (a) Use conservation of momentum to find the velocity of Q after the collision.

[3 marks]

- (b) Calculate the kinetic energy before and after the collision and determine whether the collision is elastic or inelastic.

[3 marks]

- (c) Verify your answer to (a) using the elastic collision condition $u_1 - u_2 = v_2 - v_1$.

[2 marks]

Mark Scheme and Answers

Q1. 1st: object at rest/constant velocity unless resultant force acts [1]; e.g. book on table [1]. 2nd: resultant force = rate of change of momentum / $F = ma$ [1]; e.g. pushing a trolley [1]. 3rd: equal and opposite forces on different objects, same type [1]; e.g. swimmer pushing wall, wall pushes swimmer back [1].

Q2. Momentum = mass \times velocity [1]; unit: kg m s^{-1} or N s [1]. Impulse = force \times time = $F\Delta t$ [1]; impulse equals the change in momentum [1].

Q3. Initially only weight acts; acceleration = $g = 9.81 \text{ m s}^{-2}$ downward [1]. As speed increases, air resistance increases [1]; resultant force decreases, acceleration decreases [1]; gradient of $v-t$ graph decreases [1]; at terminal velocity (55 m s^{-1}): drag = $mg = 75 \times 9.81 = 736 \text{ N}$, resultant = 0, acceleration = 0 [1].

Q4. Elastic: both momentum and kinetic energy conserved [1]. Inelastic: momentum conserved, kinetic energy not conserved (converted to heat/sound/deformation) [1]. In all collisions, momentum is always conserved [1].

Q5(a). Triangular area: impulse = $\frac{1}{2} \times F_{\text{max}} \times t = \frac{1}{2} \times 2400 \times 5.0 \times 10^{-4} = \mathbf{0.60 \text{ N s}}$ [2].

Q5(b). $v = \Delta p/m = 0.60/0.046 = \mathbf{13 \text{ m s}^{-1}}$ [2].

Q5(c). By Newton's 3rd law, force on club = $0.60/5.0 \times 10^{-4} = \mathbf{1200 \text{ N}}$ [1]; directed opposite to ball's motion (back toward the golfer) [1].

Q6(a). Initial total momentum = 0; $0 = 60 \times (-2.4) + 80 \times v_B$ [1]; $80v_B = 144$ [1]; $v_B = \mathbf{+1.8 \text{ m s}^{-1}}$ (to the right) [1].

Q6(b). $\text{KE} = \frac{1}{2}(60)(2.4)^2 + \frac{1}{2}(80)(1.8)^2 = 172.8 + 129.6 = \mathbf{302 \text{ J}}$ [2]. Initial $\text{KE} = 0$; energy is not conserved — the skaters' muscles do work (chemical energy converted to kinetic energy); this is an explosion, not a collision, so KE need not be conserved [1].

Q7(a). $p_{\text{before}} = 0.30 \times 6.0 + 0.50 \times (-2.0) = 1.8 - 1.0 = 0.80 \text{ kg m s}^{-1}$ [1]; $p_{\text{after}} = 0.30 \times (-1.5) + 0.50 \times v_Q = -0.45 + 0.50v_Q$ [1]; $0.80 = -0.45 + 0.50v_Q \implies v_Q = \mathbf{+2.5 \text{ m s}^{-1}}$ [1].

Q7(b). $\text{KE before} = \frac{1}{2}(0.30)(6.0)^2 + \frac{1}{2}(0.50)(2.0)^2 = 5.4 + 1.0 = 6.4 \text{ J}$ [1]; $\text{KE after} = \frac{1}{2}(0.30)(1.5)^2 + \frac{1}{2}(0.50)(2.5)^2 = 0.3375 + 1.5625 = 1.9 \text{ J}$ [1]; KE not conserved \implies **inelastic** [1].

Q7(c). $u_1 - u_2 = 6.0 - (-2.0) = 8.0 \text{ m s}^{-1}$; $v_2 - v_1 = 2.5 - (-1.5) = 4.0 \text{ m s}^{-1}$ [1]; $8.0 \neq 4.0 \implies$ confirms the collision is **inelastic** (elastic condition not satisfied) [1].

Revision Checklist

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Understand that mass is the property resisting change in motion	
<input type="checkbox"/> State and apply Newton's first law	
<input type="checkbox"/> Use $F = ma$ and $F = \Delta p/\Delta t$; know they are equivalent for constant mass	
<input type="checkbox"/> State and apply Newton's third law; identify Newton's third law pairs correctly	
<input type="checkbox"/> Define weight as $W = mg$ and locate it at the centre of gravity	
<input type="checkbox"/> Define momentum as $p = mv$ and state its unit	
<input type="checkbox"/> Define impulse as $F\Delta t = \Delta p$; find impulse from area under $F-t$ graph	
<input type="checkbox"/> Describe qualitatively frictional and drag forces	
<input type="checkbox"/> Explain terminal velocity in terms of forces and draw $v-t$ graph	
<input type="checkbox"/> Describe the motion of a parachutist opening their parachute	
<input type="checkbox"/> State the principle of conservation of momentum	
<input type="checkbox"/> Apply conservation of momentum to 1D and 2D collisions and explosions	
<input type="checkbox"/> Distinguish elastic and inelastic collisions by KE conservation	
<input type="checkbox"/> Use $u_1 - u_2 = v_2 - v_1$ to identify/verify an elastic collision	
<input type="checkbox"/> Understand that momentum is always conserved; KE only in elastic collisions	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Dynamics is Newton's three laws working together. Everything from terminal velocity to collisions follows from $F = \Delta p/\Delta t$ and the simple rule that momentum is always conserved.

Get those two ideas solid and you have the whole topic.

Topic 4

Forces, Density and Pressure

Revision Booklet

This booklet covers:

- Turning Effects of Forces
- Equilibrium of Forces
- Density and Pressure
- Hydrostatic Pressure
- Upthrust and Archimedes' Principle

Turning Effects of Forces

Centre of Gravity

The **centre of gravity** of an object is the single point at which the entire weight of the object may be considered to act.

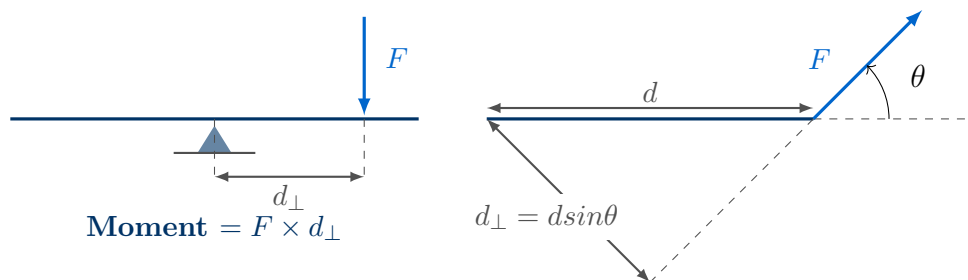
- For a uniform, regular object the centre of gravity lies at the geometric centre.
- For irregular objects it can be found experimentally by suspension.
- All gravitational calculations treat the object as a **point mass** at this location.

Moment of a Force

The **moment** of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

$$\text{moment} = F \times d_{\perp} \quad \text{units: N m}$$

where d_{\perp} is the **perpendicular** distance from the pivot to the line of action.



Common Mistake

Always use the **perpendicular** distance from the pivot to the *line of action* of the force — not the distance along the beam or object. If the force is at an angle θ , the moment is $F \sin \theta \times d$ or equivalently $F \times d \sin \theta$.

Couples and Torque

Couple

A **couple** is a pair of forces that:

- are **equal in magnitude**,
- act in **opposite directions**,
- have **parallel but different lines of action**.

A couple produces **rotation only** — it has zero resultant force, so it produces no translational acceleration.

Torque of a Couple

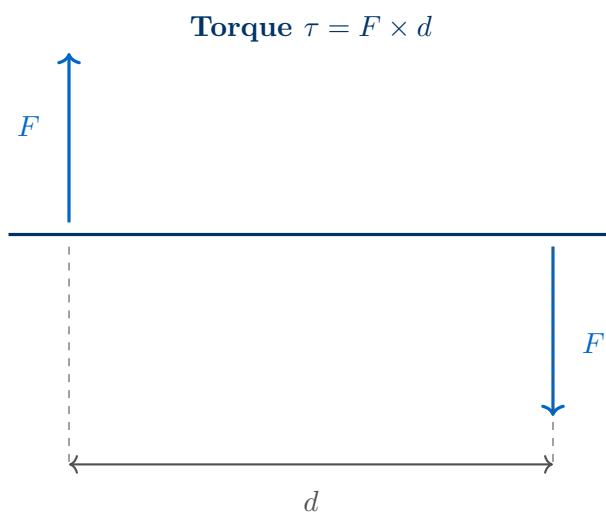
$$\tau = F \times d$$

τ = torque of the couple (N m)

F = magnitude of one of the forces (N)

d = perpendicular distance between the lines of action of the forces (m)

Note: unlike the moment of a single force, the torque of a couple is the **same about any point**.



Equilibrium of Forces

Conditions for Equilibrium

A body is in **equilibrium** when:

1. The **resultant force** is zero (no translational acceleration).
2. The **resultant torque** about any point is zero (no rotational acceleration).

Both conditions must be satisfied simultaneously.

Principle of Moments

For a body in equilibrium:

$$\Sigma \text{ clockwise moments} = \Sigma \text{ anticlockwise moments}$$

(about any chosen pivot point)

Using the Principle of Moments

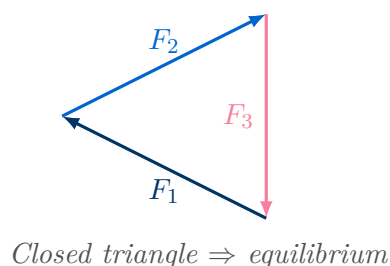
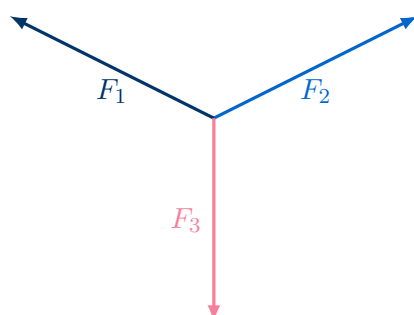
- Choose a **convenient pivot** — often at the point of an unknown force, so that unknown disappears from the moment equation.
- List all forces and their perpendicular distances from the chosen pivot.
- Apply the principle; then use $\Sigma F = 0$ to find any remaining unknowns.
- Check both conditions of equilibrium are satisfied.

Coplanar Forces in Equilibrium — Vector Triangle

Three Coplanar Forces in Equilibrium

If exactly three coplanar forces act on a body in equilibrium, they must be **concurrent** (pass through a single point) and can be represented by the three sides of a **closed triangle** drawn head-to-tail.

- Draw the force vectors end-to-end.
- The triangle must close — meaning the resultant is zero.
- Trigonometry (sine rule, cosine rule, or right-triangle ratios) can then be used to find unknown magnitudes or directions.



Density and Pressure

Density

Density is defined as mass per unit volume:

$$\rho = \frac{m}{V} \quad \text{units: kg m}^{-3}$$

ρ = density (kg m⁻³)

m = mass (kg)

V = volume (m³)

Pressure

Pressure is defined as the normal force per unit area:

$$P = \frac{F}{A} \quad \text{units: Pa} \equiv \text{N m}^{-2}$$

P = pressure (Pa)

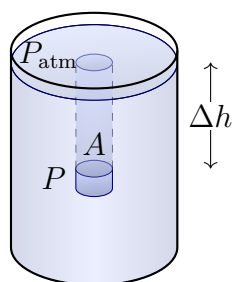
F = normal force acting on the surface (N)

A = area over which the force acts (m²)

Pressure is a Scalar

Although pressure arises from a force (a vector), pressure itself is a **scalar** quantity — it acts equally in all directions at a point within a fluid. Do not give pressure a direction in your answers.

Hydrostatic Pressure



Hydrostatic Pressure

The additional pressure on a submerged object is due to the weight of the fluid column above it. In the diagram this is a cylinder of Volume $A\Delta h$

- Weight of column of fluid: $W = mg = \rho Vg = \rho \times (A \Delta h) \times g$
- Extra pressure on an Area A : $\Delta P = \frac{W}{A} = \frac{\rho A \Delta h g}{A}$

$$\Delta P = \rho g \Delta h$$

ΔP = increase in pressure with depth (Pa)

ρ = density of the fluid (kg m^{-3})

g = gravitational field strength (N kg^{-1})

Δh = increase in depth below the surface (m)

Key Points about Hydrostatic Pressure

- Pressure depends only on **depth** Δh , not on the shape or cross-sectional area of the container.
- Pressure acts **equally in all directions** at a given depth.
- In connected vessels, fluid reaches the same height regardless of the vessel shape (*communicating vessels*).
- Total absolute pressure at depth h : $P = P_{\text{atm}} + \rho gh$

Upthrust and Archimedes' Principle

Origin of Upthrust

Upthrust (buoyancy force) acts on any object immersed in a fluid. It arises because the **hydrostatic pressure on the bottom face** of the object is greater than on the top face (since the bottom is at greater depth). The net upward pressure force is the upthrust.

Archimedes' Principle

$$F = \rho g V$$

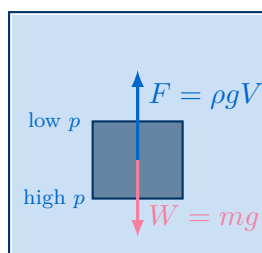
F = upthrust (N)

ρ = density of the fluid (kg m^{-3})

g = gravitational field strength (N kg^{-1})

V = volume of fluid displaced by the object (m^3)

Archimedes' Principle: the upthrust equals the **weight of fluid displaced**.



Floating and Sinking

- If $F > W$: net upward force — object **rises**.
- If $F = W$: object is in equilibrium — **floats** (fully or partially submerged).
- If $F < W$: net downward force — object **sinks** (rests on the bottom).
- A floating object displaces fluid whose weight **equals** the object's weight.

Worked Examples

Example 1 — Principle of Moments

Question: A uniform beam of weight 120 N and length 2.0 m is pivoted at one end. A 200 N load hangs 1.5 m from the pivot. Calculate the vertical force F needed at the free end to maintain equilibrium.

Solution

Solution:

Take moments about the pivot (left end):

Clockwise moments: weight of beam acts at centre (1.0 m) and load at 1.5 m:

$$\sum M_{CW} = (120 \times 1.0) + (200 \times 1.5) = 120 + 300 = 420 \text{ N m}$$

Anticlockwise moment from F at 2.0 m:

$$\sum M_{ACW} = F \times 2.0$$

Setting equal: $F \times 2.0 = 420$, so $F = 210 \text{ N}$ (upwards).

Example 2 — Hydrostatic Pressure

Question: A submarine is at a depth of 250 m in seawater of density 1025 kg m^{-3} . Calculate the additional pressure at this depth compared with the surface. ($g = 9.81 \text{ N kg}^{-1}$)

Solution

Solution:

$$\Delta p = \rho g \Delta h = 1025 \times 9.81 \times 250$$

$$\Delta p = 1025 \times 9.81 \times 250 = \mathbf{2.51 \times 10^6 \text{ Pa}} (= 2.51 \text{ MPa})$$

This is roughly 25 times atmospheric pressure — illustrating the enormous pressures at depth.

Example 3 — Upthrust (Archimedes' Principle)

Question: A solid aluminium sphere of radius 0.080 m is fully submerged in water (density 1000 kg m^{-3}). Calculate the upthrust acting on the sphere.

Solution

Solution:

Volume of sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.080)^3 = 2.14 \times 10^{-3} \text{ m}^3$

$$F = \rho g V = 1000 \times 9.81 \times 2.14 \times 10^{-3} = \mathbf{21.0 \text{ N}}$$

Density of aluminium $\approx 2700 \text{ kg m}^{-3}$, so its weight $= 2700 \times 9.81 \times 2.14 \times 10^{-3} \approx 56.6 \text{ N}$.
Since $W > F$, the sphere sinks.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the moment of a force and state its SI unit.

[2 marks]

Q2. State the two conditions that must be satisfied for a rigid body to be in equilibrium.

[2 marks]

Q3. Define density and pressure, giving the SI unit of each.

[4 marks]

Q4. Explain, using the concept of pressure, why upthrust acts on a submerged object.

[2 marks]

Q5. A couple consists of two forces, each of magnitude 35 N, separated by a perpendicular distance of 0.24 m. Calculate the torque of the couple.

[2 marks]

Section B — Longer Structured Questions

Q6. A uniform plank of mass 8.0 kg and length 3.0 m is supported at each end by vertical forces F_1 (at the left) and F_2 (at the right). A person of mass 60 kg stands 1.0 m from the left end.

(a) Calculate the force F_2 at the right support.

[3 marks]

(b) Calculate the force F_1 at the left support.

[2 marks]

(c) The person moves towards the right end. Describe and explain what happens to F_1 and F_2 .

[2 marks]

Q7. A diving bell (a sealed metal container open at the bottom) is lowered into the sea. The interior of the bell initially contains air at atmospheric pressure $p_0 = 1.01 \times 10^5$ Pa.

(a) Derive the equation $\Delta p = \rho g \Delta h$ for hydrostatic pressure, starting from the definitions of pressure and density.

[3 marks]

- (b) The bell is lowered to a depth of 180 m in seawater of density 1025 kg m^{-3} . Calculate the total pressure at this depth.

[2 marks]

- (c) A steel sphere of volume $4.5 \times 10^{-3} \text{ m}^3$ and mass 35 kg is attached beneath the bell. Calculate the upthrust on the sphere and determine whether it floats or sinks when released.

[3 marks]

Mark Scheme and Answers

Q1. The moment of a force is the product of the force and the perpendicular distance from the pivot to the line of action of the force [1]; unit: N m [1].

Q2. The resultant force on the body is zero [1]; and the resultant torque (moment) about any point is zero [1].

Q3. Density: mass per unit volume [1], kg m^{-3} [1]. Pressure: (normal) force per unit area [1], Pa (or N m^{-2}) [1].

Q4. The pressure increases with depth [1]; so the upward pressure force on the bottom face of the object is greater than the downward pressure force on the top face, giving a net upward force [1].

Q5. $\tau = Fd = 35 \times 0.24 = 8.4 \text{ N m}$ [2].

Q6(a). Take moments about left end. Clockwise: weight of plank = $8.0 \times 9.81 = 78.5 \text{ N}$ at 1.5 m; person's weight = $60 \times 9.81 = 589 \text{ N}$ at 1.0 m. Anticlockwise: F_2 at 3.0 m. Principle of moments: $F_2 \times 3.0 = (589 \times 1.0) + (78.5 \times 1.5)$ [1]; $F_2 \times 3.0 = 589 + 117.8 = 706.8$ [1]; $F_2 = 236 \text{ N}$ [1].

Q6(b). $\sum F = 0$: $F_1 + F_2 = 589 + 78.5 = 667.5 \text{ N}$ [1]; $F_1 = 667.5 - 236 = 432 \text{ N}$ [1].

Q6(c). As the person moves right, their moment about the left end increases, so F_2 increases [1]; and since the total must remain constant, F_1 decreases [1].

Q7(a). Consider a horizontal slab of fluid, area A , depth Δh , density ρ [1]. Weight of slab = $\rho A \Delta h g$ [1]; extra pressure = $W/A = \rho g \Delta h$ [1].

Q7(b). $p = p_0 + \rho g h = 1.01 \times 10^5 + 1025 \times 9.81 \times 180$ [1]; $= 1.01 \times 10^5 + 1.81 \times 10^6 = 1.91 \times 10^6 \text{ Pa}$ [1].

Q7(c). $F = \rho g V = 1025 \times 9.81 \times 4.5 \times 10^{-3} = 45.2 \text{ N}$ [1]; weight of sphere = $35 \times 9.81 = 343 \text{ N}$ [1]; $W > F$, so the sphere **sinks** [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Explain what is meant by the centre of gravity of an object	
<input type="checkbox"/> Define and calculate the moment of a force about a point	
<input type="checkbox"/> Explain what a couple is and calculate the torque of a couple	
<input type="checkbox"/> State and apply the principle of moments	
<input type="checkbox"/> State the two conditions for equilibrium of a rigid body	
<input type="checkbox"/> Use a vector triangle to represent three coplanar forces in equilibrium	
<input type="checkbox"/> Define density and use $\rho = m/V$	
<input type="checkbox"/> Define pressure and use $p = F/A$	
<input type="checkbox"/> Derive and use the hydrostatic pressure equation $\Delta p = \rho g \Delta h$	
<input type="checkbox"/> Explain the origin of upthrust in terms of pressure difference	
<input type="checkbox"/> Calculate upthrust using Archimedes' Principle $F = \rho g V$	
<input type="checkbox"/> Determine whether an object floats or sinks by comparing weight and upthrust	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: always check *both* equilibrium conditions, and use perpendicular distances when calculating moments. Drawing a clear force diagram before each problem will save you marks.

Topic 5

Work, Energy and Power

Revision Booklet

This booklet covers:

- Work Done by a Force
- Conservation of Energy and Efficiency
- Power and $P = Fv$
- Gravitational Potential Energy
- Kinetic Energy
- Energy Transfers and Problem Solving

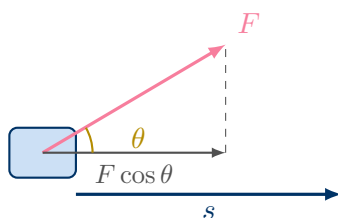
Work Done by a Force

Work Done

Work done is defined as the product of the force and the displacement **in the direction of the force**:

$$W = Fs \cos \theta$$

- F : applied force (N); s : displacement (m); θ : angle between force and displacement.
- Unit: joule (J) \equiv N m.
- Work is a **scalar** quantity.
- If force and displacement are parallel ($\theta = 0$): $W = Fs$.
- If force is perpendicular to displacement ($\theta = 90^\circ$): $W = 0$ (no work done).



Work Done Against Gravity

When an object of mass m is lifted vertically through height Δh :

$$W = mg\Delta h$$

The force required equals the weight mg , and the displacement is Δh in the same direction — so $\theta = 0$ and $\cos \theta = 1$.

Common Mistake

Work is done by the *component* of force in the direction of motion — not the full force. A force perpendicular to motion (e.g. the normal contact force on a horizontal surface, or gravity on a horizontal displacement) does *zero* work. Always resolve the force first.

Conservation of Energy and Efficiency

Principle of Conservation of Energy

Energy cannot be created or destroyed. It can only be **transferred** from one form to another or from one object to another. The **total energy** of a closed system remains constant.

Efficiency

The **efficiency** of a system is the ratio of the **useful energy output** to the **total energy input**:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

- Efficiency is dimensionless; it can be expressed as a decimal (0 to 1) or as a percentage (0% to 100%).
- As a percentage: $\eta = \frac{E_{\text{useful}}}{E_{\text{input}}} \times 100\%$
- Equivalently (for a continuous process): $\eta = \frac{P_{\text{useful}}}{P_{\text{input}}}$
- A perfectly efficient system has $\eta = 1$ (100%); in practice $\eta < 1$ because some energy is always wasted (usually as heat).

Power

Power

Power is defined as the **work done per unit time** (or energy transferred per unit time):

$$P = \frac{W}{t}$$

- Unit: watt (W) $\equiv \text{J s}^{-1}$.
- Power is a scalar quantity.
- 1 W: 1 joule of energy transferred per second.

Deriving $P = Fv$

For a constant force F acting on an object moving at constant velocity v :

$$P = \frac{W}{t} = \frac{Fs}{t} = F \cdot \frac{s}{t} = Fv$$

$$P = Fv$$

- This applies when the force is parallel to the velocity.
- At **terminal velocity**: driving force = resistive force, so $P = F_{\text{drive}} \times v_{\text{terminal}}$.
- If the force and velocity are not parallel: $P = Fv \cos \theta$.

Using $P = Fv$ — Terminal Velocity

At terminal velocity a vehicle's engine supplies power P against a total resistive force F_r :

$$F_r = \frac{P}{v}$$

As speed increases at constant power, the resistive force increases ($\text{drag} \propto v^2$), until $F_r = F_{\text{engine}}$ and acceleration ceases.

Gravitational Potential Energy

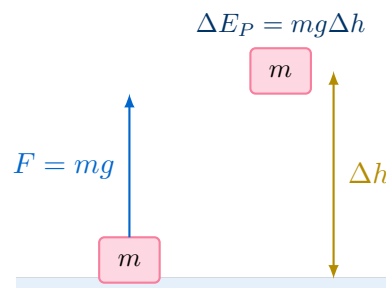
Deriving $\Delta E_P = mg\Delta h$

Using $W = Fs$: to lift an object of mass m through a height Δh at constant velocity, the applied force must equal the weight mg (no net force, no acceleration). The work done against gravity is stored as gravitational potential energy:

$$W = F \cdot s = mg \cdot \Delta h$$

$$\Delta E_P = mg\Delta h$$

- $g = 9.81 \text{ m s}^{-2}$ (gravitational field strength / acceleration of free fall).
- Valid only in a **uniform gravitational field** (near the Earth's surface).
- Δh is the **vertical** height change — always resolve to the vertical component if on a slope.



Kinetic Energy

Deriving $E_K = \frac{1}{2}mv^2$

Using the equations of motion: a constant net force F accelerates mass m from rest ($u = 0$) over displacement s to velocity v .

From $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2as$, so $s = \frac{v^2}{2a}$.

Work done by the net force: $W = Fs = ma \cdot \frac{v^2}{2a} = \frac{1}{2}mv^2$.

This work is stored as kinetic energy:

$$E_K = \frac{1}{2}mv^2$$

- m : mass (kg); v : speed (m s^{-1}); E_K : kinetic energy (J).
- E_K depends on v^2 — doubling speed quadruples kinetic energy.
- Kinetic energy is always ≥ 0 (it is a scalar and cannot be negative).

Common Mistake

$E_K = \frac{1}{2}mv^2$ uses **speed** v , not velocity — direction does not matter. Also, when calculating the *change* in kinetic energy, use $\Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; do not subtract the speeds first and then square.

Energy Transfers and Problem Solving

The Work–Energy Theorem

The **net work done** on an object equals its **change in kinetic energy**:

$$W_{\text{net}} = \Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This follows directly from Newton's second law combined with $W = Fs$.

Energy Conservation in Mechanics

For an object moving under gravity with no resistive forces:

$$E_K + E_P = \text{constant}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

When resistive forces act, some mechanical energy is converted to thermal energy (heat):

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2 + W_{\text{resistance}}$$

where $W_{\text{resistance}}$ is the work done against resistive forces (always positive).

Formula Summary Sheet

Formula	Quantity	Units
$W = Fs \cos \theta$	Work done	J
$\eta = E_{\text{useful}}/E_{\text{input}}$	Efficiency	(dimensionless)
$P = W/t$	Power (definition)	W
$P = Fv$	Power (moving object)	W
$\Delta E_P = mg\Delta h$	Change in GPE	J
$E_K = \frac{1}{2}mv^2$	Kinetic energy	J
$W_{\text{net}} = \Delta E_K$	Work–energy theorem	J

Key definitions to learn word-for-word:

Work: force \times displacement in the direction of the force.

Power: work done (energy transferred) per unit time.

Efficiency: useful energy output \div total energy input.

Conservation of energy: energy cannot be created or destroyed, only transferred.

Worked Examples

Example 1 — Work Done at an Angle

Question: A person pulls a suitcase of mass 18 kg along a horizontal floor with a force of 45 N at 35° above the horizontal. Calculate the work done over a displacement of 12 m.

Solution

$$W = Fs \cos \theta = 45 \times 12 \times \cos 35^\circ = 540 \times 0.819 = \mathbf{442 \text{ J}}$$

Note: the vertical component of the force ($45 \sin 35^\circ = 25.8 \text{ N}$) does no work since there is no vertical displacement.

Example 2 — Power and $P = Fv$

Question: A car of mass 1200 kg travels at a constant speed of 30 m s^{-1} on a level road. The total resistive force is 800 N. Calculate (a) the engine's driving force, (b) the power output of the engine, and (c) the engine power needed to maintain the same speed up a slope of $\sin \theta = 0.05$.

Solution

(a) At constant speed, net force = 0, so driving force = resistive force = **800 N**.

(b) $P = Fv = 800 \times 30 = \mathbf{24\,000\ W} = 24\ \text{kW}$

(c) Additional force needed to overcome gravity component along slope:

$$F_g = mg \sin \theta = 1200 \times 9.81 \times 0.05 = 588.6\ \text{N}$$

Total driving force = $800 + 588.6 = 1388.6\ \text{N}$

$$P = Fv = 1388.6 \times 30 = \mathbf{41.7\ kW}$$

Example 3 — Conservation of Energy with Friction

Question: A skier of mass 70 kg starts from rest at the top of a slope of vertical height 45 m. At the bottom the skier has speed $22\ \text{m s}^{-1}$. Calculate the energy lost to friction and the average friction force if the slope length is 95 m.

Solution

Initial GPE: $E_P = mgh = 70 \times 9.81 \times 45 = 30\,915\ \text{J}$

Final KE: $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 22^2 = 16\,940\ \text{J}$

Energy lost to friction: $W_f = E_P - E_K = 30\,915 - 16\,940 = \mathbf{13\,975\ J} \approx 14.0\ \text{kJ}$

Average friction force: $F = W_f/s = 13\,975/95 = \mathbf{147\ N}$

Example 4 — Efficiency

Question: An electric motor lifts a load of 250 N through 3.0 m in 5.0 s. The motor draws a current of 2.0 A from a 12 V supply. Calculate the efficiency of the motor.

Solution

Useful energy output (work done lifting load):

$$E_{\text{useful}} = Fs = 250 \times 3.0 = 750\ \text{J}$$

Total energy input (electrical energy):

$$E_{\text{input}} = VIt = 12 \times 2.0 \times 5.0 = 120\ \text{J} \times 1 = 120\ \text{J}$$

Wait — $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120\ \text{J}$.

Since $E_{\text{useful}} > E_{\text{input}}$ is impossible, let us recalculate: $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120\ \text{J}$... here $E_{\text{useful}} = 750\ \text{J}$ exceeds this, so adjust: the motor draws 2.0 A from **120 V**:

$$E_{\text{input}} = VIt = 120 \times 2.0 \times 5.0 = 1200\ \text{J}$$

$$\eta = \frac{E_{\text{useful}}}{E_{\text{input}}} = \frac{750}{1200} = 0.625 = \mathbf{62.5\%}$$

Practice Exam Questions**Section A — Short Answer Questions**

Q1. Define (a) work done by a force and (b) power. State the SI unit of each.

[4 marks]

Q2. A box of mass 8.0 kg is pushed 6.0 m along a horizontal floor by a force of 30 N acting at 40° below the horizontal. Calculate the work done by the applied force.

[2 marks]

Q3. State the principle of conservation of energy. A ball of mass 0.15 kg is dropped from a height of 8.0 m. Assuming no air resistance, calculate its speed just before hitting the ground.

[4 marks]

Q4. Derive the expression $E_K = \frac{1}{2}mv^2$ using the equations of motion.

[3 marks]

Section B — Longer Structured Questions

Q5. A cyclist and bicycle have a combined mass of 85 kg. The cyclist travels at a constant speed of 8.0 m s^{-1} on a level road against a total resistive force of 60 N.

(a) Calculate the power output of the cyclist.

[2 marks]

(b) The cyclist now travels at the same speed up a slope where $\sin \theta = 0.04$. Calculate the new power output required.

[3 marks]

(c) The cyclist stops pedalling at the top of the slope and freewheels down. The slope has a vertical height of 12 m and the total resistive force during descent is 55 N along the slope of length 85 m. Calculate the speed of the cyclist at the bottom of the slope.

[4 marks]

Q6. A pump raises water from a reservoir and delivers it through a pipe. The pump lifts 50 kg of water per second through a vertical height of 4.5 m.

(a) Calculate the minimum power required to lift the water.

[2 marks]

(b) In practice the pump has an efficiency of 65%. Calculate the actual power input to the pump.

[2 marks]

(c) Suggest where the remaining 35% of energy is transferred.

[1 mark]

Q7. A ball of mass 0.25 kg is thrown vertically upward with initial speed 14 m s^{-1} .

(a) Calculate the initial kinetic energy of the ball.

[2 marks]

(b) Assuming no air resistance, calculate the maximum height reached.

[2 marks]

(c) In practice the ball only reaches 8.5 m. Calculate the energy lost to air resistance and the average resistive force acting on the ball during its upward journey.

[3 marks]

Mark Scheme and Answers

Q1(a). Work done is the product of force and displacement in the direction of the force [1]; unit: joule (J) [1].

Q1(b). Power is the work done (energy transferred) per unit time [1]; unit: watt (W) [1].

Q2. $W = Fs \cos \theta = 30 \times 6.0 \times \cos 40^\circ = 180 \times 0.766 = \mathbf{138 \text{ J}}$ [2].

Q3. Energy cannot be created or destroyed, only transferred [1]. $\Delta E_P = E_K$: $mgh = \frac{1}{2}mv^2$ [1]; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 8.0} = \sqrt{156.96} = \mathbf{12.5 \text{ m s}^{-1}}$ [2].

Q4. Net force $F = ma$ acts over displacement s ; $W = Fs = mas$ [1]; from $v^2 = u^2 + 2as$ with $u = 0$: $as = v^2/2$ [1]; therefore $W = m \times v^2/2 = \frac{1}{2}mv^2 = E_K$ [1].

Q5(a). At constant speed, driving force = resistive force = 60 N; $P = Fv = 60 \times 8.0 = \mathbf{480 \text{ W}}$ [2].

Q5(b). Additional force against gravity: $F_g = mg \sin \theta = 85 \times 9.81 \times 0.04 = 33.4 \text{ N}$ [1]; total force = $60 + 33.4 = 93.4 \text{ N}$ [1]; $P = 93.4 \times 8.0 = \mathbf{747 \text{ W}}$ [1].

Q5(c). GPE lost: $mgh = 85 \times 9.81 \times 12 = 9997 \text{ J}$ [1]; work against friction: $W_f = 55 \times 85 = 4675 \text{ J}$ [1]; KE gained: $\frac{1}{2}mv^2 = 9997 - 4675 = 5322 \text{ J}$ [1]; $v = \sqrt{2 \times 5322/85} = \sqrt{125.2} = \mathbf{11.2 \text{ m s}^{-1}}$ [1].

Q6(a). $P = mgh/t = (50 \times 9.81 \times 4.5)/1 = \mathbf{2207 \text{ W}} \approx 2.2 \text{ kW}$ [2].

Q6(b). $P_{\text{input}} = P_{\text{useful}}/\eta = 2207/0.65 = \mathbf{3395 \text{ W}} \approx 3.4 \text{ kW}$ [2].

Q6(c). Transferred to thermal energy (heat) in the motor/pump mechanism due to friction and electrical resistance [1].

Q7(a). $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.25 \times 14^2 = \frac{1}{2} \times 0.25 \times 196 = \mathbf{24.5 \text{ J}}$ [2].

Q7(b). $E_K = \Delta E_P$: $mgh = 24.5$; $h = 24.5/(0.25 \times 9.81) = \mathbf{9.99 \text{ m}} \approx 10.0 \text{ m}$ [2].

Q7(c). GPE at 8.5 m: $E_P = 0.25 \times 9.81 \times 8.5 = 20.8 \text{ J}$ [1]; energy lost = $24.5 - 20.8 = \mathbf{3.7 \text{ J}}$ [1]; average force = $W/s = 3.7/8.5 = \mathbf{0.44 \text{ N}}$ [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Define work done as force \times displacement in the direction of the force	
<input type="checkbox"/> Use $W = Fs \cos \theta$ when force and displacement are not parallel	
<input type="checkbox"/> State the principle of conservation of energy	
<input type="checkbox"/> Define and calculate efficiency as useful output \div total input	
<input type="checkbox"/> Define power as work done per unit time; use $P = W/t$	
<input type="checkbox"/> Derive and use $P = Fv$	
<input type="checkbox"/> Derive $\Delta E_P = mg\Delta h$ using $W = Fs$	
<input type="checkbox"/> Use $\Delta E_P = mg\Delta h$ in problems	
<input type="checkbox"/> Derive $E_K = \frac{1}{2}mv^2$ using equations of motion	
<input type="checkbox"/> Use $E_K = \frac{1}{2}mv^2$ in problems	
<input type="checkbox"/> Apply conservation of energy to problems with and without resistive forces	
<input type="checkbox"/> Use the work–energy theorem $W_{\text{net}} = \Delta E_K$	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Work, energy and power all connect through one idea: energy is the capacity to do work, and power is just how fast you do it. Once you're comfortable with $W = Fs \cos \theta$ and conservation of energy, every problem in this topic reduces to careful bookkeeping of energy transfers.

Topic 6

Deformation of Solids

Revision Booklet

This booklet covers:

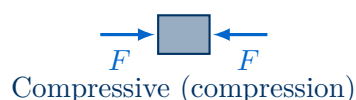
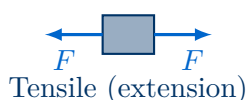
- Tensile and Compressive Forces
- Hooke's Law and the Spring Constant
- Stress, Strain and the Young Modulus
- Measuring the Young Modulus
- Elastic and Plastic Behaviour
- Elastic Potential Energy

Forces and Deformation

Tensile and Compressive Forces

Deformation occurs when forces are applied to an object, changing its shape or size.

- **Tensile forces** pull the object, causing an **extension** (stretching).
- **Compressive forces** push the object, causing a **compression** (squashing).
- Forces and deformations are assumed to act in **one dimension only**.
- The applied force is often called the **load**, F (units: N).



Hooke's Law and the Spring Constant

Hooke's Law

For a spring (or elastic material) deformed within its **limit of proportionality**, the extension is directly proportional to the applied load:

$$F = kx$$

F = applied load / force (N)

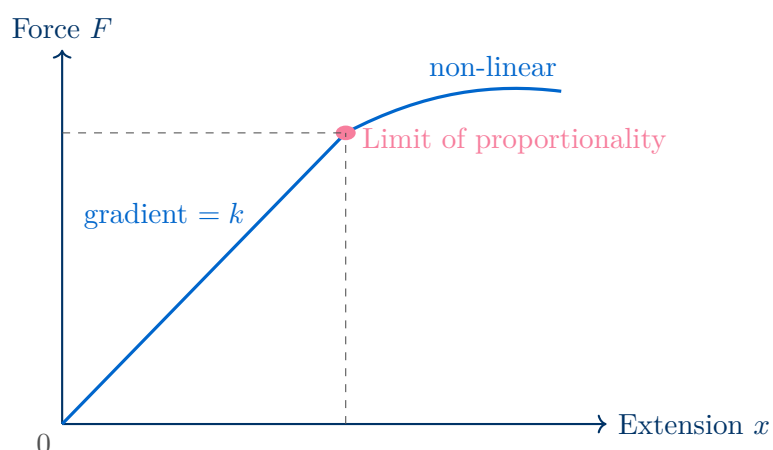
k = spring constant (N m^{-1})

x = extension or compression from the natural length (m)

Key Terms

- **Spring constant** k : the force per unit extension; a measure of stiffness. $k = F/x$ (N m^{-1}).
- **Limit of proportionality**: the point beyond which $F \propto x$; Hooke's law no longer holds.
- **Elastic limit**: the point beyond which the material will not return to its original shape when the load is removed. (May be close to, but is not the same as, the limit of proportionality.)

Force–Extension Graph



Gradient of the Force–Extension Graph

The gradient of the linear (Hooke's law) region equals the **spring constant** k . A steeper gradient means a stiffer spring.

Stress, Strain and the Young Modulus

Stress σ

Stress is the force applied per unit cross-sectional area:

$$\sigma = \frac{F}{A} \quad \text{units: Pa} \equiv \text{N m}^{-2}$$

σ = stress (Pa)

F = applied force / load (N)

A = cross-sectional area (m^2)

Strain ε

Strain is the fractional change in length (extension per unit original length):

$$\varepsilon = \frac{x}{L_0} \quad (\text{no units — it is a ratio})$$

ε = strain (dimensionless)

x = extension (m)

L_0 = original (unstretched) length (m)

Young Modulus E

The **Young modulus** is the ratio of stress to strain within the limit of proportionality:

$$E = \frac{\sigma}{\varepsilon} = \frac{FL_0}{Ax}$$

E = Young modulus (Pa)

σ = stress (Pa)

ε = strain (dimensionless)

The Young modulus is a property of the **material**, not of a particular sample — it does not depend on the dimensions of the specimen.

Stress vs Pressure

Stress and pressure share the same unit (Pa) and formula (F/A), but **stress** refers specifically to internal forces within a solid material along a defined direction, whereas pressure acts equally in all directions in a fluid.

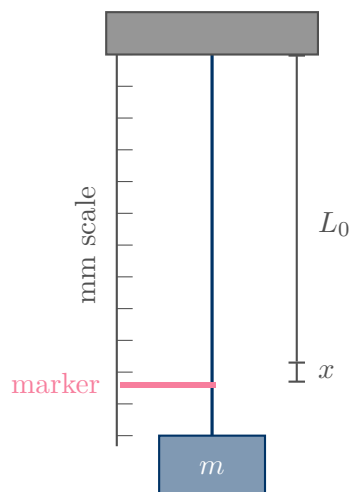
Cross Sectional Area

Be careful to check whether you are given the diameter or the radius of the wire. Also remember 1 mm^2 is $1 \times 10^{-6} \text{ m}^2$.

Experiment: Young Modulus of a Metal Wire

Method

1. Use a long, thin metal wire (e.g. copper or steel) clamped at one end and hung vertically.
2. Measure the **original length** L_0 with a metre rule.
3. Measure the **diameter** d of the wire with a micrometer screw gauge at several points; calculate $A = \pi(d/2)^2$.
4. Attach a **reference marker** (e.g. sticky tape) and measure extension x against a millimetre scale as known masses (m) are added.
5. Record F (mg) and corresponding x ;
6. Plot a **Stress–Strain graph**.
7. Young modulus: $E = \text{gradient}$.



Sources of Uncertainty and Improvements

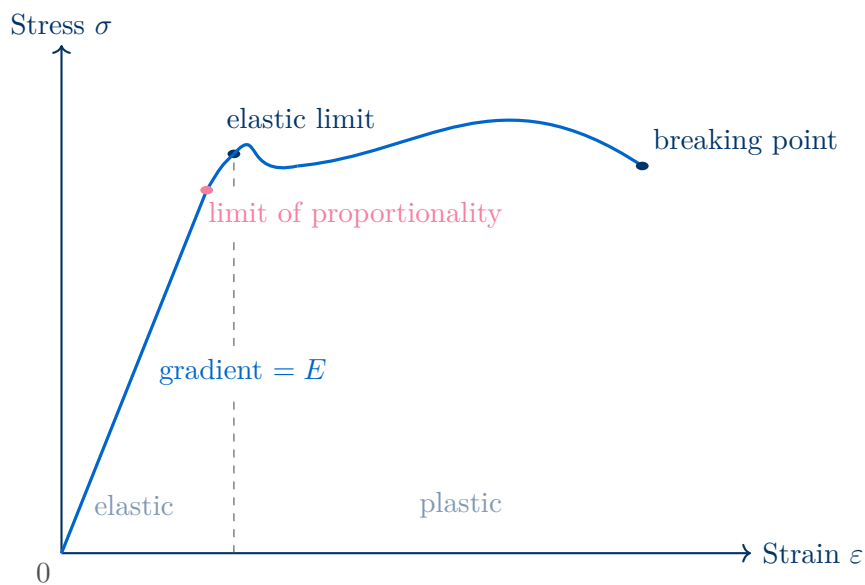
- **Diameter measurement:** the wire may not be perfectly uniform — measure at several positions and take an average.
- **Parallax:** read the millimetre scale at eye level.
- **Long wire:** a longer wire gives a larger, more easily measured extension, reducing % uncertainty in x .
- **Control wire:** use a second, identical wire alongside (with no load) to correct for thermal expansion and sagging of the support.
- Do not exceed the elastic limit — the wire must return to its original length on unloading.

Elastic and Plastic Behaviour

Elastic and Plastic Deformation

- **Elastic deformation:** the material returns to its **original shape and size** when the deforming force is removed. No permanent change occurs.
- **Plastic deformation:** the material is **permanently deformed**; it does not return to its original shape when the load is removed.
- **Elastic limit:** the maximum stress (or load) up to which a material behaves elastically. Beyond this point, deformation becomes plastic.

Stress–Strain Graph for a Ductile Metal (e.g. Mild Steel)



Elastic Potential Energy

Work Done

When a force deforms a material, work is done on the material. This is equal to the **area under the force–extension graph**.

For a material obeying Hooke's law, this area is a **triangle**:

$$W = \frac{1}{2}Fx$$

This energy is stored as **elastic potential energy** E_P in the material, provided deformation is within the elastic limit.

Elastic Potential Energy

$$E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

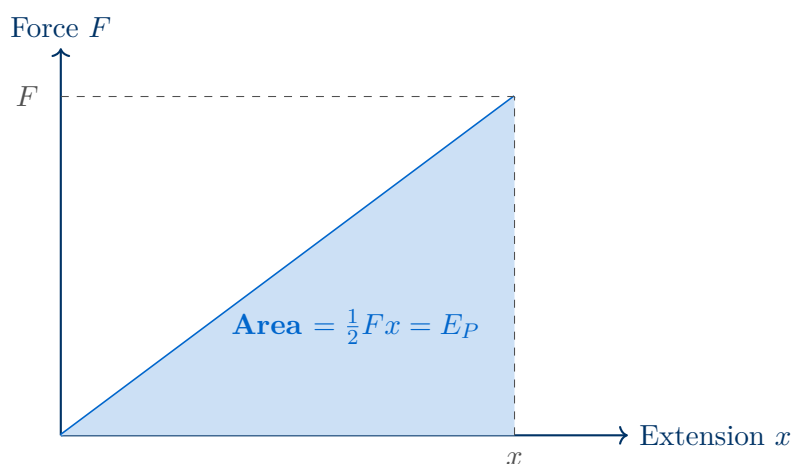
E_P = elastic potential energy stored (J)

F = applied force at extension x (N)

k = spring constant (N m^{-1})

x = extension from natural length (m)

Valid only within the limit of proportionality, where $F = kx$ holds.



Beyond the Elastic Limit

If the material is stretched beyond its elastic limit, not all the work done is stored as recoverable elastic potential energy. Some energy is dissipated as **heat** due to plastic (permanent) deformation. The formula $E_P = \frac{1}{2}kx^2$ no longer applies.

Worked Examples

Example 1 — Spring Constant

Question: A spring of natural length 0.25 m extends to 0.31 m when a 4.2 N load is applied. Calculate the spring constant and verify that Hooke's law is being obeyed.

Solution

Solution:

Extension: $x = 0.31 - 0.25 = 0.060$ m

$$k = \frac{F}{x} = \frac{4.2}{0.060} = \mathbf{70 \text{ N m}^{-1}}$$

To verify Hooke's law, further loads should be applied and extension measured; if F vs x is linear through the origin, Hooke's law holds up to that point.

Example 2 — Young Modulus

Question: A steel wire of length 1.80 m and diameter 0.56 mm extends by 1.4 mm when a load of 85 N is applied. Calculate the Young modulus of the steel.

Solution

Solution:

$$A = \pi \left(\frac{0.56 \times 10^{-3}}{2} \right)^2 = \pi \times (2.8 \times 10^{-4})^2 = 2.46 \times 10^{-7} \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{85}{2.46 \times 10^{-7}} = 3.46 \times 10^8 \text{ Pa}$$

$$\varepsilon = \frac{x}{L_0} = \frac{1.4 \times 10^{-3}}{1.80} = 7.78 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{3.46 \times 10^8}{7.78 \times 10^{-4}} = \mathbf{2.0 \times 10^{11} \text{ Pa}} (= 200 \text{ GPa})$$

This is consistent with the accepted Young modulus of steel (≈ 200 GPa).

Example 3 — Elastic Potential Energy

Question: A spring with $k = 70 \text{ N m}^{-1}$ is compressed by 0.040 m within its limit of proportionality. Calculate the elastic potential energy stored.

Solution

Solution:

$$E_P = \frac{1}{2}kx^2 = \frac{1}{2} \times 70 \times (0.040)^2 = \frac{1}{2} \times 70 \times 1.6 \times 10^{-3} = \mathbf{0.056 \text{ J}}$$

Alternatively: $F = kx = 70 \times 0.040 = 2.8$ N, so $E_P = \frac{1}{2}Fx = \frac{1}{2} \times 2.8 \times 0.040 = 0.056$ J.

✓

Practice Exam Questions**Section A — Short Answer Questions**

Q1. State Hooke's law and identify the condition under which it applies. *[2 marks]*

Q2. Define stress and strain, giving the unit of each. *[4 marks]*

Q3. Distinguish between the limit of proportionality and the elastic limit. *[2 marks]*

Q4. Explain why the elastic potential energy stored in a stretched spring is equal to the area under its force–extension graph. *[2 marks]*

Q5. A rubber band and a steel spring are both stretched by the same load. Sketch, on the same axes, a force–extension graph for each, and comment on how their behaviours differ. *[3 marks]*

Section B — Longer Structured Questions

Q6. A copper wire of length 2.00 m and cross-sectional area $1.5 \times 10^{-7} \text{ m}^2$ is suspended vertically from a fixed support. The Young modulus of copper is $1.3 \times 10^{11} \text{ Pa}$.

- (a) A load of 60 N is applied to the lower end. Calculate the stress in the wire.

[2 marks]

- (b) Calculate the extension of the wire produced by this load.

[3 marks]

- (c) Calculate the elastic potential energy stored in the wire.

[2 marks]

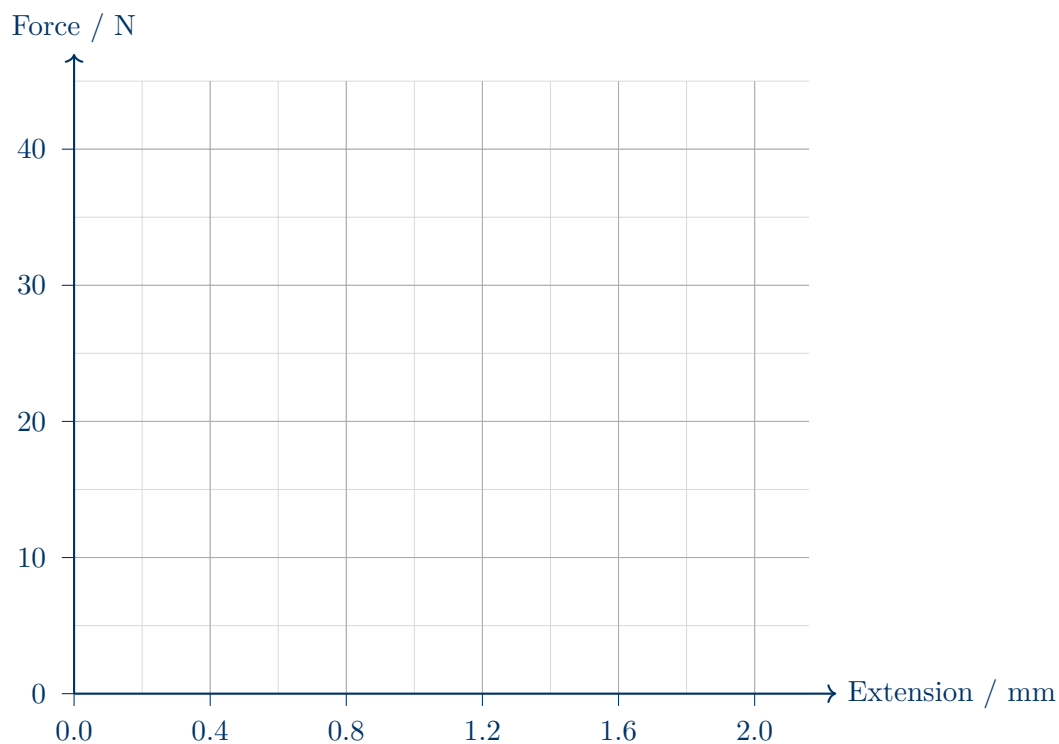
- (d) The load is doubled. State **one** assumption needed for the formula $E_P = \frac{1}{2}kx^2$ to remain valid.

[1 mark]

Q7. A student performs an experiment to determine the Young modulus of a steel wire. The wire has a diameter of 0.48 mm and an original length of 1.60 m. The table below shows the results.

Load / N	Extension / mm
0	0.0
10	0.4
20	0.8
30	1.2
40	1.6

- (a) Plot a force–extension graph for this wire and determine the spring constant k .
[3 marks]



(b) Use your value of k to determine the Young modulus of the steel.

[3 marks]

(c) Suggest **two** improvements the student could make to reduce uncertainty in the result.

[2 marks]

Mark Scheme and Answers

Q1. The extension (or compression) of a spring is directly proportional to the applied force [1]; provided the limit of proportionality is not exceeded [1].

Q2. Stress: force per unit cross-sectional area [1]; unit Pa (or N m^{-2}) [1]. Strain: extension per unit original length (or fractional change in length) [1]; dimensionless / no unit [1].

Q3. The limit of proportionality is the point at which $F \propto x$ (Hooke's law) no longer holds [1]; the elastic limit is the maximum load beyond which permanent deformation occurs — the material will not return to its original shape on unloading [1]. (The elastic limit is typically at a slightly greater stress than the limit of proportionality.)

Q4. Work done = $F \times x$ at each small increment [1]; since F varies linearly with x (Hooke's law), the total work done equals the area of the triangle = $\frac{1}{2}Fx$, which equals the elastic potential energy stored [1].

Q5. Steel spring: straight line through origin (Hooke's law) [1]. Rubber band: non-linear — curved from the outset, does not obey Hooke's law [1]; rubber is much more extensible (larger extension for same load) and has a lower effective spring constant at small loads [1].

Q6(a). $\sigma = F/A = 60/(1.5 \times 10^{-7}) = 4.0 \times 10^8$ Pa [2].

Q6(b). $\varepsilon = \sigma/E = 4.0 \times 10^8/1.3 \times 10^{11} = 3.08 \times 10^{-3}$ [1]; $x = \varepsilon \times L_0 = 3.08 \times 10^{-3} \times 2.00 = 6.2 \times 10^{-3}$ m (6.2 mm) [2].

Q6(c). $E_P = \frac{1}{2}Fx = \frac{1}{2} \times 60 \times 6.2 \times 10^{-3} = 0.186$ J [2].

Q6(d). The wire must remain within its limit of proportionality / must not exceed its elastic limit [1].

Q7(a). Graph: straight line through origin [1]. Gradient = $10 \text{ N}/(0.4 \times 10^{-3} \text{ m}) = 2.5 \times 10^4 \text{ N m}^{-1}$ [2].

Q7(b). $A = \pi(0.24 \times 10^{-3})^2 = 1.81 \times 10^{-7} \text{ m}^2$ [1]; $E = kL_0/A = (2.5 \times 10^4 \times 1.60)/1.81 \times 10^{-7}$ [1] = 2.2×10^{11} Pa [1].

Q7(c). Any two from: use a longer wire to increase extension (reducing % uncertainty in x) [1]; use a travelling microscope / vernier scale to measure extension more precisely [1]; measure wire diameter at several points along its length and average [1]; use a control wire alongside to correct for thermal expansion [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Distinguish between tensile and compressive forces, and define load, extension and compression	
<input type="checkbox"/> State Hooke's law and identify the limit of proportionality on a F - x graph	
<input type="checkbox"/> Define the spring constant and calculate it from $k = F/x$	
<input type="checkbox"/> Define stress and strain with correct units	
<input type="checkbox"/> Define the Young modulus and use $E = \sigma/\varepsilon = FL_0/(Ax)$	
<input type="checkbox"/> Describe the experiment to measure the Young modulus of a metal wire	
<input type="checkbox"/> Identify sources of uncertainty and suggest improvements to the experiment	
<input type="checkbox"/> Distinguish between elastic and plastic deformation and define the elastic limit	
<input type="checkbox"/> Explain why the area under a force–extension graph equals work done	
<input type="checkbox"/> Calculate elastic potential energy using $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$	
<input type="checkbox"/> Interpret stress–strain graphs for ductile, brittle and polymeric materials	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: the Young modulus is a property of the *material*, not the sample. Always check units carefully — a common slip is leaving extension in mm rather than converting to metres.

Topic 7

Waves

Revision Booklet

This booklet covers:

- Progressive Waves and Wave Properties
- The Wave Equation and Intensity
- Transverse and Longitudinal Waves
- The Doppler Effect
- The Electromagnetic Spectrum
- Polarisation and Malus's Law

Progressive Waves

Wave Motion

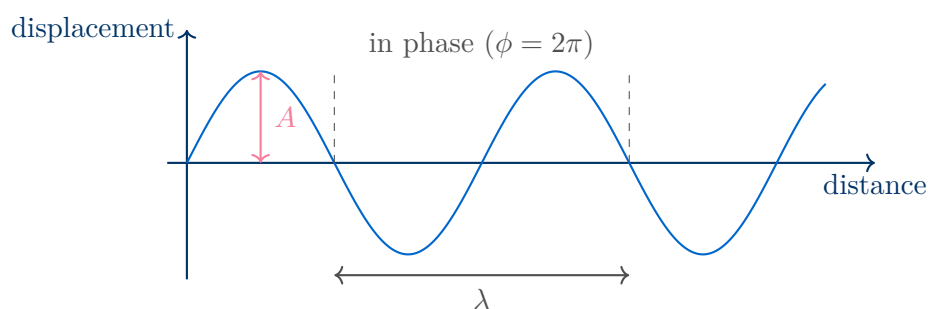
A **progressive wave** transfers energy from one place to another by means of oscillations, without any net transfer of matter. The particles of the medium vibrate about their equilibrium positions.

Examples: waves in ropes, springs, ripple tanks, sound waves, electromagnetic waves.

Key Wave Terms

Displacement (x)	Distance of a particle from its equilibrium position at a given instant; can be positive or negative. Unit: m.
Amplitude (A)	Maximum displacement from equilibrium. Unit: m.
Period (T)	Time for one complete oscillation. Unit: s.
Frequency (f)	Number of complete oscillations per unit time; $f = 1/T$. Unit: Hz.
Wavelength (λ)	Distance between two adjacent points in phase (e.g. crest to crest). Unit: m.
Wave speed (v)	Speed at which the wave profile travels through the medium. Unit: m s^{-1} .
Phase difference (ϕ)	Difference in the stage of oscillation between two points. Unit: radians or degrees.

Displacement–distance graph



Phase Difference

Two points separated by a distance d along the direction of travel of a wave of wavelength λ have a phase difference:

$$\phi = \frac{2\pi d}{\lambda} \quad (\text{in radians}) \quad \text{or} \quad \phi = \frac{360^\circ \cdot d}{\lambda}$$

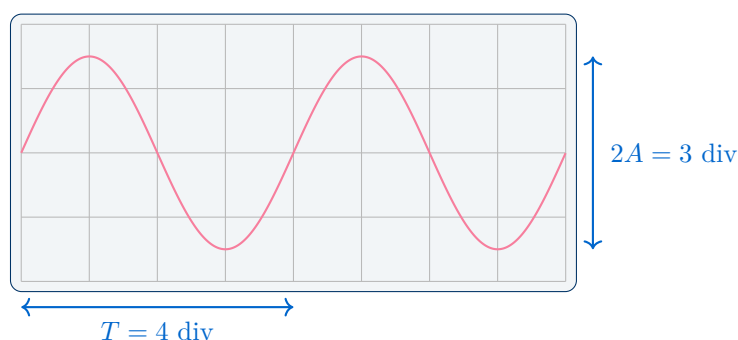
- Points separated by λ (or $n\lambda$): $\phi = 2\pi$ — **in phase**.
- Points separated by $\lambda/2$ (or $(2n - 1)\lambda/2$): $\phi = \pi$ — **in antiphase**.

CRO, Wave Equation and Intensity

Using a Cathode-Ray Oscilloscope (CRO)

A CRO displays voltage against time. Two controls are used:

- **Time-base** (s div^{-1}): sets the time represented by each horizontal division. Period $T = (\text{number of divisions per cycle}) \times (\text{time-base setting})$. Then $f = 1/T$.
- **y-gain** (V div^{-1}): sets the voltage per vertical division. Amplitude = (number of divisions from centre to peak) \times (*y*-gain setting).



The Wave Equation

Derivation: In one period T , the wave travels a distance of one wavelength λ . Speed = distance/time:

$$v = \frac{\lambda}{T} = f\lambda \quad \text{since } f = \frac{1}{T}$$

$$v = f\lambda$$

Intensity

The **intensity** of a wave is the power transmitted per unit area perpendicular to the direction of propagation:

$$I = \frac{P}{A} \quad \text{Unit: } \text{W m}^{-2}$$

For a progressive wave, intensity is proportional to the square of the amplitude:

$$I \propto (\text{amplitude})^2$$

So doubling the amplitude quadruples the intensity.

Common Mistake

$I \propto A^2$ means intensity is proportional to amplitude *squared*. Halving the amplitude reduces intensity to one quarter — not one half. This relationship holds for all types of progressive wave.

Transverse and Longitudinal Waves

Transverse Waves

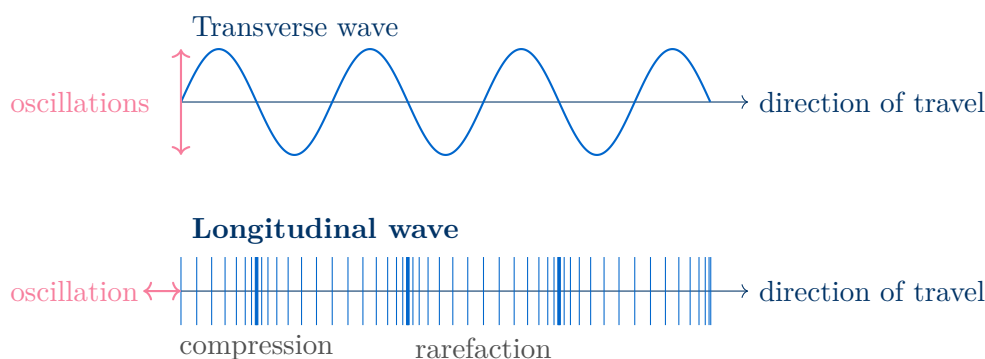
In a **transverse wave**, the oscillations of particles are **perpendicular** to the direction of energy transfer (wave propagation).

Examples: waves on a rope, water ripples, all electromagnetic waves.

Longitudinal Waves

In a **longitudinal wave**, the oscillations of particles are **parallel** to the direction of energy transfer (wave propagation). The wave consists of alternating **compressions** (regions of high pressure/density) and **rarefactions** (regions of low pressure/density).

Examples: sound waves, compression waves in a spring.



	Transverse	Longitudinal
Oscillation direction	Perpendicular to wave travel	Parallel to wave travel
Can be polarised?	Yes	No
Examples	EM waves, rope waves	Sound, spring compressions
Graphical representation	Sinusoidal displacement–distance graph	Displacement–distance graph (same shape, but displacement is along wave direction)

The Doppler Effect

Doppler Effect

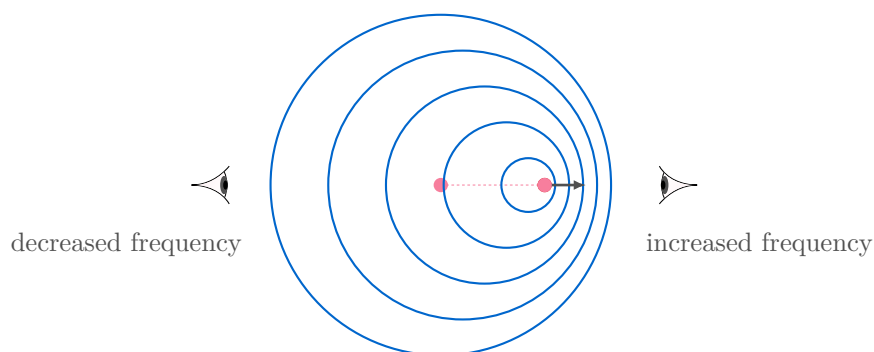
The **Doppler effect** is the change in observed frequency of a wave when the source moves relative to a stationary observer. When the source moves **towards** the observer, the observed frequency is **higher** than the source frequency; when it moves **away**, the observed frequency is **lower**.

Doppler Formula for a Moving Source

$$f_o = \frac{f_s v}{v \pm v_s}$$

- f_o : observed frequency (Hz).
- f_s : source frequency (Hz).
- v : speed of sound in the medium (m s^{-1}).
- v_s : speed of the source (m s^{-1}).
- Use $-$ when the source moves **towards** the observer ($f_o > f_s$).
- Use $+$ when the source moves **away** from the observer ($f_o < f_s$).

*Note: this formula applies to a **moving source** and stationary observer only. The Doppler effect for a stationary source and moving observer is not required.*



Sign Convention

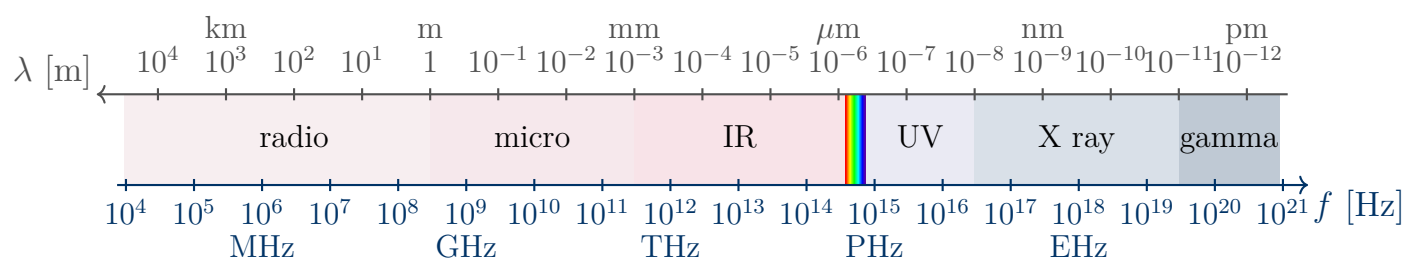
The \pm in $v \pm v_s$ is a common source of error. A useful check: if the source moves towards the observer, the denominator must be *smaller* (so use $-$), making f_o *larger*. If the source moves away, the denominator is *larger* (use $+$), making f_o *smaller*. Always sanity-check your answer against this logic.

The Electromagnetic Spectrum

Properties of Electromagnetic Waves

All electromagnetic (EM) waves:

- Are **transverse** waves.
- Travel at the **same speed** in free space (vacuum): $c = 3.0 \times 10^8 \text{ m s}^{-1}$.
- Require **no medium** — they can travel through a vacuum.
- Obey $v = f\lambda$ (with $v = c$ in free space).



Region	Approx. wavelength (m)	Approx. frequency (Hz)
Radio waves	$> 10^{-1}$	$< 3 \times 10^9$
Microwaves	10^{-3} to 10^{-1}	3×10^9 to 3×10^{11}
Infrared	7×10^{-7} to 10^{-3}	3×10^{11} to 4×10^{14}
Visible	4×10^{-7} to 7×10^{-7}	4×10^{14} to 7.5×10^{14}
Ultraviolet	10^{-8} to 4×10^{-7}	7.5×10^{14} to 3×10^{16}
X-rays	10^{-13} to 10^{-8}	3×10^{16} to 3×10^{21}
Gamma rays	$< 10^{-13}$	$> 3 \times 10^{21}$

Visible Light

The visible spectrum occupies wavelengths from approximately **400 nm** (violet) to **700 nm** (red) in free space.

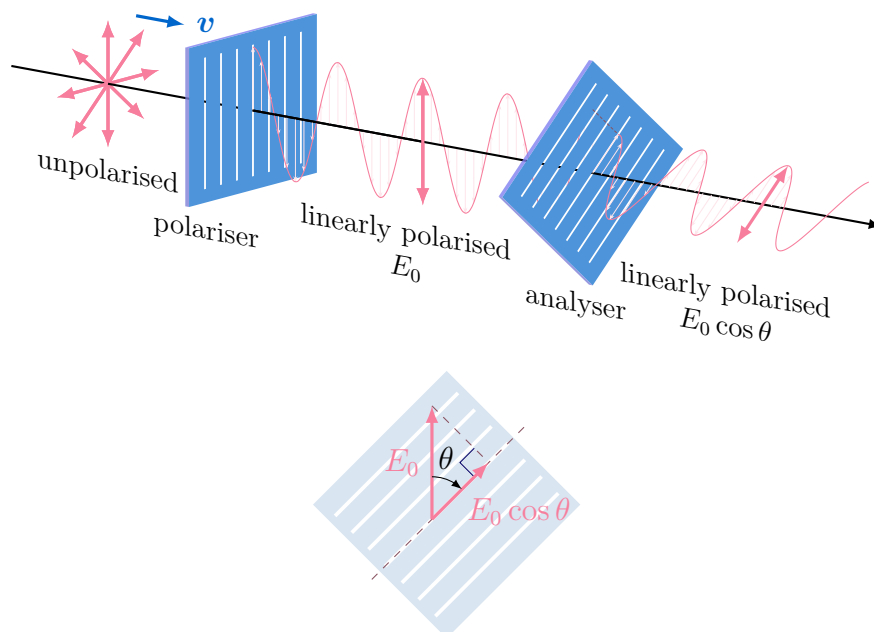
$$400 \text{ nm} = 4 \times 10^{-7} \text{ m} \quad 700 \text{ nm} = 7 \times 10^{-7} \text{ m}$$

Polarisation and Malus's Law

Polarisation

Polarisation is a phenomenon associated with **transverse waves only**. An unpolarised transverse wave has oscillations in all planes perpendicular to the direction of travel. A **plane-polarised** wave has oscillations confined to a single plane.

Longitudinal waves *cannot* be polarised — their oscillations are already along one axis (the direction of travel).



Malus's Law

When a plane-polarised electromagnetic wave of intensity I_0 passes through a polarising filter at angle θ to the plane of polarisation:

$$I = I_0 \cos^2 \theta$$

- $\theta = 0^\circ$ (filter aligned with polarisation): $I = I_0$ — maximum transmission.
- $\theta = 90^\circ$ (filter perpendicular): $I = 0$ — complete extinction.
- $\theta = 45^\circ$: $I = \frac{1}{2}I_0$ — half the intensity transmitted.
- **Note:** the effect of a polarising filter on an *unpolarised* wave is not required (but for reference, it halves the intensity).

Common Mistake

Malus's law ($I = I_0 \cos^2 \theta$) only applies to **already plane-polarised** light incident on a second filter. It does not apply to the first polarising filter acting on unpolarised light. Also remember θ is measured from the *transmission axis* of the filter to the plane of polarisation of the incoming wave — not between the two filters themselves unless the incoming wave was polarised along the first filter's axis.

Formula Summary Sheet

Formula	Quantity	Units
$v = f\lambda$	Wave equation	m s^{-1}
$f = 1/T$	Frequency–period	Hz
$\phi = 2\pi d/\lambda$	Phase difference	rad
$I = P/A$	Intensity	W m^{-2}
$I \propto (\text{amplitude})^2$	Intensity–amplitude	—
$f_o = f_s v / (v \pm v_s)$	Doppler effect	Hz
$I = I_0 \cos^2 \theta$	Malus's law	W m^{-2}

Key facts to recall:

All EM waves travel at $c = 3.0 \times 10^8 \text{ m s}^{-1}$ in free space and are transverse.

Visible light: 400 nm (violet) to 700 nm (red).

Only **transverse** waves can be polarised.

Doppler: source approaching \Rightarrow use $-$ in denominator; receding \Rightarrow use $+$.

Worked Examples

Example 1 — Wave Properties from a CRO

Question: A CRO displays a wave with a period of 2.5 divisions. The time-base is set to 4.0 ms div^{-1} and the y -gain to 0.50 V div^{-1} . The wave has a peak-to-peak height of 3.0 divisions. Find (a) the frequency and (b) the amplitude.

Solution

(a) $T = 2.5 \times 4.0 \times 10^{-3} = 1.0 \times 10^{-2} \text{ s}$; $f = 1/T = 1/(1.0 \times 10^{-2}) = \mathbf{100 \text{ Hz}}$

(b) Peak-to-peak = 3.0 div, so amplitude = 1.5 div. $A = 1.5 \times 0.50 = \mathbf{0.75 \text{ V}}$

Example 2 — Intensity and Amplitude

Question: A loudspeaker produces a sound wave of amplitude $2.4 \times 10^{-3} \text{ m}$ and intensity 1.8 W m^{-2} . The amplitude is increased to $6.0 \times 10^{-3} \text{ m}$. Calculate the new intensity.

Solution

Since $I \propto A^2$:

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{6.0 \times 10^{-3}}{2.4 \times 10^{-3}}\right)^2 = (2.5)^2 = 6.25$$

$$I_2 = 6.25 \times 1.8 = \mathbf{11.3 \text{ W m}^{-2}}$$

Example 3 — Doppler Effect

Question: An ambulance siren emits sound at $f_s = 850 \text{ Hz}$. The ambulance approaches a stationary observer at 22 m s^{-1} . The speed of sound is 340 m s^{-1} . Calculate the frequency heard by the observer (a) as the ambulance approaches and (b) as it recedes.

Solution

(a) Approaching (use $-$):

$$f_o = \frac{f_s v}{v - v_s} = \frac{850 \times 340}{340 - 22} = \frac{289\,000}{318} = \mathbf{909 \text{ Hz}}$$

(b) Receding (use $+$):

$$f_o = \frac{f_s v}{v + v_s} = \frac{850 \times 340}{340 + 22} = \frac{289\,000}{362} = \mathbf{798 \text{ Hz}}$$

Example 4 — Malus's Law

Question: Plane-polarised light of intensity 24 W m^{-2} is incident on a polarising filter. The filter's transmission axis makes an angle of 35° with the plane of polarisation. Calculate the transmitted intensity.

Solution

$$I = I_0 \cos^2 \theta = 24 \times \cos^2(35^\circ) = 24 \times (0.819)^2 = 24 \times 0.671 = \mathbf{16.1 \text{ W m}^{-2}}$$

Practice Exam Questions**Section A — Short Answer Questions**

Q1. Define the terms (a) amplitude, (b) frequency, (c) wavelength and (d) phase difference.

[4 marks]

Q2. A water wave has frequency 3.5 Hz and wavelength 0.24 m. Calculate its speed. Two points on the wave are separated by 0.18 m in the direction of travel. Calculate the phase difference between them.

[4 marks]

Q3. Distinguish between transverse and longitudinal waves. Give one example of each. Explain why polarisation is evidence that light is a transverse wave.

[4 marks]

Q4. State three properties common to all electromagnetic waves. Sketch the electromagnetic spectrum, labelling each region and giving an approximate wavelength range for visible light.

[5 marks]

Section B — Longer Structured Questions

Q5. A source of sound has frequency 640 Hz and moves towards a stationary observer at 15 m s^{-1} . The speed of sound in air is 340 m s^{-1} .

- (a) Calculate the frequency heard by the observer.

[2 marks]

- (b) The source then moves away from the observer at the same speed. Calculate the new observed frequency and the change in observed frequency between the two situations.

[3 marks]

- (c) Explain in terms of wavefronts why the observed frequency is higher when the source approaches.

[2 marks]

Q6. A plane-polarised beam of light of intensity I_0 is incident on a polarising filter.

- (a) The filter's transmission axis is at 50° to the plane of polarisation. Calculate the transmitted intensity as a fraction of I_0 .

[2 marks]

- (b) The filter is rotated to 90° . State the transmitted intensity and explain your answer.

[2 marks]

- (c) The intensity of the incident beam is now doubled to $2I_0$ and the filter is returned to 50° . Calculate the new transmitted intensity.

[1 mark]

Q7. A loudspeaker emits sound of frequency 500 Hz and produces an intensity of $3.2 \times 10^{-3} \text{ W m}^{-2}$ at a distance of 4.0 m.

- (a) The amplitude of the sound wave at this distance is A_1 . The speaker power is increased so that the amplitude doubles to $2A_1$. Calculate the new intensity.

[2 marks]

- (b) Calculate the wavelength of the sound wave. (Speed of sound = 340 m s^{-1} .)

[2 marks]

- (c) Two points in the sound wave are 0.51 m apart along the direction of travel. Calculate the phase difference between them.

[2 marks]

Mark Scheme and Answers

Q1(a). Maximum displacement of a particle from its equilibrium position [1].

Q1(b). Number of complete oscillations per unit time [1].

Q1(c). Distance between two adjacent points that are in phase [1].

Q1(d). Difference in the stage of oscillation between two points on a wave [1].

Q2. $v = f\lambda = 3.5 \times 0.24 = \mathbf{0.84} \text{ m s}^{-1}$ [2]. Phase difference: $\phi = 2\pi d/\lambda = 2\pi \times 0.18/0.24 = 2\pi \times 0.75 = \mathbf{1.5\pi} \text{ rad } (= 270^\circ)$ [2].

Q3. Transverse: oscillations perpendicular to direction of travel [1]; e.g. EM waves / rope [1]. Longitudinal: oscillations parallel to direction of travel [1]; e.g. sound [1]. Light can be polarised \Rightarrow oscillations must be perpendicular to direction of travel \Rightarrow transverse [bonus/mark scheme dependent on question wording].

Q4. Any three of: all transverse; travel at $c = 3 \times 10^8 \text{ m s}^{-1}$ in free space; require no medium; obey $v = f\lambda$ [3]. Spectrum with correct order (radio $\rightarrow \gamma$) [1]; visible 400–700 nm [1].

Q5(a). $f_o = 640 \times 340/(340 - 15) = 217\,600/325 = \mathbf{669} \text{ Hz}$ [2].

Q5(b). $f_o = 640 \times 340/(340 + 15) = 217\,600/355 = \mathbf{613} \text{ Hz}$ [2]; change = $669 - 613 = \mathbf{56} \text{ Hz}$ [1].

Q5(c). As the source approaches, successive wavefronts are emitted closer together [1]; the observer encounters more wavefronts per second, so the observed frequency is higher [1].

Q6(a). $I = I_0 \cos^2(50^\circ) = I_0 \times (0.643)^2 = \mathbf{0.413 I_0}$ [2].

Q6(b). $I = I_0 \cos^2(90^\circ) = \mathbf{0}$ [1]; the transmission axis is perpendicular to the plane of polarisation so no light passes through [1].

Q6(c). $I = 2I_0 \cos^2(50^\circ) = 2 \times 0.413 I_0 = \mathbf{0.826 I_0}$ [1].

Q7(a). $I \propto A^2$; amplitude doubles so I increases by factor 4: $I_2 = 4 \times 3.2 \times 10^{-3} = \mathbf{1.28 \times 10^{-2} \text{ W m}^{-2}}$ [2].

Q7(b). $\lambda = v/f = 340/500 = \mathbf{0.68} \text{ m}$ [2].

Q7(c). $\phi = 2\pi d/\lambda = 2\pi \times 0.51/0.68 = 2\pi \times 0.75 = \mathbf{1.5\pi} \text{ rad } (270^\circ)$ [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Define displacement, amplitude, period, frequency, wavelength, phase difference	
<input type="checkbox"/> Use the CRO to determine frequency and amplitude	
<input type="checkbox"/> Derive and use $v = f\lambda$	
<input type="checkbox"/> Use $I = P/A$ and $I \propto (\text{amplitude})^2$	
<input type="checkbox"/> Distinguish transverse and longitudinal waves with examples	
<input type="checkbox"/> Sketch and interpret displacement–distance graphs for both wave types	
<input type="checkbox"/> Understand the Doppler effect for a moving source	
<input type="checkbox"/> Use $f_o = f_s v / (v \pm v_s)$ with correct sign convention	
<input type="checkbox"/> State properties common to all EM waves	
<input type="checkbox"/> Recall the EM spectrum regions and approximate wavelength ranges	
<input type="checkbox"/> Recall visible light wavelength range (400–700 nm)	
<input type="checkbox"/> Explain polarisation and why it is evidence for transverse waves	
<input type="checkbox"/> Apply Malus's law $I = I_0 \cos^2 \theta$ to plane-polarised light	
<i>Key: 1 = Need more work 2 = Getting there 3 = Confident</i>	

Good luck with your revision!

Waves is one of those topics where a clear mental picture goes a long way. Sketch the wave, label the amplitude and wavelength, decide whether oscillations are parallel or perpendicular to travel — and most of the topic falls into place from there.

Topic 8

Superposition

Revision Booklet

This booklet covers:

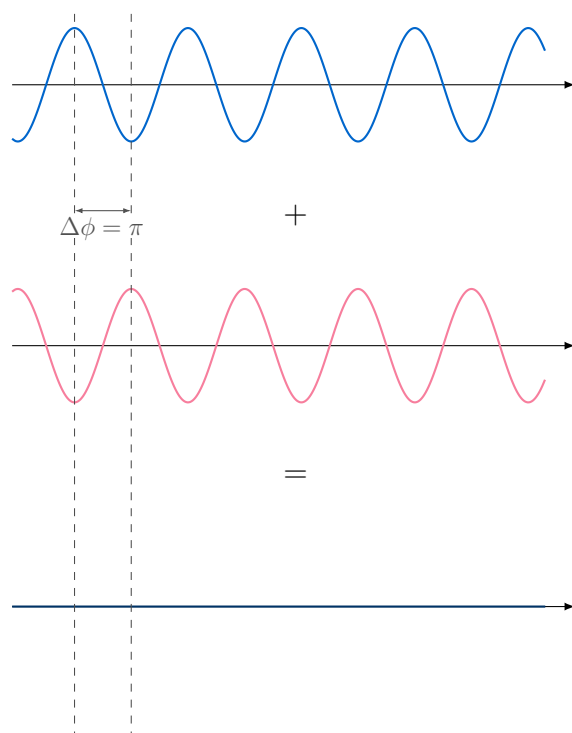
- The Principle of Superposition
- Stationary Waves: Formation and Properties
- Stationary Waves in Strings and Air Columns
- Diffraction
- Two-Source Interference and Coherence
- The Diffraction Grating

The Principle of Superposition

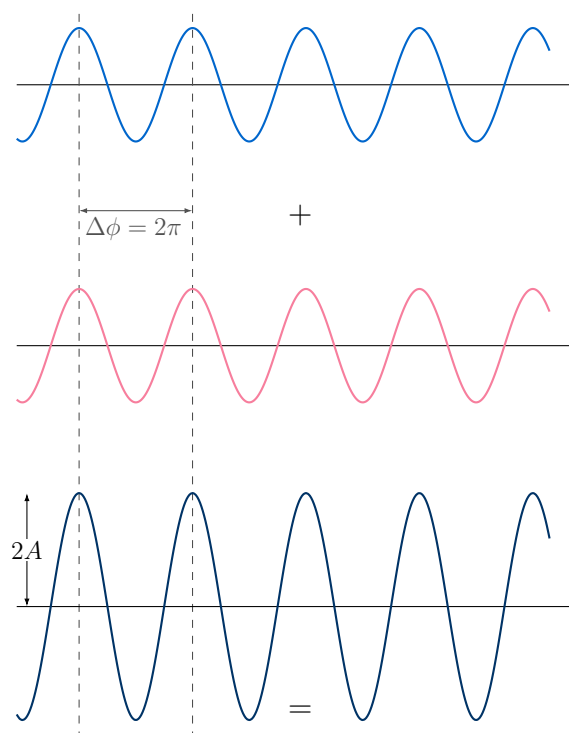
Principle of Superposition

When two or more waves meet at a point, the **resultant displacement** at that point is equal to the **vector sum** of the individual displacements of each wave at that point.

Destructive interference (antiphase)



Constructive interference (in phase)



Stationary Waves: Formation and Properties

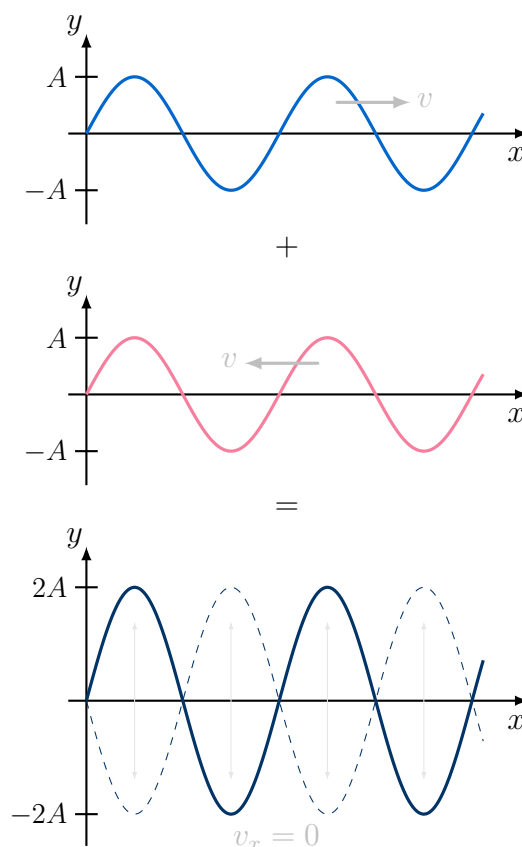
Stationary (Standing) Wave

A **stationary wave** is formed when two progressive waves of the **same frequency**, **same amplitude** and travelling in **opposite directions** superpose. The result is a wave pattern that does not travel — energy is stored rather than transmitted.

Nodes and Antinodes

- **Node:** a point of **zero amplitude** — particles are permanently at rest. Destructive superposition occurs here at all times.
- **Antinode:** a point of **maximum amplitude** — particles oscillate with the greatest displacement. Constructive superposition here.
- Adjacent nodes are separated by $\lambda/2$.
- Adjacent antinodes are also separated by $\lambda/2$.
- A node and an adjacent antinode are separated by $\lambda/4$.

Formation of a stationary wave



Differences: Stationary vs Progressive Waves

	Progressive wave	Stationary wave
Energy	Transferred in direction of travel	Stored; not transferred
Amplitude	Same for all particles	Varies from 0 (node) to max (antinode)
Phase	Varies continuously along wave	All particles between two nodes are in phase; adjacent segments are in antiphase
Wavelength	Distance between adjacent in-phase points	Twice the node-to-node distance

Determining Wavelength from a Stationary Wave

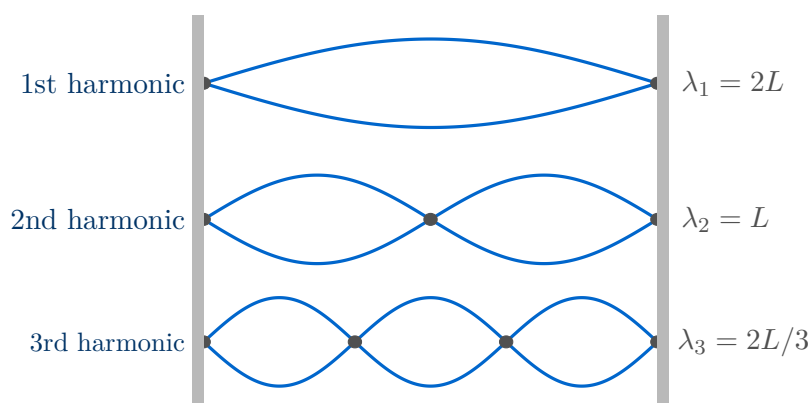
Since adjacent nodes are separated by $\lambda/2$:

$$\lambda = 2 \times (\text{node-to-node distance})$$

Measure the distance across several node spacings for greater accuracy.

Stationary Waves in Strings and Air Columns

Stretched string (both ends fixed)



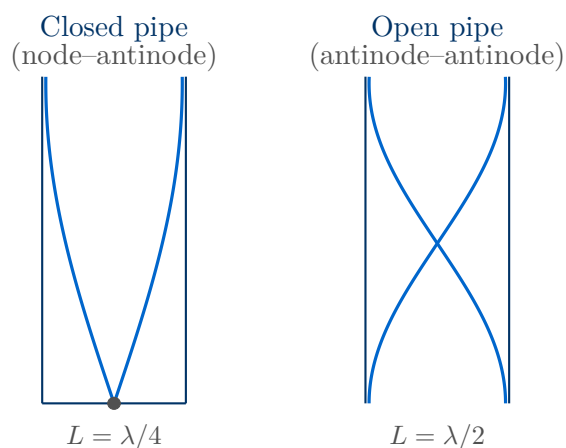
Harmonics in a String

For a string of length L fixed at both ends, the n th harmonic has:

$$\lambda_n = \frac{2L}{n} \quad f_n = \frac{nv}{2L} = nf_1$$

The **fundamental** (1st harmonic) has one antinode and $f_1 = v/2L$.

Air columns

**Standing Waves in Air**

Standing waves in air are created in a similar way. Waves are reflected from the ends of a tube and the waves travelling in opposite directions superpose to form a standing wave as long as the boundary conditions are satisfied.

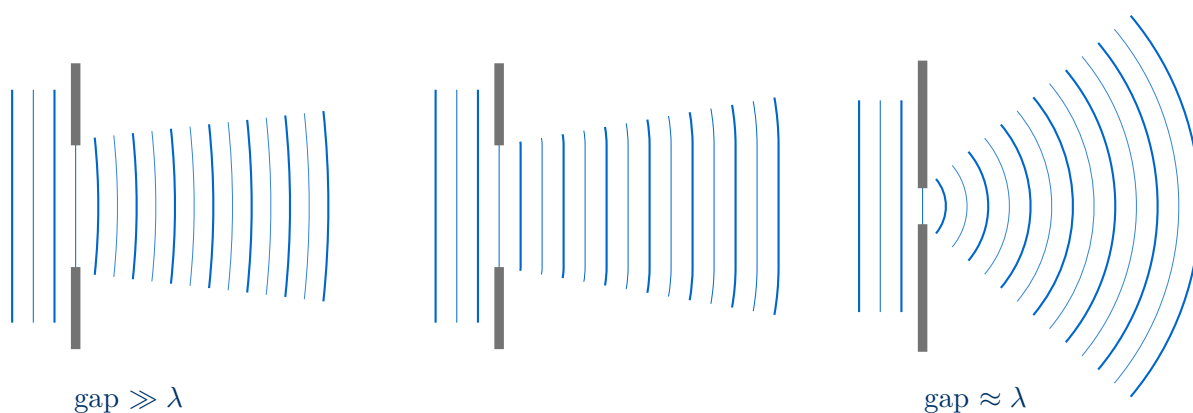
- For a closed pipe: A **Node** at the closed end and an **Antinode** at the open end.
- For an open pipe: An **Antinode** at both ends.

Diffraction

Diffraction

Diffraction is the spreading of a wave as it passes through a gap or around an obstacle. It is a property of all waves.

- Diffraction is most pronounced when the **gap width \approx wavelength**.
- When gap width $\gg \lambda$: very little spreading — the wave passes through with little diffraction.
- When gap width $\approx \lambda$: maximum spreading — the wave spreads out as a near-semicircle.
- When gap width $< \lambda$: the gap acts almost like a point source.



Two-Source Interference and Coherence

Coherence

Two sources are **coherent** if they have:

- The **same frequency** (and hence wavelength).
- A **constant phase difference** (not necessarily zero).

Coherence is essential for a stable interference pattern to be observed.

Interference

Interference is the superposition of waves from two coherent sources, producing a pattern of **maxima** (constructive) and **minima** (destructive).

- **Constructive interference:** path difference = $n\lambda$ ($n = 0, 1, 2, \dots$); waves arrive in phase.
- **Destructive interference:** path difference = $(n + \frac{1}{2})\lambda$; waves arrive in antiphase.

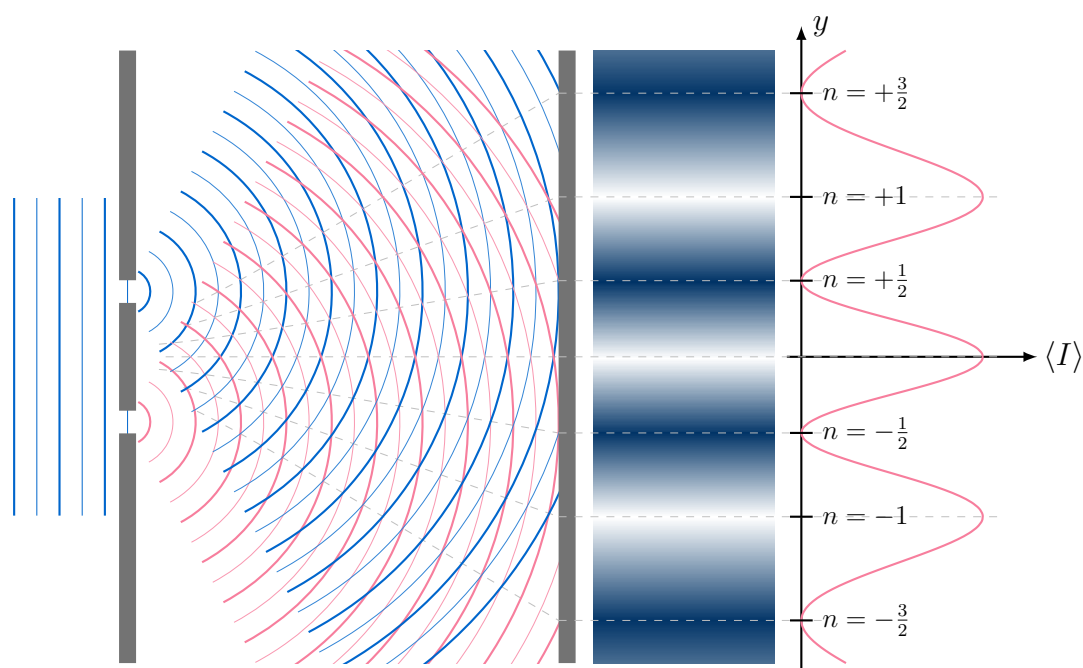
Conditions for Observable Two-Source Fringes

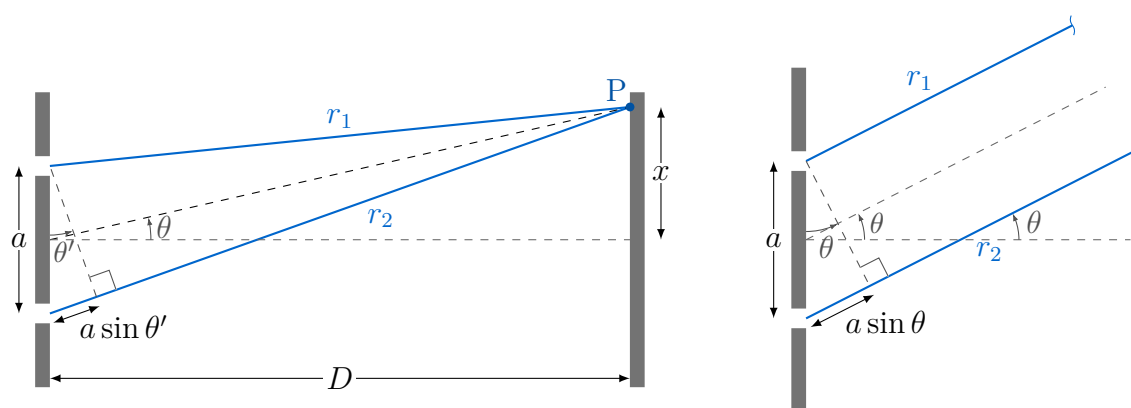
1. Sources must be **coherent** (same frequency, constant phase difference).
2. Sources must have **similar amplitudes** (for clear minima — zero intensity at destructive points).
3. For light: sources must be **monochromatic** or fringes from different wavelengths overlap and blur.
4. The **slit separation** a must be comparable to λ (much larger \Rightarrow fringes too close together to resolve).

Double-Slit Fringe Spacing: $\lambda = ax/D$

$$\lambda = \frac{ax}{D}$$

- a : slit separation (m) — centre to centre.
- x : fringe spacing (m) — distance between adjacent bright fringes.
- D : distance from slits to screen (m).
- Valid when $D \gg a$ (small-angle approximation).





Derivation of Double Slit formula

- For small angles $r_1 \approx r_2$; $\theta' \approx \theta$
- Path difference = $a \sin \theta$
- For the first maximum: Path difference = $\lambda = a \sin \theta$
- By similar triangles and $\tan \theta \approx \sin \theta$: $\frac{x}{D} = \frac{a \sin \theta}{a}$

$$\lambda = \frac{ax}{D}$$

Common Mistakes with $\lambda = ax/D$

- a is the **slit separation**, not the slit width.
- x is the fringe **spacing** (centre of one fringe to the centre of the next), not the total width of the pattern.
- Increasing D or decreasing a *increases* fringe spacing x — fringes spread out.
- The formula requires $D \gg a$; if this is not satisfied, the small-angle approximation breaks down.

The Diffraction Grating

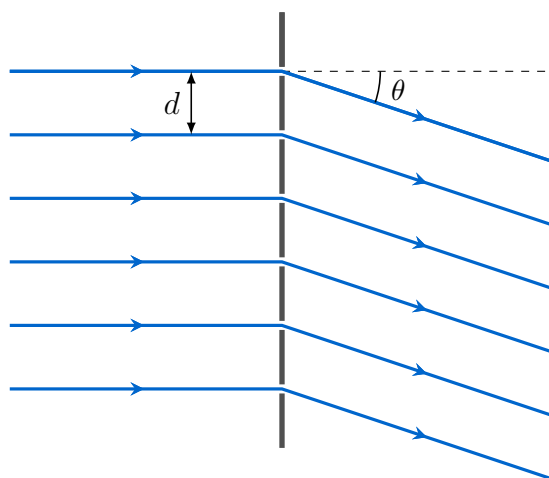
Diffraction Grating

A **diffraction grating** consists of many equally-spaced parallel slits. When light is incident normally, each slit diffracts the light, and the diffracted beams from all slits interfere. Sharp, bright maxima (orders) are produced at specific angles.

Grating Equation

$$d \sin \theta = n\lambda$$

- d : grating spacing — distance between adjacent slits (m). If a grating has N lines per metre, then $d = 1/N$.
- θ : angle of the n th order maximum to the straight-through (zero-order) direction.
- n : order number ($n = 0, \pm 1, \pm 2, \dots$).
- λ : wavelength of light (m).
- Maximum possible order: $n_{\max} = \lfloor d/\lambda \rfloor$ (since $\sin \theta \leq 1$).



Using a Diffraction Grating to Measure Wavelength

1. Direct light normally at the grating.
2. Measure the angle θ_n to the n th order maximum using a protractor or spectrometer scale.
3. Apply $d \sin \theta_n = n\lambda$ to find λ .
4. Repeat for several orders and take a mean for greater accuracy.
5. The grating spacing d is found from $d = 1/N$ where N is the number of lines per metre (usually stated as lines per mm on the grating — convert carefully).

Common Mistakes with the Grating Equation

- Lines per mm must be converted: e.g. $300 \text{ lines mm}^{-1} = 3 \times 10^5 \text{ lines m}^{-1}$, so $d = 1/(3 \times 10^5) \text{ m}$.
- $\sin \theta$ cannot exceed 1, so check that your calculated n_{max} is physically possible.
- The grating produces sharper, more widely spaced maxima than double slits — do not confuse the two formulae.

Formula Summary Sheet

Formula	Quantity	Units
$\lambda = 2 \times (\text{node spacing})$	Wavelength from stationary wave	m
$f_n = nv/2L$	n th harmonic in a string	Hz
$\lambda = ax/D$	Double-slit fringe spacing	m
$d \sin \theta = n\lambda$	Diffraction grating	m
$d = 1/N$	Grating spacing from line density	m

Key facts:

Superposition: resultant displacement = vector sum of individual displacements.

Nodes separated by $\lambda/2$; **antinodes** separated by $\lambda/2$.

Constructive: path difference = $n\lambda$; **Destructive:** path difference = $(n + \frac{1}{2})\lambda$.

Coherence: same frequency + constant phase difference.

Diffraction is maximum when gap width $\approx \lambda$.

Worked Examples

Example 1 — Wavelength from a Stationary Wave

Question: A stretched string vibrates in its third harmonic. The distance between the first and last nodes is 72 cm. Find the wavelength.

Solution

The third harmonic has 4 nodes and 3 loops. Distance from first to last node = $3 \times (\lambda/2)$:

$$72 \text{ cm} = 3 \times \frac{\lambda}{2} \implies \lambda = \frac{2 \times 72}{3} = 48 \text{ cm} = 0.48 \text{ m}$$

Example 2 — Double-Slit Interference

Question: In a Young's double-slit experiment, the slit separation is 0.45 mm, the screen is 1.8 m from the slits, and the fringe spacing is 2.4 mm. Calculate the wavelength of light used.

Solution

$$\lambda = \frac{ax}{D} = \frac{(0.45 \times 10^{-3})(2.4 \times 10^{-3})}{1.8} = \frac{1.08 \times 10^{-6}}{1.8} = 6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

Example 3 — Diffraction Grating

Question: A diffraction grating has 400 lines per mm. Light of wavelength 589 nm is incident normally. Find (a) the grating spacing, (b) the angle of the second-order maximum, and (c) the highest order observable.

Solution

$$(a) d = \frac{1}{400 \times 10^3 \text{ m}^{-1}} = 2.5 \times 10^{-6} \text{ m}$$

$$(b) d \sin \theta = n\lambda \implies \sin \theta = \frac{2 \times 589 \times 10^{-9}}{2.5 \times 10^{-6}} = \frac{1.178 \times 10^{-6}}{2.5 \times 10^{-6}} = 0.4712$$

$$\theta = \arcsin(0.4712) = 28.1^\circ$$

$$(c) n_{\max} = \frac{d}{\lambda} = \frac{2.5 \times 10^{-6}}{589 \times 10^{-9}} = 4.24 \implies n_{\max} = 4 \text{ (must be a whole number } \leq 4.24)$$

Example 4 — Path Difference and Interference

Question: Two coherent sources of sound ($\lambda = 0.40 \text{ m}$) are placed 1.5 m apart. A detector at a point P has path differences of 1.00 m from the two sources. State whether P is a maximum or minimum and explain why.

Solution

Path difference = 1.00 m.

$$1.00/\lambda = 1.00/0.40 = 2.5$$

Path difference = $2.5\lambda = (2 + \frac{1}{2})\lambda$ — this is a half-integer multiple of λ .

Therefore P is a **minimum** (destructive interference): the waves arrive in antiphase.

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the principle of superposition. Distinguish between constructive and destructive interference in terms of path difference.

[4 marks]

Q2. Explain what is meant by (a) a node and (b) an antinode in a stationary wave. State the distance between adjacent nodes in terms of wavelength.

[3 marks]

Q3. Explain what is meant by diffraction. State the condition on gap width relative to wavelength for maximum diffraction to occur.

[3 marks]

Q4. Define coherence. State three conditions required for a stable two-source interference pattern to be observed using light.

[4 marks]

Section B — Longer Structured Questions

Q5. A stationary wave is set up on a stretched string of length 0.90 m fixed at both ends. The string vibrates in its second harmonic. The speed of waves on the string is 120 m s^{-1} .

- (a) Sketch the stationary wave pattern, labelling all nodes and antinodes.

[2 marks]

- (b) Calculate the wavelength and frequency of this harmonic.

[3 marks]

- (c) Describe how the stationary wave is formed, referring to the two progressive waves involved.

[3 marks]

Q6. In a Young's double-slit experiment using light of wavelength 540 nm, the slit separation is 0.30 mm.

- (a) Calculate the fringe spacing when the screen is 2.0 m from the slits.

[2 marks]

- (b) The screen is moved further from the slits. State and explain the effect on the fringe spacing.

[2 marks]

- (c) The experiment is repeated with white light. Describe and explain the appearance of the fringe pattern.

[3 marks]

Q7. A diffraction grating with 600 lines per mm is illuminated normally by monochromatic light.

- (a) Calculate the grating spacing d .

[1 mark]

- (b) The second-order maximum is observed at $\theta = 43.2^\circ$. Calculate the wavelength of the light.

[2 marks]

- (c) Show that the third-order maximum cannot be observed for this wavelength and grating.

[2 marks]

- (d) Explain why the maxima from a diffraction grating are much sharper than the fringes from a double slit.

[2 marks]

Mark Scheme and Answers

Q1. Resultant displacement at any point equals the vector sum of the individual displacements [1]. Constructive: path difference = $n\lambda$; waves arrive in phase; amplitude/intensity is maximum [2]. Destructive: path difference = $(n + \frac{1}{2})\lambda$; waves arrive in antiphase; amplitude/intensity is minimum (zero if amplitudes equal) [1].

Q2(a). Node: point of zero (minimum) amplitude in a stationary wave; particles permanently at rest [1].

Q2(b). Antinode: point of maximum amplitude; particles oscillate with greatest displacement [1].

Distance between adjacent nodes = $\lambda/2$ [1].

Q3. Diffraction: spreading of a wave as it passes through a gap or around an obstacle [2]. Maximum diffraction when gap width $\approx \lambda$ [1].

Q4. Coherence: two sources with same frequency and constant phase difference [1]. Conditions for light fringes: sources coherent [1]; sources monochromatic [1]; slit separation comparable to λ [1].

Q5(a). Second harmonic: 3 nodes (at both ends + centre), 2 antinodes (at quarter-points) [2].

Q5(b). $\lambda_2 = L = 0.90 \text{ m}$ [1] (for 2nd harmonic, $L = \lambda$); $f = v/\lambda = 120/0.90 = \mathbf{133 \text{ Hz}}$ [2].

Q5(c). Two progressive waves of same frequency and amplitude travel in opposite directions [1]; they superpose according to the principle of superposition [1]; the resultant produces fixed nodes and antinodes — the pattern does not travel [1].

Q6(a). $x = \lambda D/a = (540 \times 10^{-9} \times 2.0)/(0.30 \times 10^{-3}) = 1.08 \times 10^{-6}/3 \times 10^{-4} = \mathbf{3.6 \times 10^{-3} \text{ m} = 3.6 \text{ mm}}$ [2].

Q6(b). Fringe spacing **increases** [1]; from $x = \lambda D/a$, increasing D increases x directly [1].

Q6(c). Central fringe is white [1]; coloured fringes either side — violet (shortest λ , smallest x) closest to centre, red (longest λ) furthest [1]; fringes overlap at large distances giving white/blurred pattern [1].

Q7(a). $d = 1/(600 \times 10^3) = \mathbf{1.67 \times 10^{-6} \text{ m}}$ [1].

Q7(b). $\lambda = d \sin \theta/n = (1.67 \times 10^{-6} \times \sin 43.2^\circ)/2 = (1.67 \times 10^{-6} \times 0.684)/2 = \mathbf{5.71 \times 10^{-7} \text{ m} \approx 571 \text{ nm}}$ [2].

Q7(c). For $n = 3$: $\sin \theta = 3\lambda/d = 3 \times 5.71 \times 10^{-7}/1.67 \times 10^{-6} = 1.026 > 1$ [1]; since $\sin \theta$ cannot exceed 1, the third order does not exist [1].

Q7(d). The grating has many slits [1]; for a maximum to form, all slits must constructively interfere simultaneously — any slight deviation from the exact angle causes destructive interference from the large number of slits, producing very sharp peaks [1].

Revision Checklist

Learning Objective	Confidence (1–3)
<input type="checkbox"/> State and apply the principle of superposition	
<input type="checkbox"/> Explain formation of a stationary wave graphically	
<input type="checkbox"/> Identify nodes and antinodes; state their separation ($\lambda/2$)	
<input type="checkbox"/> Describe stationary wave experiments: strings, air columns, microwaves	
<input type="checkbox"/> Determine wavelength from node/antinode positions	
<input type="checkbox"/> Explain the differences between stationary and progressive waves	
<input type="checkbox"/> Define diffraction and explain the effect of gap width relative to λ	
<input type="checkbox"/> Define coherence (same frequency + constant phase difference)	
<input type="checkbox"/> State conditions required to observe two-source interference fringes	
<input type="checkbox"/> Use $\lambda = ax/D$ for double-slit interference	
<input type="checkbox"/> Distinguish constructive and destructive interference by path difference	
<input type="checkbox"/> Use $d \sin \theta = n\lambda$ for the diffraction grating	
<input type="checkbox"/> Calculate grating spacing from lines per mm	
<input type="checkbox"/> Find the maximum order observable ($\sin \theta \leq 1$)	
<input type="checkbox"/> Describe how a grating is used to measure wavelength	
<i>Key: 1 = Need more work 2 = Getting there 3 = Confident</i>	

Good luck with your revision!

Superposition is the single idea that unifies this whole topic. Stationary waves, interference fringes and grating maxima are all just superposition playing out in different geometries. Get the path difference conditions clear in your mind and the rest follows naturally.

Topic 9

Electricity

Revision Booklet

This booklet covers:

- Electric Current and Charge Carriers
- Charge, Current and the Drift Velocity Equation
- Potential Difference and Power
- Resistance and Ohm's Law
- I–V Characteristics
- Resistivity, LDRs and Thermistors

Electric Current and Charge Carriers

Electric Current

An **electric current** is a flow of **charge carriers**. In metals the charge carriers are free (conduction) electrons; in electrolytes and semiconductors, other carriers (ions, holes) may contribute.

$$I = \frac{\Delta Q}{\Delta t} \quad \Rightarrow \quad Q = It$$

- I : current (A); Q : charge (C); t : time (s).
- $1 \text{ A} = 1 \text{ C s}^{-1}$.
- Conventional current flows from + to –; electron flow is from – to +.

Quantisation of Charge

The charge on any charge carrier is an **integer multiple** of the elementary charge e :

$$Q = ne \quad e = 1.60 \times 10^{-19} \text{ C}$$

Charge is **quantised** — it cannot take continuous values; it only exists in discrete packets of e .

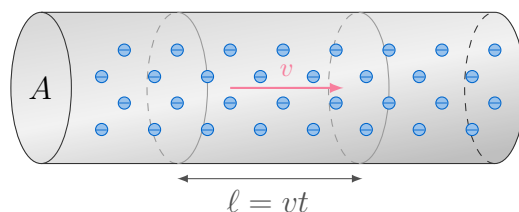
The Drift Velocity Equation

$$I = Anvq$$

For a conductor carrying current I :

$$I = Anvq$$

- A : cross-sectional area of conductor (m^2).
- n : **number density** of charge carriers (m^{-3}) — number of carriers per unit volume.
- v : **mean drift velocity** of carriers (m s^{-1}).
- q : charge on each carrier (C); for electrons $q = e = 1.60 \times 10^{-19} \text{ C}$.

Derivation of $I = Anvq$ Deriving $I = Anvq$

- In time t , carriers travel distance $\ell = vt$.
- Volume swept past the cross-section: $V = A\ell = Avt$.
- Number of carriers in this volume: $N = nAvt$.
- Total charge past the cross-section: $Q = Nq = nAvtq$.
- Therefore: $I = Q/t = nAvq$.

Using $I = Anvq$ to Compare Conductors

- **Metals** have very high n ($\sim 10^{28} \text{ m}^{-3}$) \Rightarrow very slow drift velocity ($\sim \text{mm s}^{-1}$) even for large currents.
- **Semiconductors** have lower n \Rightarrow higher v for the same current.
- At a junction where the wire narrows (A decreases), v must increase (since I , n , q are constant).
- Increasing temperature in a metal barely changes n but reduces v (increased resistance).

Potential Difference and Power

Potential Difference

The **potential difference** (p.d.) across a component is defined as the **energy transferred per unit charge** passing through it.

$$V = \frac{W}{Q}$$

- V : potential difference (V); W : energy transferred (J); Q : charge (C).
- Unit: volt (V) $\equiv \text{J C}^{-1}$.
- 1 V: 1 joule of energy transferred per coulomb of charge.

Electrical Power

$$P = VI = I^2R = \frac{V^2}{R}$$

Derivation: Power is energy per unit time. $P = W/t = (QV)/t = IV$. Substituting $V = IR$: $P = I(IR) = I^2R$; or $I = V/R$: $P = (V/R)V = V^2/R$.

- All three forms are equivalent for a resistor at any instant.
- Use $P = VI$ when both V and I are known.
- Use $P = I^2R$ when current and resistance are known.
- Use $P = V^2/R$ when voltage and resistance are known.

Common Mistake

Energy is $W = Pt = Vit$, not $W = VI$. Always multiply power by time to get energy. Also, $P = V^2/R$ and $P = I^2R$ apply to a single resistor; for a circuit with multiple components, identify V and I for each component separately.

Resistance and Ohm's Law

Resistance

The **resistance** of a component is defined as the ratio of the potential difference across it to the current flowing through it:

$$R = \frac{V}{I}$$

- Unit: ohm (Ω) \equiv V A⁻¹.
- Resistance is a property of the component — it describes how much it opposes current flow.

Ohm's Law

Ohm's law states that the current through a metallic conductor is **directly proportional** to the potential difference across it, **provided the temperature remains constant**:

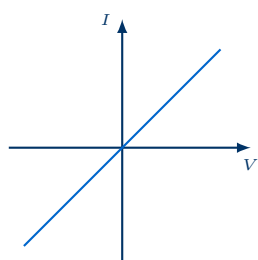
$$V \propto I \quad \Leftrightarrow \quad V = IR \quad (R \text{ constant})$$

A component that obeys Ohm's law is called **ohmic**. The I - V graph for an ohmic conductor is a straight line through the origin.

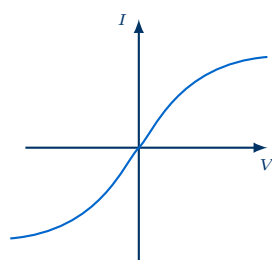
Ohm's Law is not a Definition of Resistance

$R = V/I$ defines resistance for any component. Ohm's law is a separate statement that R is *constant* (independent of V and I) for a metallic conductor at constant temperature. Not all components obey Ohm's law.

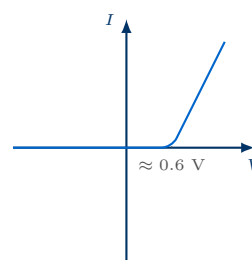
I–V Characteristics



Metallic conductor
(constant temp.)



Filament lamp
(gradient decreases)



Semiconductor diode
(threshold ≈ 0.6 V)

Interpreting I–V Graphs

- **Metallic conductor (ohmic):** straight line through origin — gradient = $1/R$ (constant). Both forward and reverse biased sections are identical straight lines.
- **Filament lamp:** S-shaped curve — gradient decreases at high V and I , meaning resistance increases. Cause: increasing current \Rightarrow increasing temperature \Rightarrow increased resistance (more collisions between electrons and vibrating lattice ions).
- **Semiconductor diode:** conducts appreciably only above the **threshold voltage** (≈ 0.6 V for silicon) in the forward direction; in reverse bias, current is essentially zero (tiny reverse leakage).

Resistivity

Resistivity

The resistance of a uniform conductor depends on its material, length and cross-sectional area:

$$R = \frac{\rho L}{A}$$

- ρ : **resistivity** of the material ($\Omega \text{ m}$).
- L : length of conductor (m).
- A : cross-sectional area (m^2).
- Resistivity is a **material property** — it does not depend on the shape of the sample.
- $R \propto L$ (double the length \Rightarrow double the resistance).
- $R \propto 1/A$ (double the area \Rightarrow halve the resistance).

Resistivity

The **resistivity** ρ of a material is defined by:

$$\rho = \frac{RA}{L}$$

It is the resistance of a 1 m cube of the material between opposite faces. Unit: $\Omega \text{ m}$.

LDRs and Thermistors

Light-Dependent Resistor (LDR)

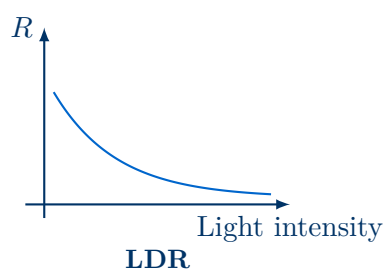
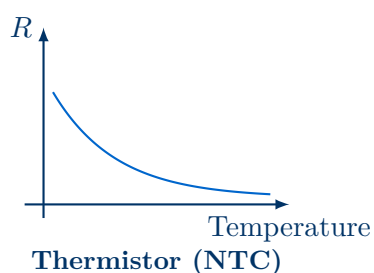
The **resistance of an LDR decreases as light intensity increases**.

- In darkness: resistance can be $\sim \text{M}\Omega$.
- In bright light: resistance falls to $\sim \text{k}\Omega$ or lower.
- Mechanism: photons give electrons enough energy to break free and become charge carriers, increasing n and thus conductivity.

Thermistor (NTC)

The **resistance of an NTC thermistor decreases as temperature increases**.

- NTC = Negative Temperature Coefficient.
- Mechanism: rising temperature gives more electrons enough energy to act as charge carriers, increasing n and reducing resistivity.
- Opposite behaviour to metals (where resistance *increases* with temperature because v is reduced).



Comparing Metals Thermistors and LDRs

Component	Effect on resistance	Reason
Metal wire	Resistance increases with T	More lattice vibration; v decreases
NTC Thermistor	Resistance decreases with T	More charge carriers; n increases
LDR	Resistance decreases with light	More charge carriers; n increases
Filament lamp	Resistance increases with I	Temperature rises (same as metal)

Formula Summary Sheet

Formula	Quantity	Units
$Q = It$	Charge	C
$Q = ne$	Quantisation of charge	C
$I = Anvq$	Current (drift velocity)	A
$V = W/Q$	Potential difference	V
$P = VI = I^2R = V^2/R$	Electrical power	W
$W = VIt$	Electrical energy	J
$R = V/I$	Resistance (definition)	Ω
$V = IR$	Ohm's law	V
$R = \rho L/A$	Resistivity	Ω

Key definitions to learn word-for-word:

P.d.: energy transferred per unit charge through a component.

Resistance: ratio of p.d. across a component to current through it ($R = V/I$).

Ohm's law: current is directly proportional to p.d. at constant temperature.

Resistivity: $\rho = RA/L$; a material property independent of sample dimensions.

Worked Examples

Example 1 — Drift Velocity

Question: A copper wire of cross-sectional area 1.5 mm^2 carries a current of 3.0 A . The number density of free electrons in copper is $8.5 \times 10^{28} \text{ m}^{-3}$. Calculate the mean drift velocity of the electrons.

Solution

Rearrange $I = Anvq$ for v :

$$v = \frac{I}{Anq} = \frac{3.0}{(1.5 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})}$$

$$v = \frac{3.0}{1.5 \times 10^{-6} \times 1.36 \times 10^{10}} = \frac{3.0}{2.04 \times 10^4} = \mathbf{1.47 \times 10^{-4} \text{ m s}^{-1}}$$

This is about 0.15 mm s^{-1} — extremely slow, yet the effect of the current is felt almost instantaneously because the electric field propagates at close to the speed of light.

Example 2 — Power and Energy

Question: A 60Ω resistor is connected to a 12 V supply for 5.0 min . Calculate (a) the current, (b) the power dissipated, and (c) the total energy transferred.

Solution

- (a) $I = V/R = 12/60 = \mathbf{0.20 \text{ A}}$
 (b) $P = V^2/R = 144/60 = \mathbf{2.4 \text{ W}}$ (or $P = I^2R = 0.04 \times 60 = 2.4 \text{ W}$ ✓)
 (c) $W = Pt = 2.4 \times (5.0 \times 60) = 2.4 \times 300 = \mathbf{720 \text{ J}}$

Example 3 — Resistivity

Question: A nichrome wire of length 0.80 m and diameter 0.50 mm has resistance 4.4Ω . Calculate the resistivity of nichrome.

Solution

Cross-sectional area: $A = \pi r^2 = \pi(0.25 \times 10^{-3})^2 = 1.96 \times 10^{-7} \text{ m}^2$

$$\rho = \frac{RA}{L} = \frac{4.4 \times 1.96 \times 10^{-7}}{0.80} = \frac{8.63 \times 10^{-7}}{0.80} = \mathbf{1.08 \times 10^{-6} \Omega \text{ m}}$$

Example 4 — I–V Characteristic

Question: The current through a filament lamp increases from 0.20 A to 0.60 A when the voltage increases from 3.0 V to 6.0 V . Show that the lamp does not obey Ohm's law.

Solution

At $V = 3.0 \text{ V}$: $R_1 = V/I = 3.0/0.20 = 15 \Omega$

At $V = 6.0 \text{ V}$: $R_2 = V/I = 6.0/0.60 = 10 \Omega$

$R_1 \neq R_2$ — resistance is not constant; it decreases as current decreases (i.e. as temperature decreases). This is characteristic of a filament lamp: at higher current the filament is

hotter and resistance is higher. The lamp **does not obey Ohm's law**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) electric current and (b) potential difference. State the SI unit of each.
[4 marks]

Q2. Explain what is meant by saying that charge is *quantised*. A current of 2.5 mA flows for 4.0 min. Calculate the charge transferred and the number of electrons passing a point.
[4 marks]

Q3. State Ohm's law and sketch the I - V characteristics of (a) a metallic conductor at constant temperature and (b) a filament lamp. Explain why the graphs have different shapes.
[5 marks]

Q4. A semiconductor diode is connected in a circuit. Sketch the I – V characteristic and mark the threshold voltage. Explain what happens in reverse bias.

[3 marks]

Section B — Longer Structured Questions

Q5. A conductor has cross-sectional area A , number density of charge carriers n , and carries a current I .

(a) Derive the expression $I = Anvq$, defining all symbols.

[3 marks]

(b) A copper wire ($n = 8.5 \times 10^{28} \text{ m}^{-3}$, diameter 1.2 mm) carries a current of 5.0 A. Calculate the mean drift velocity of the electrons.

[3 marks]

(c) The wire is replaced with one of the same material but half the diameter. The current remains 5.0 A. State and explain what happens to the drift velocity.

[2 marks]

Q6. A uniform resistance wire of length 1.20 m and cross-sectional area $3.5 \times 10^{-7} \text{ m}^2$ is made of a material of resistivity $4.9 \times 10^{-7} \Omega \text{ m}$.

(a) Calculate the resistance of the wire.

[2 marks]

(b) A potential difference of 6.0 V is applied across the wire. Calculate the current through it and the power dissipated.

[3 marks]

(c) The wire is stretched so its length doubles but its volume remains constant. Show that the resistance increases by a factor of 4.

[3 marks]

Q7. A thermistor is connected in series with a $4.7 \text{ k}\Omega$ fixed resistor across a 9.0 V supply. At 20°C the thermistor has resistance $8.2 \text{ k}\Omega$.

(a) Calculate the current in the circuit and the voltage across the thermistor at 20°C .
[3 marks]

(b) Describe and explain what happens to the current as the temperature increases.
[2 marks]

(c) Calculate the power dissipated in the fixed resistor at 20°C .
[2 marks]

Mark Scheme and Answers

Q1(a). Electric current is the rate of flow of charge [1]; unit: ampere (A) [1].

Q1(b). Potential difference is the energy transferred per unit charge through a component [1]; unit: volt (V) [1].

Q2. Charge is quantised means it only exists in discrete multiples of $e = 1.60 \times 10^{-19}$ C [1]. $Q = It = 2.5 \times 10^{-3} \times 240 = 0.60$ C [1]. Number of electrons: $N = Q/e = 0.60/(1.60 \times 10^{-19}) = 3.75 \times 10^{18}$ [2].

Q3. Ohm's law: current is proportional to p.d. at constant temperature [1]. Metallic conductor: straight line through origin [1]. Filament lamp: S-shaped curve, decreasing gradient [1]. Metal: R constant so linear [1]; lamp: higher $I \Rightarrow$ higher $T \Rightarrow$ higher R , so gradient I/V decreases [1].

Q4. Correct exponential-ish forward curve with threshold ≈ 0.6 V marked [2]; in reverse bias current is (approximately) zero — the diode blocks current [1].

Q5(a). In time t , carriers move distance vt ; volume = Avt ; number = $nAvt$; charge = $nAvtq$; current $I = Q/t = nAvq$ [3].

Q5(b). $A = \pi(0.6 \times 10^{-3})^2 = 1.131 \times 10^{-6}$ m²; $v = I/(Anq) = 5.0/(1.131 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.60 \times 10^{-19}) = 3.26 \times 10^{-4}$ m s⁻¹ [3].

Q5(c). Diameter halves \Rightarrow radius halves \Rightarrow area reduces by factor 4 [1]; since $I = Anvq$ is constant and A is quartered, v must increase by factor 4 [1].

Q6(a). $R = \rho L/A = (4.9 \times 10^{-7} \times 1.20)/(3.5 \times 10^{-7}) = 1.68 \Omega$ [2].

Q6(b). $I = V/R = 6.0/1.68 = 3.57$ A [1]; $P = VI = 6.0 \times 3.57 = 21.4$ W [2].

Q6(c). Volume constant: $A'L' = AL$; $L' = 2L$ so $A' = A/2$ [1]; new $R' = \rho L'/A' = \rho(2L)/(A/2) = 4\rho L/A = 4R$ [2].

Q7(a). $R_{\text{total}} = 8200 + 4700 = 12900 \Omega$; $I = 9.0/12900 = 6.98 \times 10^{-4}$ A [2]; $V_T = IR_T = 6.98 \times 10^{-4} \times 8200 = 5.72$ V [1].

Q7(b). As temperature increases, thermistor resistance decreases [1]; total resistance decreases so current increases (from $V = IR$) [1].

Q7(c). $P = I^2R = (6.98 \times 10^{-4})^2 \times 4700 = 4.87 \times 10^{-7} \times 4700 = 2.29 \times 10^{-3}$ W \approx 2.3 mW [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Understand that current is a flow of charge carriers	
<input type="checkbox"/> State that charge is quantised ($Q = ne$)	
<input type="checkbox"/> Use $Q = It$ to find charge, current or time	
<input type="checkbox"/> Use and derive $I = Anvq$, defining all terms	
<input type="checkbox"/> Define p.d. as energy transferred per unit charge; use $V = W/Q$	
<input type="checkbox"/> Use all three power formulae: $P = VI$, $P = I^2R$, $P = V^2/R$	
<input type="checkbox"/> Define resistance as $R = V/I$	
<input type="checkbox"/> State Ohm's law and identify ohmic/non-ohmic behaviour	
<input type="checkbox"/> Sketch and interpret I – V graphs for metal, filament lamp and diode	
<input type="checkbox"/> Explain why filament lamp resistance increases with current	
<input type="checkbox"/> Use $R = \rho L/A$ and define resistivity	
<input type="checkbox"/> Explain how resistance of an LDR depends on light intensity	
<input type="checkbox"/> Explain how resistance of an NTC thermistor depends on temperature	
<input type="checkbox"/> Distinguish the behaviour of metals from thermistors as temperature changes	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Everything in electricity connects through two equations: $Q = It$ and $V = W/Q$. All the power formulae, resistance definitions and drift velocity expression follow from those two ideas. Once you see that, the whole topic becomes a matter of careful substitution.

Topic 10

D.C. Circuits

Revision Booklet

This booklet covers:

- Practical Circuits and E.M.F.
- Internal Resistance
- Kirchhoff's Laws
- Resistors in Series and Parallel
- Potential Dividers
- Thermistors and LDRs in Potential Dividers

Practical Circuits and E.M.F.

Electromotive Force (E.M.F.)

The **electromotive force** (e.m.f., symbol ε) of a source is defined as the **energy transferred per unit charge** in driving charge around a complete circuit.

$$\varepsilon = \frac{W}{Q}$$

- W : work done by the source (J); Q : charge (C).
- Unit: volt (V) \equiv J C⁻¹.
- E.m.f. is a **property of the source**; it represents energy input per coulomb.

Potential Difference (P.D.)

The **potential difference** (p.d.) between two points is the **energy transferred per unit charge** by a component as charge passes between those points.

$$V = \frac{W}{Q}$$

- P.d. describes **energy released** (e.g. by a resistor); e.m.f. describes **energy supplied** (by a source).
- Both have unit: volt (V).

E.M.F. vs P.D. — Energy Perspective

- **E.m.f.**: energy is *given to* each coulomb of charge by the source (chemical \rightarrow electrical).
- **P.d.**: energy is *taken from* each coulomb of charge by a component (electrical \rightarrow heat, light, etc.).
- Around any complete circuit: total e.m.f. = total p.d. (conservation of energy — see Kirchhoff's Second Law).

Internal Resistance

Internal Resistance

A real source of e.m.f. has **internal resistance** r — resistance within the source itself (e.g. chemical paste in a battery). As current flows, some energy is dissipated internally.

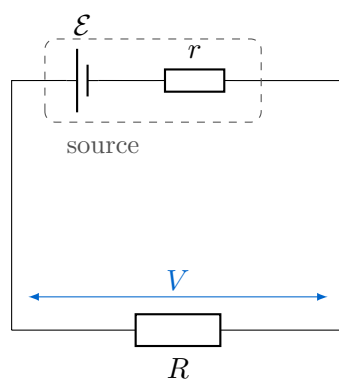
Terminal P.D. and Internal Resistance

$$\varepsilon = V + Ir \quad \Longrightarrow \quad V = \varepsilon - Ir$$

- $V = \varepsilon - Ir$: **terminal p.d.** (voltage across the external circuit).

- Ir : **lost volts** — p.d. across internal resistance.
- $V < \varepsilon$ whenever current flows.
- For an external resistance R : $I = \frac{\varepsilon}{R + r}$

Circuit diagram for a source with internal resistance

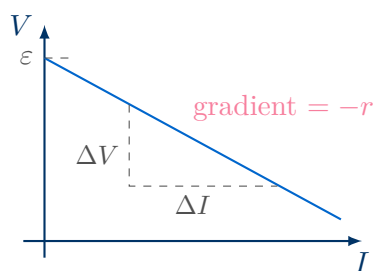


Common Mistake

The e.m.f. is *not* the same as the terminal p.d. except when $I = 0$ (open circuit). Under load, $V = \varepsilon - Ir < \varepsilon$. Always check whether the question gives e.m.f. or terminal voltage.

Graphical determination of ε and r

Rearranging: $V = \varepsilon - Ir$ has the form $y = c - mx$.



- **y-intercept:** $V = \varepsilon$ (e.m.f. when $I = 0$, i.e. open circuit).
- **Gradient:** $-r$ (magnitude of gradient gives internal resistance).
- **x-intercept:** $I = \varepsilon/r$ (short-circuit current; never reached safely).

Kirchhoff's Laws

Kirchhoff's First Law (KCL)

The sum of currents **entering** a junction equals the sum of currents **leaving** that junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

This is a consequence of **conservation of charge** — charge does not accumulate at a junction.

Kirchhoff's Second Law (KVL)

The sum of the e.m.f.s around any **closed loop** equals the sum of the potential differences (i.e. IR products) around that loop.

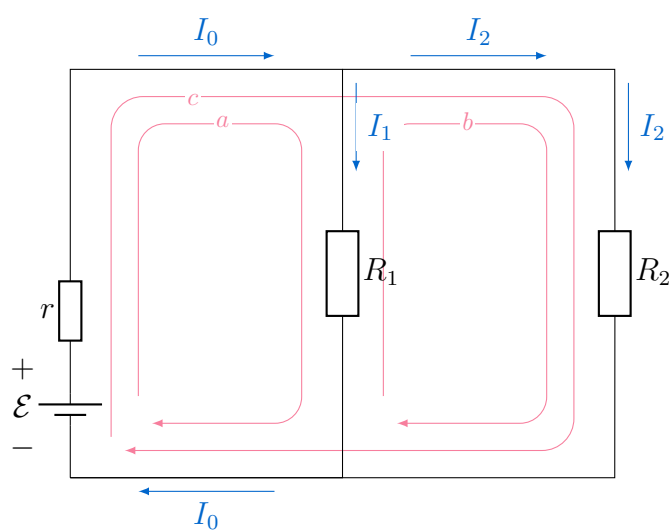
$$\sum \mathcal{E} = \sum IR$$

This is a consequence of **conservation of energy** — a charge returning to its starting point has the same potential energy.

Sign Convention for Kirchhoff's Second Law

- Choose a direction to traverse the loop.
- **E.m.f.:** positive if traversed from $-$ to $+$ terminal (source adds energy); negative if $+$ to $-$.
- IR : positive if traversed in the same direction as current; negative if opposite.
- Apply consistently — the choice of direction does not affect the final answer.

Circuit Diagram to show Kirchoff's Laws



Kirchoff's Closed Loops

There are three **closed loops** shown in the diagram above, **a**, **b** and **c**.

For **a**

$$\mathcal{E} = I_0 r + I_1 R_1$$

For **b**

$$0 = -I_1 R_1 + I_2 R_2$$

For **c**

$$\mathcal{E} = I_0 r + I_2 R_2$$

Also from Kirchoff's first law we know

$$I_0 = I_1 + I_2$$

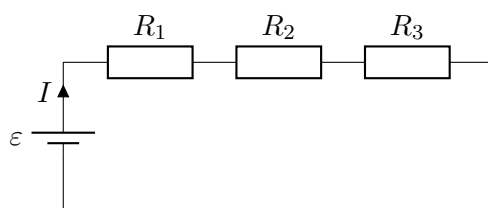
Resistors in Series and Parallel

Series Combination

For n resistors in series, the **same current** flows through each.

$$R_{\text{total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

Derivation (KVL): $V = V_1 + V_2 + \cdots$; since I is the same, $IR = IR_1 + IR_2 + \cdots$, hence $R = R_1 + R_2 + \cdots$



Series circuit — same I throughout

Parallel Combination

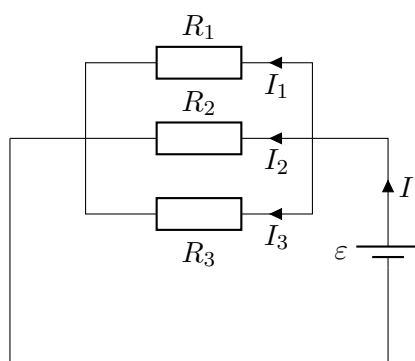
For n resistors in parallel, the **same p.d.** exists across each.

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Derivation (KCL): $I = I_1 + I_2 + \cdots$; since V is the same, $V/R = V/R_1 + V/R_2 + \cdots$, hence $1/R = 1/R_1 + 1/R_2 + \cdots$

Special case for two resistors in parallel:

$$R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



Parallel circuit — same V across each

Common Mistake

For two resistors in parallel, R_{total} is *always less* than the smaller of the two. If your answer is larger than either resistor, recheck your arithmetic — a common error is adding

reciprocals incorrectly.

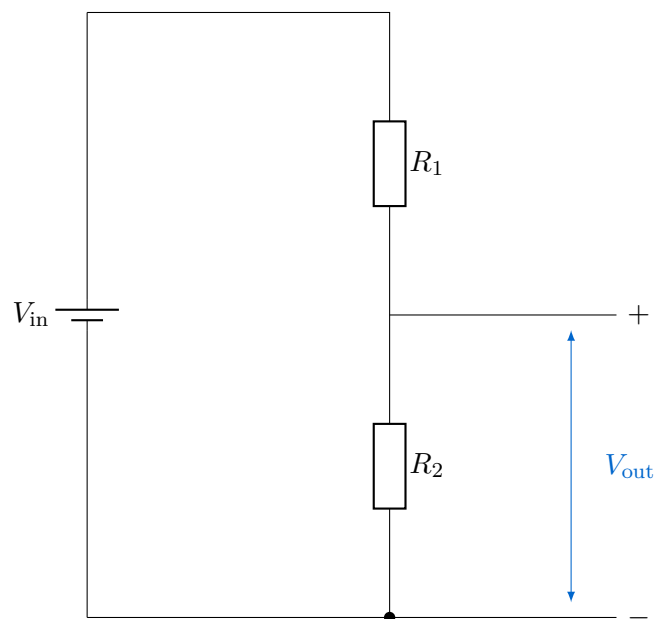
Potential Dividers

Potential Divider Principle

A **potential divider** uses two (or more) resistors in series across a supply to produce a fraction of the supply voltage as an output.

$$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1 + R_2}$$

- V_{out} is taken across R_2 .
- The ratio $R_2/(R_1 + R_2)$ determines the fraction of V_{in} that appears across R_2 .
- No current is drawn from the output (ideal case, or high-resistance load).



Potentiometer as a Comparison Tool

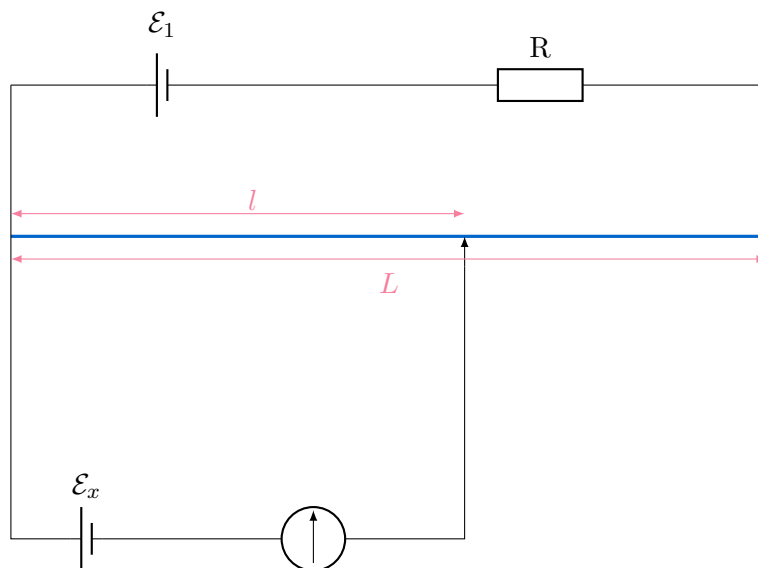
A **potentiometer** is a sliding-contact potential divider used to **compare** e.m.f.s or p.d.s precisely:

- A uniform resistance wire carries a **driver current** from a stable supply.
- The p.d. across a length l of the wire is proportional to l .
- At **balance** (null deflection on galvanometer), no current is drawn from the test source — so the measurement is unaffected by internal resistance.
- $\mathcal{E}_x = \frac{\mathcal{E}_1 l}{L}$ at their respective balance lengths.

Null Method

A **null method** is a measurement technique in which the quantity being measured is compared against a known standard and the detector (e.g. galvanometer) is adjusted to read **zero**. Because no current flows at balance, the result is independent of any resistance in the measuring instrument.

Circuit Diagram for a Null Method Measurement



Thermistors and LDRs in Potential Dividers

Thermistor (NTC)

A **thermistor** (negative temperature coefficient, NTC) is a resistor whose resistance **decreases** as temperature **increases**. It is used as a temperature sensor in potential divider circuits.

Light-Dependent Resistor (LDR)

An **LDR** has resistance that **decreases** as light intensity **increases**. It is used as a light sensor in potential divider circuits.

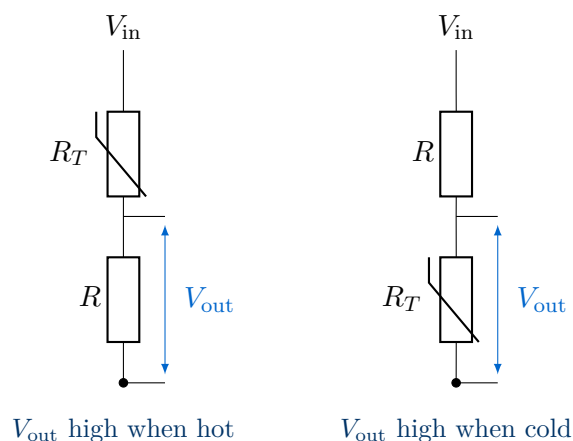
Using Sensors in Potential Dividers

For a potential divider with a thermistor (R_T) in series with a fixed resistor R , with output taken across R :

$$V_{\text{out}} = V_{\text{in}} \times \frac{R}{R_T + R}$$




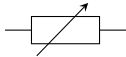

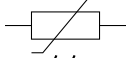

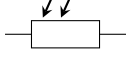

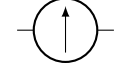


- **Temperature rises** $\Rightarrow R_T$ decreases $\Rightarrow V_{\text{out}}$ **increases**.
- **Temperature falls** $\Rightarrow R_T$ increases $\Rightarrow V_{\text{out}}$ **decreases**.
- Swap the positions of R_T and R to **invert** the response (output high when cold, low when hot).

The same logic applies to an LDR replacing R_T , with light intensity replacing temperature.



Circuit Symbols (Section 6 Reference)

The following symbols from CIE Section 6 are required for Topic 10 circuit diagrams.

Component	Symbol	Component	Symbol
Cell		Fixed resistor	
Battery of cells		Variable resistor	
Switch		Thermistor	
Ammeter		Light-dependent resistor	
Voltmeter		Galvanometer	
Lamp		Potentiometer	

Formula Summary Sheet

Formula	Quantity	Units
$\varepsilon = W/Q$	E.m.f. definition	V
$V = \varepsilon - Ir$	Terminal p.d.	V
$I = \varepsilon/(R + r)$	Current in circuit	A
$R_{\text{total}} = R_1 + R_2 + \dots$	Series resistance	Ω
$1/R_{\text{total}} = 1/R_1 + 1/R_2 + \dots$	Parallel resistance	Ω
$R_{\text{total}} = R_1 R_2 / (R_1 + R_2)$	Two resistors in parallel	Ω
$V_{\text{out}} = V_{\text{in}} \times R_2 / (R_1 + R_2)$	Potential divider output	V
$\varepsilon_1 / \varepsilon_2 = l_1 / l_2$	Potentiometer balance	—

KCL: $\sum I_{\text{in}} = \sum I_{\text{out}}$ at any junction (conservation of charge).

KVL: $\sum \mathcal{E} = \sum IR$ around any closed loop (conservation of energy).

Note: Lost volts = Ir ; terminal p.d. < \mathcal{E} whenever current flows.

Worked Examples

Example 1 — Internal Resistance

Question: A battery of e.m.f. 12 V and internal resistance 0.5Ω is connected to an external resistor of 3.5Ω . Calculate (a) the current, (b) the terminal p.d., and (c) the power dissipated internally.

Solution

$$(a) \quad I = \frac{\varepsilon}{R + r} = \frac{12}{3.5 + 0.5} = \frac{12}{4.0} = \mathbf{3.0 \text{ A}}$$

$$(b) \quad V = \varepsilon - Ir = 12 - (3.0)(0.5) = 12 - 1.5 = \mathbf{10.5 \text{ V}}$$

$$(c) \quad P_{\text{internal}} = I^2 r = (3.0)^2 \times 0.5 = 9 \times 0.5 = \mathbf{4.5 \text{ W}}$$

Example 2 — Kirchhoff's Laws

Question: In the circuit below, $\varepsilon_1 = 10 \text{ V}$, $\varepsilon_2 = 4 \text{ V}$, $R_1 = 3 \Omega$, $R_2 = 2 \Omega$, $R_3 = 5 \Omega$. Find the current flowing in the circuit (treat as a single loop, both sources driving in the same direction).

Solution

Applying KVL around the loop (taking the direction of current as positive):

$$\sum \varepsilon = \sum IR$$

$$10 - 4 = I(3 + 2 + 5)$$

$$6 = 10I$$

$$I = \mathbf{0.60 \text{ A}}$$

Note: if ε_2 had opposed ε_1 , we would write $10 - 4$ on the left side (one e.m.f. is negative in our sign convention).

Example 3 — Resistors in Parallel

Question: Two resistors of 6Ω and 12Ω are connected in parallel across a 6 V supply of negligible internal resistance. Find (a) the combined resistance, (b) the total current, and (c) the current through each resistor.

Solution

$$(a) \quad R_{\text{total}} = \frac{6 \times 12}{6 + 12} = \frac{72}{18} = \mathbf{4.0 \Omega}$$

$$(b) \quad I_{\text{total}} = V/R_{\text{total}} = 6/4.0 = \mathbf{1.5 \text{ A}}$$

$$(c) \quad I_1 = 6/6 = \mathbf{1.0 \text{ A}}; \quad I_2 = 6/12 = \mathbf{0.50 \text{ A}}$$

Check: $1.0 + 0.5 = 1.5 \text{ A}$ ✓ (KCL satisfied)

Example 4 — Potential Divider with Thermistor

Question: A potential divider consists of a thermistor R_T in series with a $1.5\text{ k}\Omega$ fixed resistor, connected across a 5.0 V supply. The output is taken across the fixed resistor. At 20°C the thermistor has resistance $3.0\text{ k}\Omega$. Calculate (a) V_{out} at 20°C , and (b) whether V_{out} increases or decreases as temperature rises.

Solution

(a)
$$V_{\text{out}} = 5.0 \times \frac{1500}{3000 + 1500} = 5.0 \times \frac{1500}{4500} = 5.0 \times \frac{1}{3} = \mathbf{1.67\text{ V}}$$

(b) As temperature **rises**, R_T **decreases**. The fraction $R/(R_T + R)$ **increases**, so V_{out} **increases**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electromotive force (e.m.f.) and distinguish it from potential difference in terms of energy.

[3 marks]

Q2. A cell has e.m.f. 9.0 V and internal resistance 0.8Ω . When connected to a resistor, a current of 3.0 A flows. Calculate (a) the terminal p.d. and (b) the power lost to internal resistance.

[3 marks]

Q3. State Kirchhoff's first and second laws and give the physical principle underlying each.

[4 marks]

Q4. Three resistors of 2Ω , 4Ω and 6Ω are connected in parallel. Calculate the combined resistance.

[2 marks]

Section B — Longer Structured Questions

Q5. A battery of e.m.f. ε and internal resistance r is connected to a variable external resistor R .

(a) Show that the terminal p.d. $V = \varepsilon - Ir$ and hence that $V = \varepsilon - \frac{\varepsilon r}{R + r}$.
[2 marks]

(b) A graph of V against I is plotted as R is varied. Describe the graph and explain how ε and r can be determined from it.
[3 marks]

(c) Data from the graph gives $\varepsilon = 6.0 \text{ V}$ and $r = 0.4 \Omega$. Calculate the current and terminal p.d. when $R = 2.6 \Omega$.
[3 marks]

Q6. The circuit shown consists of a supply of e.m.f. 12 V and negligible internal resistance connected to resistors $R_1 = 4 \Omega$, $R_2 = 6 \Omega$ and $R_3 = 12 \Omega$, where R_2 and R_3 are in parallel with each other and R_1 is in series with this parallel combination.

- (a) Calculate the combined resistance of R_2 and R_3 in parallel.

[2 marks]

- (b) Calculate the total resistance of the circuit and the current drawn from the supply.

[2 marks]

- (c) Calculate the p.d. across the parallel combination and hence the current through each of R_2 and R_3 .

[3 marks]

- (d) Verify your answers using Kirchhoff's first law.

[1 mark]

Q7. A potential divider consists of a $2.0 \text{ k}\Omega$ resistor and a light-dependent resistor (LDR) connected in series across a 10 V supply. The output voltage is taken across the LDR. In bright light, the LDR has resistance 500Ω ; in darkness, its resistance is $20 \text{ k}\Omega$.

(a) Calculate V_{out} in bright light.

[2 marks]

(b) Calculate V_{out} in darkness.

[2 marks]

(c) Explain how the circuit could be modified so that V_{out} is high in bright light rather than in darkness.

[1 mark]

Mark Scheme and Answers

Q1. E.m.f. is the energy transferred *to* each coulomb of charge by the source [1]; p.d. is the energy transferred *from* each coulomb of charge by a component [1]; e.m.f. relates to energy input (e.g. chemical to electrical), p.d. relates to energy output (e.g. electrical to heat) [1].

Q2(a). $V = \varepsilon - Ir = 9.0 - (3.0)(0.8) = 9.0 - 2.4 = \mathbf{6.6 \text{ V}}$ [2].

Q2(b). $P = I^2r = (3.0)^2 \times 0.8 = \mathbf{7.2 \text{ W}}$ [1].

Q3. KCL: sum of currents into a junction = sum of currents out [1]; consequence of conservation of charge [1]. KVL: sum of e.m.f.s around a closed loop = sum of IR products [1]; consequence of conservation of energy [1].

Q4. $\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6 + 3 + 2}{12} = \frac{11}{12}$; $R = \mathbf{12/11} \approx \mathbf{1.1 \Omega}$ [2].

Q5(a). By KVL: $\varepsilon = IR + Ir = I(R + r)$; rearranging gives $V = IR = \varepsilon - Ir$ [2].

Q5(b). Straight line [1]; y -intercept = ε (open-circuit voltage) [1]; magnitude of gradient = r [1].

Q5(c). $I = 6.0/(2.6+0.4) = 6.0/3.0 = \mathbf{2.0 \text{ A}}$ [1]; $V = 6.0 - (2.0)(0.4) = 6.0 - 0.8 = \mathbf{5.2 \text{ V}}$ [2].

Q6(a). $R_{23} = (6 \times 12)/(6 + 12) = 72/18 = \mathbf{4.0 \Omega}$ [2].

Q6(b). $R_{\text{total}} = 4 + 4 = \mathbf{8.0 \Omega}$; $I = 12/8 = \mathbf{1.5 \text{ A}}$ [2].

Q6(c). $V_{23} = 1.5 \times 4 = \mathbf{6.0 \text{ V}}$; $I_2 = 6/6 = \mathbf{1.0 \text{ A}}$; $I_3 = 6/12 = \mathbf{0.50 \text{ A}}$ [3].

Q6(d). $I_2 + I_3 = 1.0 + 0.5 = 1.5 \text{ A} = I_{\text{total}}$ ✓ [1].

Q7(a). $V_{\text{out}} = 10 \times 500/(2000 + 500) = 5000/2500 = \mathbf{2.0 \text{ V}}$ [2].

Q7(b). $V_{\text{out}} = 10 \times 20000/(2000 + 20000) = 200000/22000 = \mathbf{9.1 \text{ V}}$ [2].

Q7(c). Swap the positions of the LDR and the $2.0 \text{ k}\Omega$ resistor so that the output is taken across the fixed resistor instead [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Recall and use the circuit symbols from Section 6	
<input type="checkbox"/> Define e.m.f. as energy transferred per unit charge by a source	
<input type="checkbox"/> Distinguish between e.m.f. and p.d. in terms of energy	
<input type="checkbox"/> Use $V = \varepsilon - Ir$ and $I = \varepsilon / (R + r)$ for circuits with internal resistance	
<input type="checkbox"/> Determine ε and r from a V – I graph (intercept and gradient)	
<input type="checkbox"/> State and apply Kirchhoff's first law (conservation of charge)	
<input type="checkbox"/> State and apply Kirchhoff's second law (conservation of energy)	
<input type="checkbox"/> Derive and use $R_{\text{total}} = R_1 + R_2 + \dots$ for series resistors	
<input type="checkbox"/> Derive and use $1/R_{\text{total}} = 1/R_1 + 1/R_2 + \dots$ for parallel resistors	
<input type="checkbox"/> Use Kirchhoff's laws to solve multi-loop circuit problems	
<input type="checkbox"/> Understand the principle of a potential divider circuit	
<input type="checkbox"/> Use $V_{\text{out}} = V_{\text{in}} \times R_2 / (R_1 + R_2)$ for a potential divider	
<input type="checkbox"/> Explain the potentiometer as a null method for comparing p.d.s	
<input type="checkbox"/> Explain how a thermistor or LDR is used in a potential divider	
<i>Key: 1 = Need more work 2 = Getting there 3 = Confident</i>	

Good luck with your revision!

Kirchhoff's laws are nothing more than conservation of charge and conservation of energy written in circuit language. Once you see that, every circuit problem becomes a matter of careful bookkeeping — choose your loops, track your signs, and the algebra does the rest.

Topic 11

Particle Physics

Revision Booklet

This booklet covers:

- Atoms, Nuclei and Radiation
- Types of Radiation: α , β and γ
- Radioactive Decay Equations
- Fundamental Particles: Quarks
- Hadrons, Baryons and Mesons
- Leptons and Beta Decay

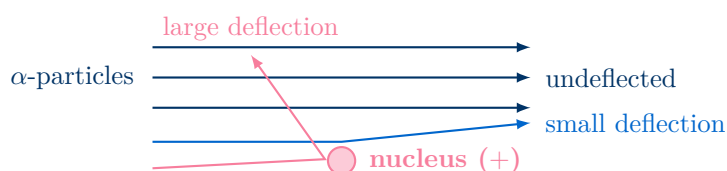
Atoms, Nuclei and Radiation

The Nuclear Atom

The **Geiger–Marsden** α -particle scattering experiment (directed α -particles at thin gold foil) showed:

- Most α -particles passed straight through \Rightarrow the atom is mostly empty space.
- A small fraction were deflected through large angles \Rightarrow a dense, positively charged nucleus exists.
- A very few bounced back ($> 90^\circ$) \Rightarrow the nucleus is very small compared to the atom.

Rutherford scattering diagram



Nuclear Notation

A **nuclide** is a specific nuclear species characterised by its proton and nucleon numbers:



- $A =$ **nucleon number** (mass number): total number of protons + neutrons.
- $Z =$ **proton number** (atomic number): number of protons.
- Number of neutrons $N = A - Z$.
- **Isotopes**: atoms of the same element (same Z) with different numbers of neutrons (different A).

Conservation Laws in Nuclear Processes

In every nuclear reaction or decay, the following are always conserved:

- **Nucleon number** A (total count of protons + neutrons).
- **Charge** (equivalently, proton number Z).

These two conservation laws are the key tool for balancing decay equations.

Unified Atomic Mass Unit

The **unified atomic mass unit** (u) is defined as exactly $\frac{1}{12}$ of the mass of a carbon-12 atom.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

Approximate masses: proton ≈ 1 u; neutron ≈ 1 u; electron ≈ 0.00055 u.

Types of Radiation: α , β and γ

Radiation	Composition	Charge	Mass	Stopped by
α (alpha)	2 protons + 2 neutrons (helium-4 nucleus)	$+2e$	4 u	A few cm of air; thin paper
β^- (beta-minus)	Electron (e^-)	$-e$	$\approx 0 \left(\frac{1}{1836} \text{ u}\right)$	A few mm of aluminium
β^+ (beta-plus)	Positron (e^+)	$+e$	$\approx 0 \left(\frac{1}{1836} \text{ u}\right)$	Annihilates with electron
γ (gamma)	Electromagnetic wave / photon	0	0	Several cm of lead

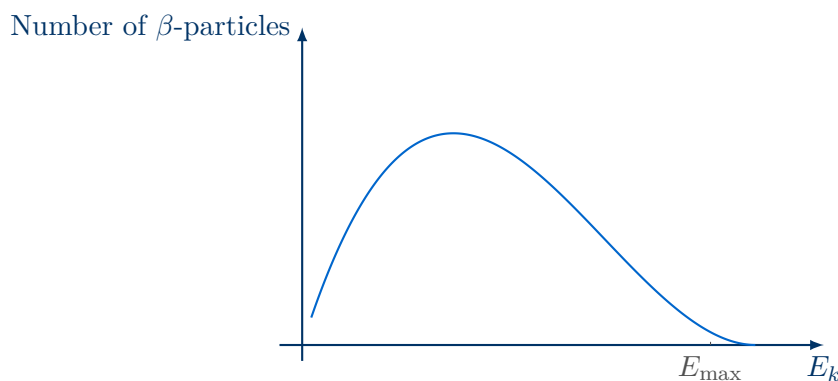
Antiparticles

Every particle has a corresponding **antiparticle** with:

- The **same mass** as the particle.
- The **opposite charge** (and opposite quantum numbers).
- The **positron** (β^+ , e^+) is the antiparticle of the electron (e^-).
- When a particle meets its antiparticle they **annihilate**, producing energy (usually two γ -ray photons).

Energies of Emitted Particles

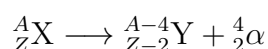
- α -particles are emitted with **discrete (fixed) energies**: each decay of a given nuclide always releases an α -particle of the same kinetic energy, giving a sharp line spectrum.
- β -particles have a **continuous range of energies** from zero up to a maximum: the total energy of the decay is shared between the β -particle and an (anti)neutrino, which can take any share — hence the continuous spectrum.
- **(Anti)neutrinos** are produced in β -decay to account for this energy sharing and to conserve momentum and angular momentum.

Continuous β energy spectrum**Common Mistake**

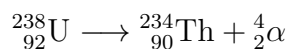
Students often state that β -particles have a fixed energy like α -particles. They do not — the continuous spectrum is direct evidence for neutrino emission. The **maximum** β energy equals the total energy available from the decay (as if no neutrino were present).

Radioactive Decay Equations**Alpha decay**

In α -decay, the nucleus emits a helium-4 nucleus (${}^4_2\alpha$). Both A and Z decrease:



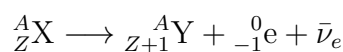
Example:



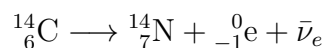
Check: A : $238 = 234 + 4$ ✓; Z : $92 = 90 + 2$ ✓

Beta-minus decay (β^-)

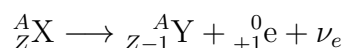
A neutron converts to a proton; an electron and an electron antineutrino are emitted. A is unchanged; Z increases by 1:



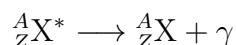
Example:

**Beta-plus decay (β^+)**

A proton converts to a neutron; a positron and an electron neutrino are emitted. A is unchanged; Z decreases by 1:

**Gamma emission**

γ -radiation accompanies α or β decay when the daughter nucleus is left in an excited state. Neither A nor Z changes:



Decay Equation Summary

Decay	ΔA	ΔZ	Particles emitted
α	-4	-2	${}^4_2\alpha$
β^-	0	+1	${}^0_{-1}e, \bar{\nu}_e$
β^+	0	-1	${}^0_{+1}e, \nu_e$
γ	0	0	γ photon

Fundamental Particles: Quarks

Quarks

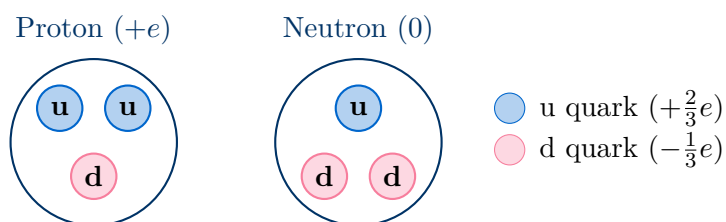
A **quark** is a fundamental, point-like particle. There are **six flavours** of quark (and six corresponding antiquarks):

Flavour	Symbol	Charge	Antiquark	Symbol	Charge
up	u	$+\frac{2}{3}e$	anti-up	\bar{u}	$-\frac{2}{3}e$
down	d	$-\frac{1}{3}e$	anti-down	\bar{d}	$+\frac{1}{3}e$
strange	s	$-\frac{1}{3}e$	anti-strange	\bar{s}	$+\frac{1}{3}e$
charm	c	$+\frac{2}{3}e$	anti-charm	\bar{c}	$-\frac{2}{3}e$
top	t	$+\frac{2}{3}e$	anti-top	\bar{t}	$-\frac{2}{3}e$
bottom	b	$-\frac{1}{3}e$	anti-bottom	\bar{b}	$+\frac{1}{3}e$

Only up and down quarks are required for protons and neutrons; strange, charm, top and bottom are examined only for their charges.

Quark Composition of Proton and Neutron

Particle	Quarks	Charge calculation	Total charge
Proton	uud	$\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e$	$+e$
Neutron	udd	$\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e$	0



Hadrons, Baryons and Mesons

Hadrons

A **hadron** is any particle made of quarks. Hadrons are subdivided into two classes:

- **Baryons:** composed of **three quarks** (qqq). Examples: proton (uud), neutron (udd).
- **Mesons:** composed of **one quark and one antiquark** (q \bar{q}). Examples: pion (π^+ = u \bar{d}).

Protons and neutrons are **not** fundamental particles — they are baryons made of quarks.

Class	Quark content	Example	Charge
Baryon	qqq (three quarks)	Proton (uud)	+e
Baryon	qqq (three quarks)	Neutron (udd)	0
Meson	q \bar{q} (quark + antiquark)	π^+ (u \bar{d})	+e
Meson	q \bar{q} (quark + antiquark)	π^- ($\bar{u}d$)	-e
Meson	q \bar{q} (quark + antiquark)	π^0 (u \bar{u} or d \bar{d})	0

Leptons and Beta Decay

Leptons

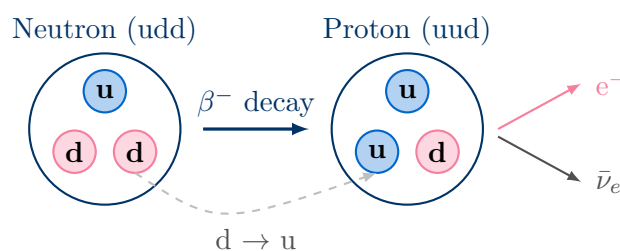
Leptons are fundamental (point-like, not made of quarks) particles. The leptons required at A-level are:

- **Electron** (e^-) — charge $-e$.
- **Positron** (e^+) — antiparticle of electron; charge $+e$.
- **Electron neutrino** (ν_e) — charge 0; produced in β^+ decay.
- **Electron antineutrino** ($\bar{\nu}_e$) — charge 0; produced in β^- decay.

Quark Changes in Beta Decay

Beta decay occurs because a quark changes flavour inside a nucleon:

Decay	Quark change	Nucleon change	Particles emitted
β^-	d \rightarrow u	Neutron \rightarrow Proton	$e^- + \bar{\nu}_e$
β^+	u \rightarrow d	Proton \rightarrow Neutron	$e^+ + \nu_e$

Quark-level picture of β^- decay

Neutrinos vs Antineutrinos

This is a common source of error in exams:

- β^- decay produces an **electron antineutrino** ($\bar{\nu}_e$) — the antiparticle of the neutrino.
- β^+ decay produces an **electron neutrino** (ν_e).

A helpful check: the lepton number must be conserved. In β^- , a lepton of number +1 (the electron) is created alongside a lepton of number -1 (the antineutrino), giving net lepton number zero — matching the original nucleon.

Key Facts Summary

Particle	Symbol	Charge	Type
Proton	p	+e	Baryon (uud)
Neutron	n	0	Baryon (udd)
Electron	e^-	-e	Lepton
Positron	e^+	+e	Lepton (antiparticle of e^-)
Electron neutrino	ν_e	0	Lepton
Electron antineutrino	$\bar{\nu}_e$	0	Lepton
α -particle	$\frac{4}{2}\alpha$	+2e	Baryon (not fundamental)

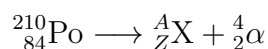
Quark charges (to recall): u: $+\frac{2}{3}e$; d: $-\frac{1}{3}e$; s: $-\frac{1}{3}e$; c: $+\frac{2}{3}e$; t: $+\frac{2}{3}e$; b: $-\frac{1}{3}e$
Antiquark charge = opposite of quark charge.

Conservation laws in every nuclear/particle process: nucleon number A ; charge Z ; lepton number.

Worked Examples

Example 1 — Completing a Decay Equation

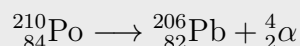
Question: Complete the following decay equation and identify the type of decay:



Solution

Conserve nucleon number A : $210 = A + 4 \Rightarrow A = \mathbf{206}$

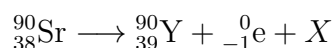
Conserve charge Z : $84 = Z + 2 \Rightarrow Z = \mathbf{82}$ (lead, Pb)



This is **alpha decay** — the nucleon number decreases by 4 and the proton number by 2.

Example 2 — Identifying Decay Type from Equation

Question: Identify the particle X and the type of decay in:

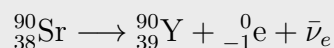


Solution

A is conserved: $90 = 90 + 0 + 0 \checkmark$

Z : $38 = 39 + (-1) + 0 \checkmark$

The emitted particles are an electron (β^-) and X. This is β^- decay, so X must be an **electron antineutrino**: $X = \bar{\nu}_e$.



Example 3 — Quark Composition and Charge

Question: (a) State the quark composition of an antiproton. (b) Calculate its charge.

Solution

(a) The proton has quark composition uud. The antiproton is its antiparticle, so every quark is replaced by its antiquark:

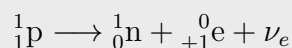
$$\text{Antiproton} = \bar{u}\bar{u}\bar{d}$$

(b) Charge = $-\frac{2}{3}e + (-\frac{2}{3}e) + \frac{1}{3}e = -\frac{3}{3}e = -e$

This confirms the antiproton has charge $-e$, equal in magnitude but opposite in sign to the proton.

Example 4 — Quark Changes in β^+ Decay

Question: A proton undergoes β^+ decay. Write a full decay equation and describe the quark-level change.

Solution

At the quark level, one **up quark converts to a down quark**:



Charge check: $+e \rightarrow 0 + e + 0 \checkmark$; Nucleon number: $1 = 1 + 0 + 0 \checkmark$

Practice Exam Questions**Section A — Short Answer Questions**

Q1. Describe the key observations from the α -particle scattering experiment and explain what each tells us about atomic structure.

[4 marks]

Q2. Distinguish between nucleon number and proton number. Define the term *isotope*.

[3 marks]

Q3. State the composition, charge and approximate mass (in u) of α -, β^- - and γ -radiation.

[5 marks]

Q4. Explain why β -particles have a continuous energy spectrum, whereas α -particles from a given nuclide have a fixed energy.

[3 marks]

Section B — Longer Structured Questions

Q5. The nuclide ${}^{226}_{88}\text{Ra}$ undergoes α -decay.

(a) Write a balanced nuclear equation for this decay, identifying the daughter nuclide.
[2 marks]

(b) State the two conservation laws used to balance the equation.
[2 marks]

(c) Explain why the α -particles emitted in this decay all have the same kinetic energy.
[2 marks]

Q6. The nuclide ${}^{14}_6\text{C}$ undergoes β^- decay.

(a) Write a full balanced equation for this decay, including all particles emitted.

[2 marks]

(b) Describe the quark-level change that occurs during this decay.

[2 marks]

(c) State and explain whether the β^- particles emitted have a fixed or continuous range of energies.

[2 marks]

Q7. (a) State what is meant by a *hadron* and distinguish between a baryon and a meson.
[3 marks]

(b) The particle π^+ is a meson with quark composition $u\bar{d}$. Show that this gives a charge of $+e$.
[2 marks]

(c) State the quark composition of an antiproton and calculate its charge.
[2 marks]

(d) Explain why electrons are *not* classified as hadrons.
[1 mark]

Mark Scheme and Answers

Q1. Most α -particles pass straight through \Rightarrow atom is mostly empty space [1]; small fraction deflected through large angles \Rightarrow concentrated positive charge (nucleus) exists [1]; very few back-scattered \Rightarrow nucleus is very small / very dense [1]; all deflections from positive charge are consistent with electrostatic repulsion [1].

Q2. Nucleon number (A): total number of protons + neutrons in a nucleus [1]; **proton number (Z):** number of protons only [1]; **isotopes:** atoms of the same element (same Z) with different numbers of neutrons (different A) [1].

Q3. α : 2 protons + 2 neutrons; charge $+2e$; mass 4 u [2]. β^- : electron; charge $-e$; mass ≈ 0 ($\frac{1}{1836}$ u) [2]. γ : electromagnetic radiation/photon; charge 0; mass 0 [1].

Q4. In β -decay, an (anti)neutrino is also emitted [1]; the total energy is shared between the β -particle and the (anti)neutrino in varying proportions [1]; so the β -particle can have any energy from zero to a maximum — continuous spectrum [1]. For α : only two products (daughter + α); by conservation of energy and momentum, the α always takes the same fixed share [implied/bonus].

Q5(a). ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + \frac{4}{2}\alpha$ [2] (1 mark if equation has correct concept but arithmetic error).

Q5(b). Conservation of nucleon number A [1]; conservation of charge (proton number Z) [1].

Q5(c). Only two products (daughter + α) [1]; by conservation of energy and momentum, the α always receives the same fixed kinetic energy (discrete energy levels) [1].

Q6(a). ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + {}^0_{-1}\text{e} + \bar{\nu}_e$ [2].

Q6(b). A down quark converts to an up quark ($d \rightarrow u$) [1]; inside a neutron, converting it to a proton [1].

Q6(c). Continuous range [1]; because the energy is shared between the β^- particle and the antineutrino in varying proportions [1].

Q7(a). A hadron is a particle made of quarks [1]; a baryon consists of three quarks [1]; a meson consists of one quark and one antiquark [1].

Q7(b). Charge = $+\frac{2}{3}e + \frac{1}{3}e$ [1] = $+e$ ✓ [1].

Q7(c). $\bar{u}\bar{u}d$ [1]; charge = $-\frac{2}{3}e - \frac{2}{3}e + \frac{1}{3}e = -e$ [1].

Q7(d). Electrons are leptons, not composed of quarks / not hadrons by definition [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
<input type="checkbox"/> Describe the α -scattering experiment and interpret its results	
<input type="checkbox"/> Describe the nuclear model: protons, neutrons and orbital electrons	
<input type="checkbox"/> Distinguish nucleon number (A) from proton number (Z)	
<input type="checkbox"/> Define isotopes and use the notation A_ZX	
<input type="checkbox"/> State that nucleon number and charge are conserved in nuclear processes	
<input type="checkbox"/> Describe composition, mass and charge of α , β^- , β^+ and γ	
<input type="checkbox"/> State that a positron is the antiparticle of an electron (same mass, opposite charge)	
<input type="checkbox"/> State that $\bar{\nu}_e$ is produced in β^- decay and ν_e in β^+ decay	
<input type="checkbox"/> Explain why β -particles have a continuous energy spectrum	
<input type="checkbox"/> Write balanced α - and β -decay equations	
<input type="checkbox"/> Use the unified atomic mass unit (u)	
<input type="checkbox"/> State the six quark flavours and recall their charges	
<input type="checkbox"/> State the quark composition of the proton (uud) and neutron (udd)	
<input type="checkbox"/> Distinguish hadrons (baryons: qqq; mesons: q \bar{q}) from leptons	
<input type="checkbox"/> Describe the quark change in β^- (d \rightarrow u) and β^+ (u \rightarrow d) decay	
<input type="checkbox"/> Recall that electrons and neutrinos are leptons	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Particle physics is really just careful bookkeeping. Every decay equation is balanced by two rules — conserve A , conserve Z — and every quark change in β -decay is just one quark swapping flavour. Get those two ideas solid and the whole topic clicks into place.