CIE International A Level Physics

CIE A-Level Physics — Year 13

Revision Booklet

This booklet covers:

- Topic 12 — Motion in a Circle
- Topic 13 — Gravitational Fields
- Topics 14, 15 & 16 — Thermal Physics
- Topic 17 — Oscillations
- Topic 18 — Electric Fields
- Topic 19 — Capacitance
- Topic 20 — Magnetic Fields
- Topic 21 — Alternating Currents
- Topic 22 — Quantum Physics
- Topic 23 — Nuclear Physics
- Topic 24 — Medical Physics
- Topic 25 — Astronomy and Cosmology

CIE International A Level Physics

Topic 12

Motion in a Circle

Revision Booklet

This booklet covers:

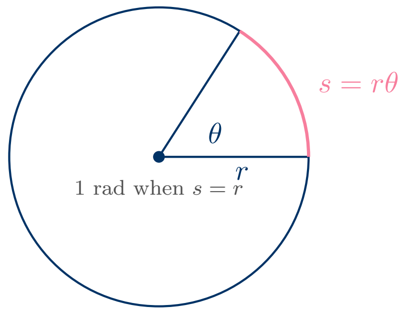
- Angular Displacement and the Radian
- Angular Speed and Period
- Centripetal Acceleration
- Centripetal Force
- Examples of Circular Motion

Angular Displacement and the Radian

The Radian

The **radian** (rad) is the SI unit of angle. One radian is the angle subtended at the centre of a circle by an arc whose length equals the radius.

$$\theta \text{ (rad)} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$



Key Angle Conversions

Full circle:	360ř $= 2\pi$ rad
Half circle:	180ř $= \pi$ rad
Quarter circle:	90ř $= \pi/2$ rad
To convert degrees to radians:	multiply by $\pi/180$
To convert radians to degrees:	multiply by $180/\pi$

Angular Speed and Linear Speed

Angular Speed

The **angular speed** ω of an object moving in a circle is the rate of change of angular displacement:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \text{units: rad s}^{-1}$$

For uniform circular motion (constant speed), ω is constant.

Angular Speed and Period

$$\omega = \frac{2\pi}{T} = 2\pi f$$

ω = angular speed (rad s^{-1})
T = period — time for one complete revolution (s)
f = frequency (Hz = s^{-1})

Linear Speed and Angular Speed

$$v = r\omega$$

v = linear (tangential) speed (m s^{-1})
r = radius of the circular path (m)
ω = angular speed (rad s^{-1})

Although ω is constant for uniform circular motion, the **velocity** is not — its direction changes continuously. The speed v is constant but the direction of motion is always **tangential** to the circle.

Speed vs Velocity in Circular Motion

In uniform circular motion:

- **Speed** is constant — the magnitude of velocity does not change.
- **Velocity** is not constant — its direction changes at every point.
- Because velocity changes, the object is **accelerating**, even though its speed is constant.

Centripetal Acceleration

Centripetal Acceleration

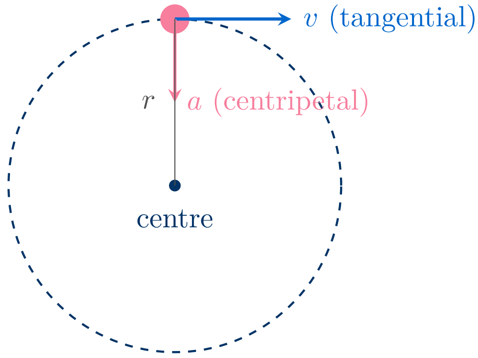
For an object moving in a circle at constant speed, the acceleration is directed **towards the centre** of the circle. This is called **centripetal acceleration**.

- It arises because the **direction** of velocity is continuously changing.
- A force of **constant magnitude** that is always **perpendicular to the velocity** produces centripetal acceleration.
- Because the force is perpendicular to motion, it does **no work** — kinetic energy (and hence speed) remains constant.

Centripetal Acceleration

$$a = r\omega^2 = \frac{v^2}{r}$$

a = centripetal acceleration, directed towards centre (m s^{-2})
r = radius of circular path (m)
ω = angular speed (rad s^{-1})
v = linear speed (m s^{-1})



Centripetal Force

Centripetal Force

The **centripetal force** is the resultant force directed towards the centre of the circle that causes centripetal acceleration. It is not a new type of force — it is whatever force (or combination of forces) acts towards the centre in a given situation.

Centripetal Force

Applying Newton's second law ($F = ma$) with $a = r\omega^2 = v^2/r$:

$$F = mr\omega^2 = \frac{mv^2}{r}$$

F = centripetal force, directed towards centre (N)
m = mass of the object (kg)
r = radius of circular path (m)
ω = angular speed (rad s^{-1})
v = linear speed (m s^{-1})

What Provides the Centripetal Force?

The centripetal force is provided by different physical forces depending on the situation:

Situation	**Force providing centripetal force**
Planet/satellite orbiting a star	Gravitational attraction
Car turning on a flat road	Friction between tyres and road
Ball on a string, horizontal circle	Tension in the string
Electron orbiting nucleus (Bohr model)	Electrostatic (Coulomb) attraction
Object on the inside of a curved loop	Normal contact force

"Centrifugal Force" is Not Real

There is no outward "centrifugal force" acting on the object. In the reference frame of the ground (inertial frame), the only horizontal force on an object in circular motion is the **inward** centripetal force. The sensation of being "pushed outward" is the result of inertia — the body's tendency to continue in a straight line.

Examples of Circular Motion

Conical Pendulum

A mass on a string of length L makes angle θ with the vertical, moving in a horizontal circle of radius $r = L\sin\theta$.

Conical Pendulum

Resolving forces:

$$\text{Vertical:} \quad T\cos\theta = mg$$
$$\text{Horizontal (centripetal):} \quad T\sin\theta = \frac{mv^2}{r} = mr\omega^2$$

Dividing: $\tan\theta = \dfrac{r\omega^2}{g} = \dfrac{v^2}{rg}$

Car on a Banked Track

On a banked track (angle θ), the horizontal component of the normal force provides the centripetal force, reducing the need for friction.

Ideal Banking Angle

For a vehicle travelling at speed v on a track banked at angle θ, with no friction:

$$\tan\theta = \frac{v^2}{rg}$$

Vertical Circular Motion

For an object on the inside of a vertical loop of radius r at the **top** of the loop:

$$mg + N = \frac{mv^2}{r}$$

The minimum speed for the object to maintain contact: $N \geq 0 \Rightarrow v_{\text{min}} = \sqrt{gr}$.

Formula Summary Sheet

Formula	Quantity	Units
$\theta = s/r$	Angular displacement	rad
$\omega = \Delta\theta/\Delta t$	Angular speed	rad s^{-1}
$\omega = 2\pi/T = 2\pi f$	Angular speed from period	rad s^{-1}
$v = r\omega$	Linear speed	m s^{-1}
$a = r\omega^2 = v^2/r$	Centripetal acceleration	m s^{-2}
$F = mr\omega^2 = mv^2/r$	Centripetal force	N

Useful: 2π rad = 360ř; 1 revolution = 2π rad; $v = r\omega$ links linear and angular quantities.

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy

1. **Find** ω: use $\omega = 2\pi/T$ or $\omega = 2\pi f$ — convert rpm or revolutions per second to rad s^{-1} first.
2. **Find** v: use $v = r\omega$ if needed.
3. **Identify the centripetal force**: decide which physical force (tension, gravity, normal force, friction) provides it.
4. **Apply** $F = mr\omega^2$ **or** $F = mv^2/r$: equate to the expression for that force and solve.

Common Errors — Avoid These!

- Using **degrees** instead of radians in $v = r\omega$ or $a = r\omega^2$ — ω must always be in rad s^{-1}.
- Confusing **period** T with frequency f — remember $T = 1/f$.
- Forgetting to **identify the centripetal force** physically — in a free-body diagram, only real forces appear; there is no "centripetal force" arrow separate from, say, tension or gravity.
- In vertical circle problems, failing to account for **the component of gravity** that contributes to (or subtracts from) the centripetal force.
- Confusing r (radius) with **diameter** — always halve the diameter.

Worked Examples

Example 1 — Angular and Linear Speed

Question: A CD rotates at 500 rpm. Calculate (a) the angular speed, and (b) the linear speed of a point 6.0 cm from the centre.

Solution

Solution:
(a) Convert rpm to rad s^{-1}:

$$\omega = \frac{500 \times 2\pi}{60} = \mathbf{52.4} \text{ rad s}^{-1}$$

(b) $v = r\omega = 0.060 \times 52.4 = \mathbf{3.14}$ m s^{-1}

Example 2 — Centripetal Force on a Car

Question: A car of mass 1200 kg travels at 18 m s^{-1} around a flat circular bend of radius 45 m. Calculate the centripetal force required and state what provides it.

Solution

Solution:

$$F = \frac{mv^2}{r} = \frac{1200 \times 18^2}{45} = \frac{1200 \times 324}{45} = \mathbf{8640} \text{ N}$$

This force is provided by **friction** between the tyres and the road surface, acting towards the centre of the bend.

Example 3 — Satellite Orbit

Question: A satellite orbits Earth at a radius of 7.5×10^6 m. The gravitational field strength at this altitude is 7.1 N kg^{-1}. Calculate the orbital period.

Solution

Solution:
Gravitational force provides centripetal force:

$$mg = mr\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{7.1}{7.5 \times 10^6}} = 9.73 \times 10^{-4} \text{ rad s}^{-1}$$

Period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{9.73 \times 10^{-4}} = 6454 \text{ s} \approx \mathbf{1.79} \text{ hours}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the radian.

[1 mark]

Q2. A wheel of radius 0.35 m completes 120 revolutions per minute. Calculate (a) the angular speed and (b) the linear speed of a point on the rim.

[3 marks]

Q3. Explain why an object moving in a circle at constant speed is accelerating, and state the direction of this acceleration.

[2 marks]

Q4. A stone of mass 0.15 kg is attached to a string and swung in a horizontal circle of radius 0.80 m at 3.0 rev s^{-1}. Calculate the tension in the string. (Assume the string is horizontal.)

[3 marks]

Q5. Explain why the centripetal force does no work on an object moving in a circle at constant speed.

[2 marks]

Section B — Longer Structured Questions

Q6. A car of mass 950 kg travels over the top of a hill that has a circular cross-section of radius 120 m.

(a) Draw a free-body diagram for the car at the top of the hill, showing and labelling the forces acting.

[2 marks]

(b) Write an equation relating the forces at the top of the hill to the centripetal acceleration. Hence find the speed at which the car just loses contact with the road.

[4 marks]

(c) At a speed of 20 m s^{-1}, calculate the normal contact force on the car at the top of the hill.

[2 marks]

Q7. A conical pendulum consists of a mass of 0.25 kg on a string of length 0.60 m, rotating so that the string makes an angle of 30ř with the vertical.

(a) Calculate the radius of the circular path.

[1 mark]

(b) Calculate the tension in the string.

[2 marks]

(c) Calculate the angular speed of the mass.

[3 marks]

Mark Scheme and Answers

Q1. The radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle [1].

Q2. (a) $\omega = 120 \times 2\pi/60 = \mathbf{4\pi = 12.6}$ rad s^{-1} [2]. (b) $v = r\omega = 0.35 \times 12.6 = \mathbf{4.4}$ m s^{-1} [1].

Q3. The direction of the velocity changes continuously [1]; acceleration is the rate of change of velocity — a change in direction (even at constant speed) constitutes acceleration, directed towards the centre of the circle [1].

Q4. $\omega = 3.0 \times 2\pi = 18.85$ rad s^{-1} [1]; $T = F = mr\omega^2 = 0.15 \times 0.80 \times 18.85^2$ [1] $= \mathbf{42.6}$ N [1].

Q5. The centripetal force is always perpendicular to the velocity (direction of motion) [1]; work done $= F \cos 90ř \times d = 0$ — a perpendicular force does no work [1].

Q6(a). Diagram showing weight mg downward and normal force N upward, with $mg > N$ at speed [2].

Q6(b). At top of hill, net downward force provides centripetal force: $mg - N = mv^2/r$ [1]; at the point of losing contact, $N = 0$: $mg = mv^2/r$ [1]; $v = \sqrt{gr} = \sqrt{9.81 \times 120}$ [1] $= \mathbf{34.3}$ m s^{-1} [1].

Q6(c). $mg - N = mv^2/r$; $N = m(g - v^2/r) = 950(9.81 - 20^2/120) = 950(9.81 - 3.33)$ [1] $= 950 \times 6.48 = \mathbf{6160}$ N [1].

Q7(a). $r = L \sin\theta = 0.60 \sin 30ř = \mathbf{0.30}$ m [1].

Q7(b). $T\cos\theta = mg$; $T = mg/\cos 30ř = (0.25 \times 9.81)/\cos 30ř$ [1] $= \mathbf{2.83}$ N [1].

Q7(c). $T \sin\theta = mr\omega^2$ [1]; $\omega^2 = T\sin\theta/(mr) = 2.83 \times \sin 30$ř$/(0.25 \times 0.30)$ [1]; $\omega = \sqrt{2.83 \times 0.5/0.075} = \sqrt{18.9} = \mathbf{4.34}$ rad $\mathrm{s}^{-\mathbf{1}}$ [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define the radian and convert between degrees and radians	
☐ Define angular speed ω and use $\omega = 2\pi/T = 2\pi f$	
☐ Use $v = r\omega$ to relate linear and angular speed	
☐ Explain why an object in uniform circular motion is accelerating	
☐ State that centripetal acceleration is directed towards the centre	
☐ Use $a = r\omega^2$ and $a = v^2/r$	
☐ Use $F = mr\omega^2$ and $F = mv^2/r$	
☐ Identify the physical force providing centripetal force in a given situation	
☐ Explain why centripetal force does no work	
☐ Solve problems involving objects at the top/bottom of vertical circles	
☐ Solve conical pendulum problems by resolving tension components	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Circular motion is the bridge between mechanics and fields — once you are confident with centripetal force and acceleration, gravitational and electric field problems fall into place naturally. Always ask: what real force is pointing towards the centre?

CIE International A Level Physics

Topic 13

Gravitational Fields

Revision Booklet

This booklet covers:

- Newton's Law of Gravitation
- Gravitational Field Strength
- Gravitational Potential
- Orbital Motion & Kepler's Third Law
- Escape Velocity

Core Concepts and Definitions

Gravitational Field

A **gravitational field** is a region of space in which a mass experiences a force due to the gravitational attraction of another mass.

- Gravitational fields are always **attractive** — there is no gravitational repulsion.
- Any mass creates a gravitational field in the space around it.
- A **test mass** placed in the field will experience a force towards the source mass.

Radial vs Uniform Fields

- **Radial field** (e.g., around a planet): field lines point towards the centre; g decreases with distance.
- **Uniform field** (e.g., near Earth's surface over small distances): field lines are parallel and equally spaced; g is approximately constant at 9.81 N kg^{-1}.

Radial Field **Uniform Field**

Newton's Law of Gravitation

Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

F = gravitational force between the two masses (N)
G = gravitational constant = 6.67×10^{-11} N m^{2}kg^{-2}
m_1, m_2 = the two point masses (or uniform spheres) (kg)
r = separation between centres of mass (m)

Key Points

- The force is always **attractive**.
- It applies strictly to **point masses**, but also works for uniform spheres if r is measured from the centre.
- r is the **centre-to-centre** distance — not surface to surface.

- The force obeys an **inverse-square law**: double the distance, quarter the force.

Gravitational Field Strength

Definition of Gravitational Field Strength g

The **gravitational field strength** at a point is the gravitational force exerted per unit mass on a small test mass placed at that point.

$$g = \frac{F}{m} \qquad \text{units: N kg}^{-1} \equiv \text{m s}^{-2}$$

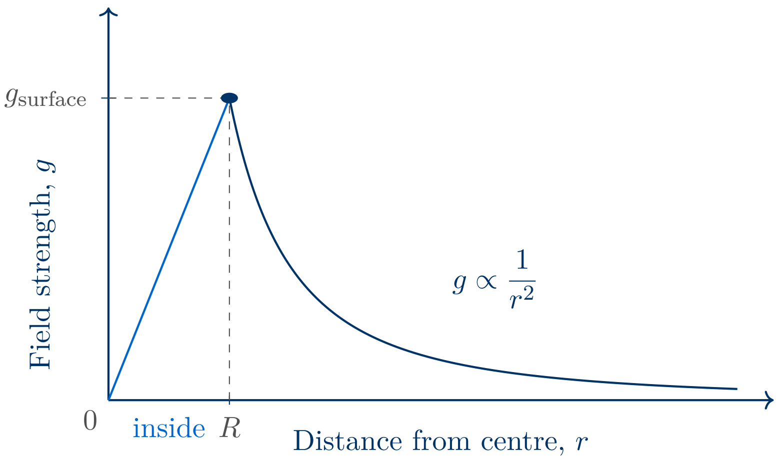
g is a **vector** quantity, directed towards the source mass.

Field Strength Formulae

$$g = \frac{GM}{r^2}$$

g = gravitational field strength at distance r (N kg^{-1})
G = gravitational constant (N m^2 kg^{-2})
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Variation of g with distance r



Common Mistake

Do not confuse g (field strength, N kg^{-1}) with G (the universal gravitational constant, N m^2 kg^{-2}). They are completely different quantities!

Gravitational Potential

Definition of Gravitational Potential ϕ

The **gravitational potential** at a point is the work done per unit mass to move a small test mass from infinity to that point.

$$\phi = \frac{W}{m} \qquad \text{units: J kg}^{-1}$$

Gravitational potential is always **negative** (zero at infinity; work is done *by* gravity as mass moves inwards, so the system loses potential energy).

Gravitational Potential Formulae

$$\phi = -\frac{GM}{r} \qquad E_p = m\phi = -\frac{GMm}{r}$$

ϕ = gravitational potential at distance r (J kg^{-1})
E_p = gravitational potential energy (J)
G = 6.67×10^{-11} N m^2 kg^{-2}
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Relationship Between g and ϕ

Field Strength from Potential Gradient

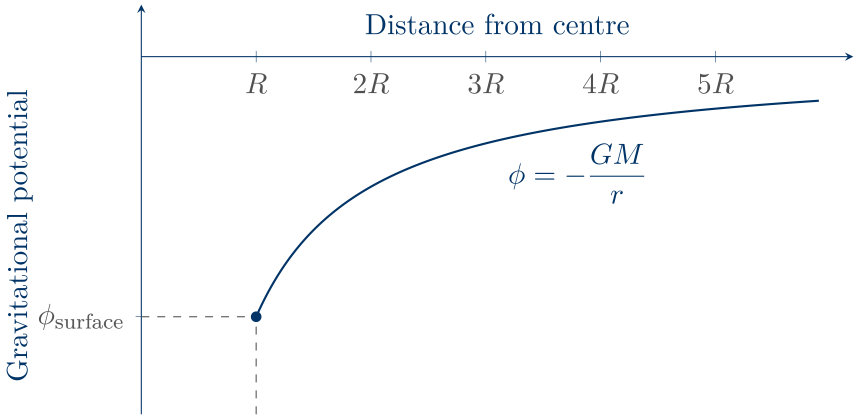
$$g = -\frac{\Delta\phi}{\Delta r}$$

Interpreting the Gradient

g is the **negative gradient** of the potential–distance graph. On a ϕ–r graph:

- A steeper gradient $\Rightarrow$ stronger field (larger g)
- The gradient becomes shallower as r increases $\Rightarrow$ g decreases
- The area under a g–r graph gives the change in potential $\Delta\phi$

Graph of ϕ against r



$\phi \to 0$ as $r \to \infty$

Equipotential Surfaces

An **equipotential surface** is a surface on which the gravitational potential is the same everywhere.

- No work is done moving a mass along an equipotential surface.
- Equipotentials are always **perpendicular** to field lines.
- For a point mass (or uniform sphere), equipotentials are concentric spheres.

Circular Orbits and Satellites

For an object of mass m in a circular orbit of radius r around a mass M, the gravitational force provides the centripetal force:

Orbital Mechanics

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\therefore \quad v = \sqrt{\frac{GM}{r}} \qquad \text{(orbital speed)}$$

$$T^2 = \frac{4\pi^2}{GM} r^3 \qquad \text{(Kepler's Third Law)}$$

Kepler's Third Law

$$T^2 \propto r^3$$

The square of the orbital period is proportional to the cube of the orbital radius. The constant of proportionality is $\frac{4\pi^2}{GM}$.

Energy in Circular Orbits

Energy of a Satellite in Circular Orbit

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r} \qquad \text{(kinetic energy — always positive)}$$
$$E_p = -\frac{GMm}{r} \qquad \text{(potential energy — always negative)}$$
$$E_{total} = E_k + E_p = -\frac{GMm}{2r} \qquad \text{(total energy — always negative)}$$

As r increases: v decreases, E_k decreases, E_p increases, E_{total} increases (becomes less negative).

Geostationary Orbits

Geostationary Satellite

A **geostationary** satellite has:

- An orbital period of exactly **24 hours**
- An orbit in the **equatorial plane**
- Movement in the **same direction** as Earth's rotation (west to east)
- An orbital radius of approximately 4.2×10^7 m ($\approx 36\,000$ km altitude)

It appears **stationary** relative to a point on Earth's surface.
Uses: satellite TV, telecommunications, weather monitoring.

Escape Velocity

Definition of Escape Velocity

The **escape velocity** is the minimum speed at which an object must be projected from the surface of a body so that it can escape to infinity against the gravitational field, without any further energy input.

Escape Velocity

Setting total energy = 0 at infinity:

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Note: $v_{esc} = \sqrt{2}\,v_{orbital}$ at the same radius.

Formula Summary Sheet

Formula	Quantity	Units
$F = \frac{Gm_1m_2}{r^2}$	Gravitational force	N
$g = \frac{F}{m}$	Field strength (definition)	N kg^{-1}
$g = \frac{GM}{r^2}$	Field strength (point mass)	N kg^{-1}
$\phi = -\frac{GM}{r}$	Gravitational potential	J kg^{-1}
$E_p = -\frac{GMm}{r}$	Gravitational PE	J
$g = -\frac{\Delta\phi}{\Delta r}$	Field from potential gradient	N kg^{-1}
$v = \sqrt{\frac{GM}{r}}$	Orbital speed	m s^{-1}
$T^2 = \frac{4\pi^2 r^3}{GM}$	Kepler's Third Law	s^2, m^3
$v_{esc} = \sqrt{\frac{2GM}{R}}$	Escape velocity	m s^{-1}

Constants: $G = 6.67 \times 10^{-11}$ N m^{2}kg^{-2}, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Calculation Questions

1. **Identify** what you are asked to find.
2. **List** the quantities given; convert units if needed (e.g. days → seconds, km → m).
3. **Select** the appropriate formula.
4. **Substitute** values carefully, showing all working.
5. **Check** units and significant figures in your answer.

Common Errors — Avoid These!

- Using **diameter** instead of radius in $g = GM/r^2$.
- Forgetting the **negative sign** in $\phi = -GM/r$.
- Not converting **hours/days to seconds** before applying Kepler's Law.
- Confusing g (field strength, vector) with ϕ (potential, scalar).
- Saying $\phi = 0$ at the surface — it is zero only **at infinity**.

- Thinking higher orbit ⇒ faster orbital speed — it is **slower**.

Worked Examples

Example 1 — Field Strength at Altitude

Question: Calculate the gravitational field strength at a height of 400 km above Earth's surface. ($M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m)

Solution

Solution:
$r = R_E + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6$ m

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.77 \times 10^6)^2}$$

$$g = \frac{3.98 \times 10^{14}}{4.58 \times 10^{13}} = \mathbf{8.69} \text{ N kg}^{-1}$$

Note: this is the field strength at the ISS orbit — astronauts are *not* weightless because gravity is zero!

Example 2 — Orbital Period Using Kepler's Third Law

Question: A satellite orbits Earth at a radius of 4.2×10^7 m. Calculate its orbital period.

Solution

Solution:

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 \times (4.2 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}$$

$$T^2 = \frac{4\pi^2 \times 7.41 \times 10^{22}}{3.98 \times 10^{14}} = 7.36 \times 10^9 \text{ s}^2$$

$$T = \sqrt{7.36 \times 10^9} = 8.58 \times 10^4 \text{ s} \approx \mathbf{23.8} \text{ hours}$$

This is a geostationary orbit ($T \approx 24$ h).

Example 3 — Change in Gravitational Potential Energy

Question: A spacecraft of mass 2500 kg is moved from the Earth's surface to an altitude of 800 km. Calculate the increase in gravitational potential energy.

Solution

Solution:
$r_1 = 6.37 \times 10^6$ m, $\quad r_2 = 6.37 \times 10^6 + 8.00 \times 10^5 = 7.17 \times 10^6$ m

$$\Delta E_p = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta E_p = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 2500 \times \left(\frac{1}{6.37 \times 10^6} - \frac{1}{7.17 \times 10^6}\right)$$

$$\Delta E_p = 9.95 \times 10^{17} \times 1.75 \times 10^{-8} = \mathbf{1.74 \times 10^{10}}\ \text{J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Newton's Law of Gravitation.

[2 marks]

Q2. Define gravitational field strength and state its units.

[2 marks]

Q3. Explain why gravitational potential is always a negative quantity.

[2 marks]

Q4. The gravitational field strength at Earth's surface is 9.81 N kg^{-1}. Calculate the field strength at a distance of $3R_E$ from Earth's centre, where R_E is Earth's radius.

[2 marks]

Q5. Two planets, X and Y, orbit the same star. Planet X has an orbital radius twice that of planet Y. Determine the ratio T_X/T_Y.

[3 marks]

Section B — Longer Structured Questions

Q6. A satellite of mass m orbits a planet of mass M in a circular orbit of radius r.

(a) Show that the orbital speed of the satellite is given by $v = \sqrt{GM/r}$.

[2 marks]

(b) Show that the period of the orbit satisfies $T^2 = \dfrac{4\pi^2 r^3}{GM}$.

[2 marks]

(c) The satellite is moved to a lower orbit. Explain what happens to its:

- speed
- kinetic energy
- total energy

[3 marks]

Q7. The Moon orbits the Earth with a period of 27.3 days at a mean orbital radius of 3.84×10^8 m.

(a) Use this data to calculate the mass of the Earth.

[3 marks]

(b) Calculate the gravitational potential at the Moon's orbital radius.

[2 marks]

(c) A 75 kg astronaut travels from Earth's surface to the Moon's orbital radius. Calculate the change in gravitational potential energy.

[3 marks]

Mark Scheme and Answers

Q1. Any two masses exert an attractive force on each other [1]; the force is proportional to the product of their masses and inversely proportional to the square of their separation [1].

Q2. Gravitational field strength is the gravitational force per unit mass acting on a (small test) mass placed at that point [1]; units: N kg^{-1} (or m s^{-2}) [1].

Q3. Gravitational potential is defined as zero at infinity [1]; work is done by gravity as mass moves from infinity inward, so the potential decreases below zero — it is always negative [1].

Q4. $g \propto 1/r^2$; at $3R_E$, $g = 9.81/3^2 = 9.81/9 = \mathbf{1.09}$ N kg^{-1} [2].

Q5. By Kepler's Third Law: $T^2 \propto r^3$, so $\left(\frac{T_X}{T_Y}\right)^2 = \left(\frac{2r_Y}{r_Y}\right)^3 = 8$ [2]; $\frac{T_X}{T_Y} = \sqrt{8} = \mathbf{2\sqrt{2} \approx 2.83}$ [1].

Q6(a). Gravitational force = centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]; cancel m, rearrange: $v = \sqrt{GM/r}$ [1].

Q6(b). Substitute $v = 2\pi r/T$ into result from (a) [1]; rearrange to get $T^2 = 4\pi^2 r^3/GM$ [1].

Q6(c). Speed **increases** [1]; KE **increases** [1]; total energy **decreases** (becomes more negative) [1].

Q7(a). $T = 27.3 \times 24 \times 3600 = 2.36 \times 10^6$ s; $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84 \times 10^8)^3}{6.67 \times 10^{-11}(2.36 \times 10^6)^2}$ [1] $= \mathbf{6.02 \times 10^{24}}$ kg [2].

Q7(b). $\phi = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24}}{3.84 \times 10^8} = \mathbf{-1.05 \times 10^6}$ J kg^{-1} [2].

Q7(c). $\Delta E_p = m\Delta\phi = m(\phi_{\text{Moon orbit}} - \phi_{\text{surface}})$ [1]; $\phi_{\text{surface}} = -GM/R_E = -6.25 \times 10^7$ J kg^{-1}; $\Delta\phi = -1.05 \times 10^6 - (-6.25 \times 10^7) = 6.14 \times 10^7$ J kg^{-1}; $\Delta E_p = 75 \times 6.14 \times 10^7 = \mathbf{4.6 \times 10^9}$ J [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	State and apply Newton's Law of Gravitation	
☐	Define gravitational field strength; use $g = F/m$ and $g = GM/r^2$	
☐	Sketch field line and equipotential diagrams	
☐	Define gravitational potential and explain why it is negative	
☐	Use $\phi = -GM/r$ and $E_p = -GMm/r$	
☐	Apply the relationship $g = -\Delta\phi/\Delta r$	
☐	Derive the expression for orbital speed	
☐	State and apply Kepler's Third Law ($T^2 \propto r^3$)	
☐	Analyse energy changes in satellite orbits	
☐	Describe the properties of a geostationary satellite	
☐	Derive and apply the formula for escape velocity	
☐	Interpret graphs of g vs r and ϕ vs r	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: understanding *why* formulas work is more powerful than memorising them. Practice drawing diagrams and deriving key results from first principles.

CIE International A Level Physics

Topic 14, 15 & 16

Thermal Physics

Revision Booklet

This booklet covers:

- Temperature and Thermal Equilibrium
- Temperature Scales
- Specific Heat Capacity and Latent Heat
- The Mole and Ideal Gases
- Kinetic Theory of Gases
- Internal Energy
- The First Law of Thermodynamics

Thermal Equilibrium

Thermal Energy Transfer

Thermal (heat) energy is transferred from a region of **higher temperature** to a region of **lower temperature**. Transfer continues until the two regions reach the same temperature.

Thermal Equilibrium

Two objects are in **thermal equilibrium** when there is no net transfer of thermal energy between them. This occurs when they are at the **same temperature**.

Zeroth Law of Thermodynamics: If object A is in thermal equilibrium with object C, and object B is also in thermal equilibrium with object C, then A and B are in thermal equilibrium with each other.

Temperature Scales

Thermometric Property

A **thermometric property** is a physical property that varies continuously and measurably with temperature. Examples include:

- Resistance of a metal wire (e.g. platinum resistance thermometer)
- e.m.f. of a thermocouple
- Volume of a gas at constant pressure
- Density of a liquid

Thermodynamic Temperature Scale

The **thermodynamic (Kelvin) temperature scale** does not depend on the property of any particular substance. It is an absolute scale with:

- **Absolute zero** (0 K) — the lowest possible temperature
- **Triple point of water** (273.16 K) — fixed reference point

Temperature Conversion

$$T/\text{K} = \theta/^\circ\text{C} + 273.15$$

T = thermodynamic temperature (K)
θ = Celsius temperature (°C)

Note: a temperature *difference* of 1 K equals a difference of 1 °C.

Absolute Zero

At absolute zero (0 K = −273.15 °C):

- Molecules have minimum possible kinetic energy
- It is impossible to reach in practice, only to approach asymptotically
- All ideal gases would have zero volume (zero pressure at constant volume)

Specific Heat Capacity and Latent Heat

Specific Heat Capacity

The **specific heat capacity** c of a substance is the energy required to raise the temperature of **unit mass** by **one kelvin** (or one degree Celsius), without change of state.

$$c = \frac{Q}{m\,\Delta T} \qquad \text{units: J kg}^{-1}\text{K}^{-1}$$

Specific Heat Capacity

$$Q = mc\Delta T$$

Q = thermal energy transferred (J)
m = mass (kg)
c = specific heat capacity ($\text{J kg}^{-1}\text{ K}^{-1}$)
ΔT = temperature change (K or °C)

Specific Latent Heat

The **specific latent heat** L of a substance is the energy required to change the state of **unit mass** at constant temperature.

$$L = \frac{Q}{m} \qquad \text{units: J kg}^{-1}$$

- **Specific latent heat of fusion** L_f: solid → liquid (melting)
- **Specific latent heat of vaporisation** L_v: liquid → gas (boiling)

Note: $L_v > L_f$ for any given substance, since greater work is done separating molecules completely during vaporisation.

Specific Latent Heat

$$Q = mL$$

Q = thermal energy transferred (J)
m = mass (kg)
L = specific latent heat (J kg^{-1})

Common Mistake

During a change of state, temperature remains **constant** even though energy is being supplied. The energy goes into breaking intermolecular bonds, not increasing kinetic energy.

The Mole and Ideal Gases

Amount of Substance

The **mole** (mol) is the SI base unit for amount of substance. One mole of any substance contains exactly $N_A = 6.02 \times 10^{23}$ particles (the Avogadro constant).

Equation of State for an Ideal Gas

$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

p = pressure (Pa)
V = volume (m^3)
n = number of moles (mol)
R = molar gas constant = $8.31\ \mathrm{J\,mol^{-1}K^{-1}}$
T = thermodynamic temperature (K)
N = number of molecules
k = Boltzmann constant = $1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$

The Boltzmann constant: $k = R/N_A$

Ideal Gas

An **ideal gas** is one that obeys $pV \propto T$ (where T is thermodynamic temperature) under all conditions. Real gases approximate ideal behaviour at low pressures and high temperatures.

Gas Laws — special cases of $pV = nRT$

- **Boyle's Law** (constant T): pV = constant, so $p_1V_1 = p_2V_2$
- **Charles' Law** (constant p): V/T = constant, so $V_1/T_1 = V_2/T_2$
- **Pressure Law** (constant V): p/T = constant, so $p_1/T_1 = p_2/T_2$

Common Mistake

Always use **thermodynamic temperature in kelvin** in gas law calculations — never degrees Celsius.

Kinetic Theory of Gases

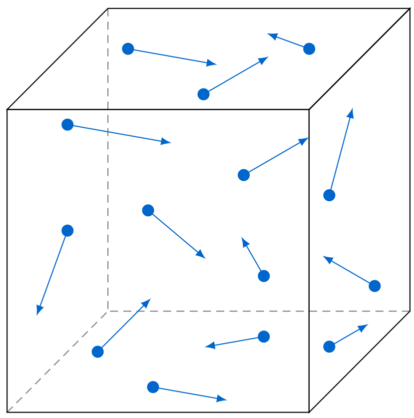
Assumptions of the Kinetic Model

Basic Assumptions of Kinetic Theory

1. The gas contains a **large number** of molecules moving in **random directions** with a range of speeds.
2. The molecules occupy **negligible volume** compared to the volume of the gas.
3. All collisions are **perfectly elastic** (no kinetic energy lost).
4. **Intermolecular forces are negligible** except during collisions.
5. The duration of collisions is **negligible** compared to the time between collisions.
6. Molecules obey **Newton's laws of motion**.

Pressure from Kinetic Theory

The pressure exerted by a gas arises from molecules colliding with the container walls. Each collision exerts a force; the average of many such collisions gives a steady pressure.



Kinetic Theory Equation

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

p = pressure (Pa)
V = volume (m^3)
N = total number of molecules
m = mass of one molecule (kg)
$\langle c^2\rangle$ = mean square speed ($m^2\,s^{-2}$)

The **root-mean-square speed**: $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$

Note: $c_{\text{r.m.s.}}$ is not the same as the mean speed $\bar{c}$. Because we square first then average, faster molecules contribute more. In general $c_{\text{r.m.s.}} > \bar{c}$.

Derivation of $pV = \frac{1}{3}Nm\langle c^2\rangle$

Consider one molecule of mass m moving with speed c_x in the x-direction inside a box of side L.

Step 1 — change in momentum at one wall: The molecule hits the right wall and bounces back elastically:

$$\Delta p = mc_x - (-mc_x) = 2mc_x$$

Step 2 — time between collisions with the same wall: The molecule must travel $2L$ before returning:

$$\Delta t = \frac{2L}{c_x}$$

Step 3 — force from one molecule:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mc_x}{2L/c_x} = \frac{mc_x^2}{L}$$

Step 4 — pressure from N molecules: Summing over all N molecules and dividing by area L^2:

$$p = \frac{Nm\langle c_x^2\rangle}{L^3} = \frac{Nm\langle c_x^2\rangle}{V}$$

Step 5 — extend to 3 dimensions: By symmetry of random motion: $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle = \frac{1}{3}\langle c^2\rangle$

$$\boxed{pV = \frac{1}{3}Nm\langle c^2\rangle}$$

Molecular Kinetic Energy

Derivation of $\langle E_k\rangle = \frac{3}{2}kT$

Start with the two equations for an ideal gas:

$$pV = \frac{1}{3}Nm\langle c^2\rangle \qquad \text{and} \qquad pV = NkT$$

Equating the right-hand sides:

$$\frac{1}{3}Nm\langle c^2\rangle = NkT$$

Divide both sides by N and multiply by $\frac{3}{2}$:

$$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

Since $\langle E_k\rangle = \frac{1}{2}m\langle c^2\rangle$:

$$\boxed{\langle E_k\rangle = \frac{3}{2}kT}$$

Average Translational Kinetic Energy

$$\langle E_k\rangle = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

The average kinetic energy of a molecule is **directly proportional to thermodynamic temperature**.

Key Deductions

- At absolute zero, molecules have minimum kinetic energy
- Molecules of different gases at the same temperature have the **same average KE**
- Heavier molecules move more **slowly** on average at the same temperature: $c_{\text{r.m.s.}} = \sqrt{3kT/m}$
- $c_{\text{r.m.s.}} \propto \sqrt{T}$ — doubling thermodynamic temperature increases r.m.s. speed by factor $\sqrt{2}$

Internal Energy

Internal Energy

The **internal energy** of a system is the sum of the **random kinetic energies** and **potential energies** of all the molecules in the system.

$$U = E_{k,\text{total}} + E_{p,\text{total}}$$

- Internal energy is a **function of state**.
- For an **ideal gas**: $E_p = 0$, so $U = E_{k,\text{total}}$ only.
- A rise in temperature $\Rightarrow$ increase in average KE $\Rightarrow$ increase in U.
- During a change of state: temperature (and KE) constant; E_p increases as bonds break; U increases.

The First Law of Thermodynamics

Work Done by/on a Gas

When a gas changes volume at **constant pressure**:

$$W = p\Delta V$$

W = work done **by** the gas (J)
p = pressure (Pa)
ΔV = change in volume (m^3)

First Law of Thermodynamics

$$\Delta U = q + W$$

ΔU = increase in internal energy of the system (J)
q = energy supplied **to** the system by heating (J)
W = work done **on** the system (J)

Sign Convention

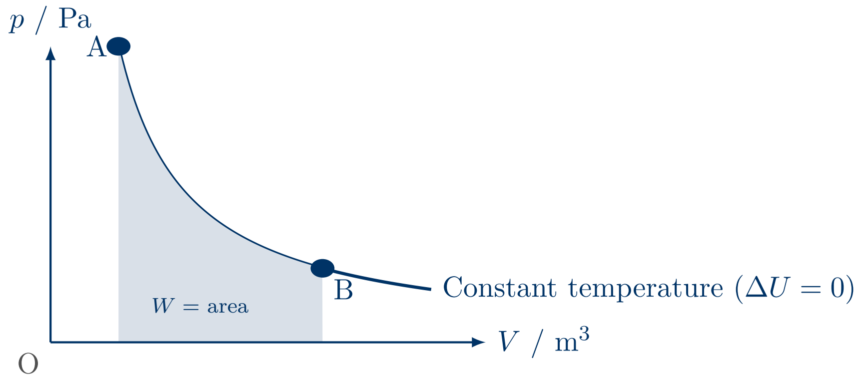
CIE uses $\Delta U = q + W$ where W is work done **on** the system. Some textbooks use $\Delta U = q - W$ where W is work done **by** the system. Always check which convention is being used.

Special Cases of the First Law

Process	Condition	Result
Constant temperature	$\Delta T = 0 \Rightarrow \Delta U = 0$	$q = -W$
No heat transfer	$q = 0$	$\Delta U = W$
Constant volume	$\Delta V = 0 \Rightarrow W = 0$	$\Delta U = q$
Constant pressure	p constant	$\Delta U = q - p\Delta V$

p–V Diagrams

The **area under a p–V curve** equals the work done by the gas during that process.



Formula Summary

Formula	Quantity	Units
$T/\text{K} = \theta/°\text{C} + 273.15$	Temperature conversion	K
$Q = mc\Delta T$	Sensible heat	J
$Q = mL$	Latent heat	J
$pV = nRT$	Ideal gas (moles)	Pa, m^3, K
$pV = NkT$	Ideal gas (molecules)	Pa, m^3, K
$k = R/N_A$	Boltzmann constant	J K^{-1}
$pV = \frac{1}{3}Nm\langle c^2\rangle$	Kinetic theory	Pa
$\langle E_k\rangle = \frac{3}{2}kT$	Mean molecular KE	J
$W = p\Delta V$	Work done by gas	J
$\Delta U = q + W$	First law	J

Constants: $R = 8.31\ \text{J mol}^{-1}\text{K}^{-1}$ $k = 1.38 \times 10^{-23}\ \text{J K}^{-1}$ $N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$

Worked Examples

Example 1 — Specific Heat Capacity

Question: A 2.0 kg block of aluminium ($c = 900\ \text{J kg}^{-1}\text{K}^{-1}$) is heated from 20 °C to 80 °C. Calculate the energy supplied.

Solution

$$Q = mc\Delta T = 2.0 \times 900 \times (80 - 20) = \mathbf{1.08 \times 10^5}\ \text{J}$$

Example 2 — Ideal Gas Law

Question: A fixed mass of ideal gas has pressure 1.2×10^5 Pa and volume 3.0×10^{-3} m^3 at 27 °C. It is heated at constant pressure until its volume doubles. Find the final temperature.

Solution

$T_1 = 27 + 273.15 = 300.15$ K. At constant pressure, $V \propto T$:

$$T_2 = T_1 \times \frac{V_2}{V_1} = 300.15 \times 2 = \mathbf{600}\ \text{K}\ (\mathbf{327}\ °\text{C})$$

Example 3 — r.m.s. Speed

Question: Calculate the r.m.s. speed of nitrogen molecules ($M = 0.028\ \text{kg mol}^{-1}$) at 300 K.

Solution

Mass of one molecule: $m = 0.028/6.02 \times 10^{23} = 4.65 \times 10^{-26}$ kg

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.65 \times 10^{-26}}} = \sqrt{2.67 \times 10^5} = \mathbf{517}\ \text{m s}^{-1}$$

Example 4 — First Law

Question: A gas is compressed. 650 J of work is done on the gas and 200 J of heat flows out. Find the change in internal energy.

Solution

$W = +650$ J (work done *on* gas), $\quad q = -200$ J (heat leaves)

$$\Delta U = q + W = -200 + 650 = +\mathbf{450}\ \text{J}$$

Internal energy **increases** by 450 J.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define *specific latent heat of vaporisation* and explain why it is greater than the specific latent heat of fusion for the same substance.

[3 marks]

Q2. State the basic assumptions of the kinetic theory of gases.

[4 marks]

Q3. Explain what is meant by *internal energy* and state how it differs for an ideal gas compared to a real gas.

[3 marks]

Q4. A gas at pressure 2.0×10^5 Pa and temperature 300 K occupies a volume of 0.40 m^3. Calculate the number of molecules present.

[2 marks]

Q5. Show that the mean kinetic energy of a gas molecule is $\frac{3}{2}kT$ by combining the kinetic theory equation with the ideal gas equation.

[3 marks]

Section B — Longer Structured Questions

Q6. A sample of water of mass 0.50 kg is heated from 20 °C to 100 °C and then completely vaporised.
($c_{\text{water}} = 4200\ \text{J kg}^{-1}\text{K}^{-1}$, $L_v = 2.26 \times 10^6\ \text{J kg}^{-1}$)

(a) Calculate the energy needed to heat the water from 20 °C to 100 °C.

[2 marks]

(b) Calculate the energy needed to vaporise the water at 100 °C.

[2 marks]

(c) Explain in terms of molecular behaviour why energy is needed to vaporise water even though the temperature does not change.

[2 marks]

Q7. A fixed mass of ideal gas undergoes the cycle A → B → C → A on a p–V diagram where A → B is constant temperature expansion, B → C is constant volume pressure decrease, and C → A is constant pressure compression.

(a) State the change in internal energy during A → B and justify your answer.
[2 marks]

(b) Using the first law, find q for A → B given that the work done by the gas is 440 J.
[2 marks]

(c) Describe the energy changes during the constant volume process B → C.
[2 marks]

Mark Scheme and Answers

Q1. Specific latent heat of vaporisation is the energy per unit mass required to change a substance from liquid to gas at constant temperature [1]; greater than L_f because molecules must be completely separated, requiring work against intermolecular forces over a much greater distance than in melting [2].

Q2. Large number of molecules moving randomly in all directions with a range of speeds [1]; occupying negligible volume compared to container [1]; collisions perfectly elastic [1]; intermolecular forces negligible except during collisions; collisions of negligible duration; obey Newton's laws [1].

Q3. Internal energy is the sum of random kinetic and potential energies of all molecules [1]; for an ideal gas, $E_p = 0$ so internal energy is kinetic energy only [1]; for a real gas, molecules have potential energy due to intermolecular forces [1].

Q4. $N = pV/kT = (2.0 \times 10^5 \times 0.40)/(1.38 \times 10^{-23} \times 300) = \mathbf{1.93 \times 10^{25}}$ molecules [2].

Q5. $pV = \frac{1}{3}Nm\langle c^2\rangle$ [1]; equate with $pV = NkT$: $\frac{1}{3}m\langle c^2\rangle = kT$ [1]; so $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ [1].

Q6(a). $Q = 0.50 \times 4200 \times 80 = \mathbf{1.68 \times 10^5}$ J [2].

Q6(b). $Q = 0.50 \times 2.26 \times 10^6 = \mathbf{1.13 \times 10^6}$ J [2].

Q6(c). KE of molecules unchanged (temperature constant) [1]; energy breaks intermolecular bonds as molecules separate completely, increasing potential energy [1].

Q7(a). $\Delta U = 0$ [1]; temperature is constant and for an ideal gas internal energy depends only on temperature [1].

Q7(b). $\Delta U = 0$, work done on gas $= -440$ J; $q = +440$ J — heat flows into gas [2].

Q7(c). No work done ($\Delta V = 0$) [1]; temperature and KE decrease; heat flows out; internal energy decreases [1].

Revision Checklist

Learning Objective	Confidence (1–3)
☐ Explain thermal equilibrium and direction of heat flow	
☐ Convert between Celsius and Kelvin	
☐ State examples of thermometric properties	
☐ Define and use specific heat capacity: $Q = mc\Delta T$	
☐ Define and use specific latent heat: $Q = mL$	
☐ State the assumptions of kinetic theory	
☐ Use $pV = nRT$ and $pV = NkT$	
☐ Derive and use $pV = \frac{1}{3}Nm\langle c^2\rangle$	
☐ Show that mean KE of a molecule is $\frac{3}{2}kT$	
☐ Define internal energy and relate to temperature	
☐ Apply the first law $\Delta U = q + W$	
☐ Interpret and use p–V diagrams	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: understanding *why* formulas work is more powerful than memorising them.
Practice drawing p–V diagrams and deriving key results from first principles.

CIE International A Level Physics

Topic 17

Oscillations

Revision Booklet

This booklet covers:

- Simple Harmonic Motion
- Displacement, Velocity & Acceleration
- Energy in SHM
- Damped Oscillations
- Forced Oscillations & Resonance

Core Concepts and Definitions

Oscillatory Motion — Key Terms

- **Displacement** x: the distance from the equilibrium position, with direction (m).
- **Amplitude** x_0: the maximum displacement from equilibrium (m).
- **Period** T: the time for one complete oscillation (s).
- **Frequency** f: the number of complete oscillations per second (Hz); $f = 1/T$.
- **Angular frequency** ω: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}).
- **Phase difference** ϕ: the fraction of a cycle by which one oscillation leads or lags another, expressed in radians.

Simple Harmonic Motion (SHM)

An oscillation is **simple harmonic** if the acceleration of the object is:

- **proportional** to its displacement from a fixed equilibrium point, and
- always directed **towards** that equilibrium point (opposite to displacement).

$$a \propto -x$$

Equations of Simple Harmonic Motion

Defining Equation of SHM

$$a = -\omega^2 x$$

a = acceleration (m s^{-2})
ω = angular frequency (rad s^{-1})
x = displacement from equilibrium (m)

The negative sign confirms acceleration is always directed **opposite** to displacement.

Displacement; Velocity and Acceleration

$$x = x_0 \sin \omega t \qquad \text{(starting from equilibrium)}$$

$$v = v_0 \cos \omega t \qquad \text{where } v_0 = \omega x_0$$

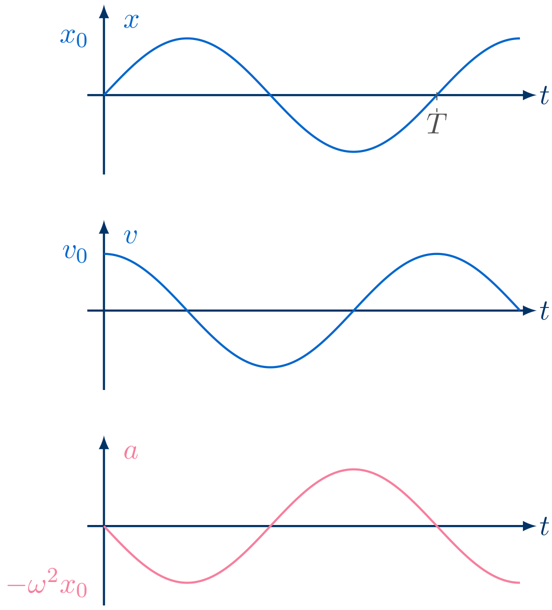
$$v = \pm\,\omega\sqrt{x_0^2 - x^2}$$

x_0 = amplitude (m)
v_0 = maximum speed = ωx_0 (m s^{-1})
t = time (s)

Maxima and Minima

- Acceleration is **maximum** at maximum displacement ($x = \pm x_0$): $|a_{max}| = \omega^2 x_0$
- Speed is **maximum** at the equilibrium position ($x = 0$): $v_{max} = \omega x_0$
- Speed is **zero** at the turning points ($x = \pm x_0$).

Graphs of x, v and a against time



Phase Relationships

- v leads x by $\frac{\pi}{2}$ rad (quarter period ahead).
- a leads v by $\frac{\pi}{2}$ rad, so a is **antiphase** (π rad ahead) with x.

Common Mistake

The equation $x = x_0 \sin \omega t$ assumes the object starts at the **equilibrium position** at $t = 0$. If it starts at maximum displacement, use $x = x_0 \cos \omega t$ instead. Always check the initial conditions.

Energy in Simple Harmonic Motion

Total Energy of a SHM System

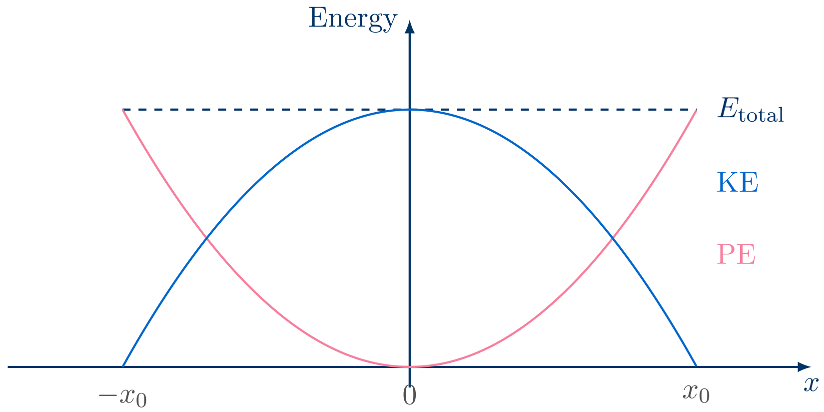
$$E = \frac{1}{2}m\omega^2 x_0^2$$

E = total mechanical energy (J)
m = mass of the oscillating object (kg)
ω = angular frequency (rad s^{-1})
x_0 = amplitude (m)

Energy Interchange During SHM

- At the **equilibrium position** ($x = 0$): KE is maximum, PE is zero.
- At the **turning points** ($x = \pm x_0$): KE is zero, PE is maximum.
- The **total energy remains constant** throughout (assuming no damping).
- KE $= \frac{1}{2}m\omega^2(x_0^2 - x^2)$ PE $= \frac{1}{2}m\omega^2 x^2$

Energy–displacement graph



Damped and Forced Oscillations

Damping

Damping

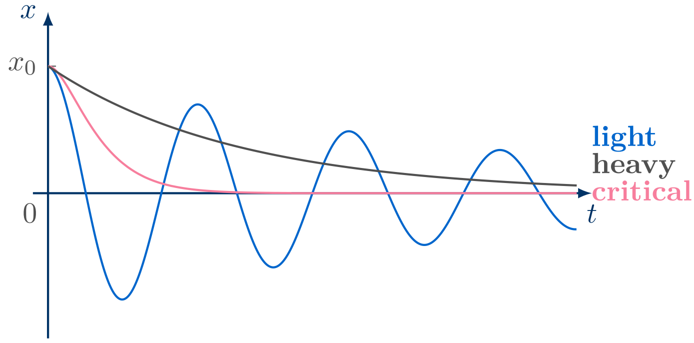
Damping occurs when a resistive force (e.g., air resistance, viscosity) acts on an oscillating system. Energy is removed from the system, causing the amplitude to **decrease over time**. The frequency is approximately unchanged for light damping.

Types of Damping

- **Light damping**: amplitude decreases gradually over many oscillations; the system oscillates for a long time before coming to rest.

- **Critical damping**: the system returns to equilibrium in the **shortest possible time** without oscillating. Used in car suspension and door closers.
- **Heavy (overdamping)**: the system returns to equilibrium **slowly** without oscillating; slower return than critical damping.

Displacement–time graphs for the three types of damping



Forced Oscillations and Resonance

Forced Oscillations and Natural Frequency

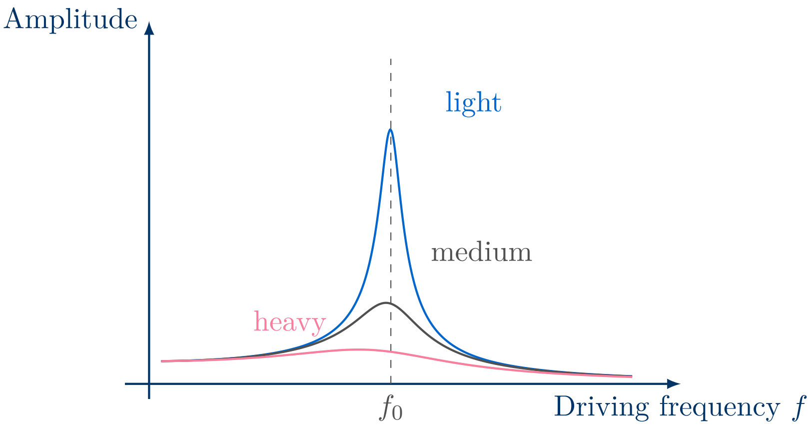
- Every oscillating system has a **natural frequency** f_0 at which it oscillates freely when displaced and released.
- A **forced oscillation** occurs when a periodic **driving force** is applied to the system at a **driving frequency** f.
- The system oscillates at the **driving frequency**, not necessarily at its natural frequency.

Resonance

Resonance occurs when the driving frequency equals the natural frequency of the system ($f = f_0$). At resonance:

- The amplitude of oscillation is a **maximum**.
- Energy transfer from the driver to the system is most efficient.
- The degree of damping determines how sharp the resonance peak is and the maximum amplitude reached.

Amplitude–frequency graph (resonance curves)



Effect of Damping on Resonance

- More damping ⇒ lower and broader resonance peak.
- More damping ⇒ peak shifts **below** f_0 (towards lower frequencies).
- Less damping ⇒ sharper, taller peak with maximum amplitude closer to f_0.
- With no damping the peak would be infinite at exactly f_0.

Resonance in Real Life

Resonance can be **useful** (e.g. MRI scanners, musical instruments, microwave ovens) or **destructive** (e.g. bridges vibrating in wind, buildings in earthquakes). Engineers use damping to control unwanted resonance.

Formula Summary Sheet

Formula	Quantity	Units
$a = -\omega^2 x$	SHM defining equation	m s^{-2}
$\omega = 2\pi f = \dfrac{2\pi}{T}$	Angular frequency	rad s^{-1}
$x = x_0 \sin \omega t$	Displacement (from equilibrium at $t = 0$)	m
$v = v_0 \cos \omega t$	Velocity	m s^{-1}
$v = \pm\omega\sqrt{x_0^2 - x^2}$	Speed at displacement x	m s^{-1}
$v_0 = \omega x_0$	Maximum speed	m s^{-1}
$a_{max} = \omega^2 x_0$	Maximum acceleration	m s^{-2}
$E = \frac{1}{2}m\omega^2 x_0^2$	Total energy of SHM system	J
$\text{KE} = \frac{1}{2}m\omega^2(x_0^2 - x^2)$	Kinetic energy at displacement x	J
$\text{PE} = \frac{1}{2}m\omega^2 x^2$	Potential energy at displacement x	J

Key relationships: $T = 1/f$, $\omega = 2\pi f$, $v_{max} = \omega x_0$, $a_{max} = \omega^2 x_0$
Phase: v leads x by $\pi/2$; a is antiphase with x (leads by π)

Worked Examples

Example 1 — Finding Angular Frequency and Max Speed

Question: A mass oscillates with SHM, amplitude 4.0 cm and period 0.80 s. Calculate (a) the angular frequency and (b) the maximum speed.

Solution

(a) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = \mathbf{7.85}$ rad s^{-1}

(b) $v_{max} = \omega x_0 = 7.85 \times 0.040 = \mathbf{0.31}$ m s^{-1}

Example 2 — Speed at a Given Displacement

Question: The same mass ($\omega = 7.85$ rad s^{-1}, $x_0 = 4.0$ cm) has displacement $x = 2.5$ cm. Find its speed.

Solution

$$v = \omega\sqrt{x_0^2 - x^2} = 7.85 \times \sqrt{(0.040)^2 - (0.025)^2}$$

$$v = 7.85 \times \sqrt{1.60 \times 10^{-3} - 6.25 \times 10^{-4}} = 7.85 \times \sqrt{9.75 \times 10^{-4}}$$

$$v = 7.85 \times 0.0312 = \mathbf{0.245}\ \text{m s}^{-1}$$

Example 3 — Total Energy of SHM System

Question: A 0.20 kg mass oscillates with $\omega = 7.85$ rad s^{-1} and amplitude 4.0 cm. Calculate the total energy.

Solution

$$E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.20 \times (7.85)^2 \times (0.040)^2$$

$$E = 0.10 \times 61.6 \times 1.60 \times 10^{-3} = \mathbf{9.9 \times 10^{-3}}\ \text{J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the two conditions that define simple harmonic motion.

[2 marks]

Q2. Define the terms *amplitude*, *period* and *angular frequency* for an oscillating system.

[3 marks]

Q3. An object undergoes SHM with frequency 2.5 Hz and amplitude 6.0 cm.

(a) Calculate the angular frequency. *[1 mark]*

(b) Calculate the maximum acceleration. *[2 marks]*

(c) Calculate the speed when $x = 4.0$ cm. *[2 marks]*

Q4. Sketch displacement–time graphs for *light*, *critical* and *heavy* damping on the same axes, starting from the same initial displacement. Label each curve.

[3 marks]

Q5. Explain what is meant by *resonance* and state the condition under which it occurs.

[2 marks]

Section B — Longer Structured Questions

Q6. A 0.15 kg mass is attached to a spring and oscillates vertically with SHM. Its displacement is given by $x = 0.050\sin(12t)$, where x is in metres and t in seconds.

(a) Write down the amplitude and angular frequency of the motion.

[2 marks]

(b) Show that the maximum speed is 0.60 m s^{-1}.

[2 marks]

(c) Calculate the total energy of the oscillation.

[2 marks]

(d) Sketch graphs on the same axes showing how the kinetic energy and potential energy vary with displacement x. Label your axes clearly.

[3 marks]

Q7. A child on a swing is pushed periodically by a parent.

(a) Explain why the amplitude of the swing increases when the pushing frequency equals the natural frequency of the swing.

[2 marks]

(b) The parent now pushes at a frequency higher than the natural frequency. Describe and explain what happens to the amplitude.

[2 marks]

(c) Air resistance acts on the swing. Describe how this affects the resonance curve compared to an undamped system.

[2 marks]

Mark Scheme and Answers

Q1. Acceleration is proportional to displacement from a fixed (equilibrium) point [1]; acceleration is always directed towards that point / opposite to displacement [1].

Q2. Amplitude: maximum displacement from equilibrium [1]. Period: time for one complete oscillation [1]. Angular frequency: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}) [1].

Q3(a). $\omega = 2\pi f = 2\pi \times 2.5 = \mathbf{15.7}$ rad s$^{\mathbf{-1}}$ [1].

Q3(b). $a_{\max} = \omega^2 x_0 = (15.7)^2 \times 0.060$ [1] $= \mathbf{14.8}$ m s$^{\mathbf{-2}}$ [1].

Q3(c). $v = \omega\sqrt{x_0^2 - x^2} = 15.7 \times \sqrt{(0.060)^2 - (0.040)^2}$ [1] $= 15.7 \times 0.0447 = \mathbf{0.70}$ m s$^{\mathbf{-1}}$ [1].

Q4. All three start from same x_0 [1]; light damping: decaying sinusoid, multiple oscillations; critical: returns to zero smoothly in shortest time without oscillating; heavy: slower return to zero, no oscillation — all three correctly labelled [2].

Q5. Resonance: when the driving frequency equals the natural frequency of the system [1]; the amplitude of oscillation reaches a maximum [1].

Q6(a). Amplitude $x_0 = 0.050$ m [1]; angular frequency $\omega = 12$ rad s^{-1} [1].

Q6(b). $v_{\text{max}} = \omega x_0 = 12 \times 0.050 = 0.60$ m s^{-1} [2] (must show substitution).

Q6(c). $E = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} \times 0.15 \times 144 \times 2.5 \times 10^{-3}$ [1] $= \mathbf{2.7 \times 10^{-2}}$ J [1].

Q6(d). PE: upward parabola, zero at $x = 0$, maximum E at $x = \pm x_0$ [1]; KE: inverted parabola, maximum E at $x = 0$, zero at $x = \pm x_0$ [1]; axes correctly labelled with E, $-x_0$, 0, x_0 [1].

Q7(a). Energy is transferred most efficiently from driver to system when $f_{\text{drive}} = f_0$ [1]; each push is in phase with the motion so energy is added each cycle, increasing amplitude [1].

Q7(b). Amplitude decreases [1]; driving frequency is not matched to natural frequency so energy transfer is less efficient / pushes are out of phase with motion [1].

Q7(c). The resonance peak is lower (smaller maximum amplitude) [1]; the peak is broader and shifts slightly to a frequency below f_0 [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define displacement, amplitude, period, frequency and angular frequency	
☐ State and apply the two conditions for SHM	
☐ Use $a = -\omega^2 x$ to identify and analyse SHM	
☐ Use $x = x_0 \sin \omega t$ and $v = v_0 \cos \omega t$	
☐ Use $v = \pm\omega\sqrt{x_0^2 - x^2}$ to find speed at any displacement	
☐ Sketch and interpret graphs of x, v and a against t	
☐ Describe the phase relationships between x, v and a	
☐ Describe the interchange between KE and PE during SHM	
☐ Use $E = \frac{1}{2}m\omega^2 x_0^2$ for total energy	
☐ Sketch and interpret KE and PE against displacement graphs	
☐ Explain and distinguish light, critical and heavy damping	
☐ Sketch displacement–time graphs for the three types of damping	
☐ Explain resonance and the condition for it to occur	
☐ Describe the effect of damping on the resonance curve	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Remember: in SHM, acceleration and displacement are always linked by $a = -\omega^2 x$. If you can start from this equation and derive everything else, you truly understand the topic.

CIE International A Level Physics

Topic 18

Electric Fields

Revision Booklet

This booklet covers:

- Electric Fields and Field Lines
- Uniform Electric Fields
- Coulomb's Law
- Electric Field of a Point Charge
- Electric Potential

Core Concepts and Definitions

Electric Field

An **electric field** is a region of space in which a charged object experiences a force.

- Electric field is defined as the **force per unit positive charge** acting on a small stationary test charge placed at that point.
- It is a **vector** quantity, directed along the force on a positive test charge.
- Units: $\mathrm{N\ C^{-1} \equiv V\ m^{-1}}$.

$$E = \frac{F}{q}$$

Radial vs Uniform Fields

- **Radial field** (around a point charge or sphere): field lines point radially inward (negative charge) or outward (positive charge); E decreases with distance.
- **Uniform field** (between parallel plates): field lines are parallel and equally spaced; E is constant throughout.

Radial Field (+) **Uniform Field**

Uniform Electric Fields

Field Strength Between Parallel Plates

$$E = \frac{\Delta V}{\Delta d}$$

E = electric field strength ($\mathrm{V\ m^{-1}}$)
ΔV = potential difference between the plates (V)
Δd = separation of the plates (m)

Motion of Charged Particles in a Uniform Field

- A charge q in a uniform field E experiences a constant force $F = qE$.
- The motion is analogous to projectile motion in a gravitational field:
  - Along the field: **uniform acceleration** $a = qE/m$.

  - Perpendicular to the field: **constant velocity** (if no other forces).
- The path of the charge is **parabolic**.

Common Mistake

$E = \Delta V/\Delta d$ applies **only** to uniform fields (parallel plates). Do not apply it to the field around a point charge — use $E = Q/(4\pi\varepsilon_0 r^2)$ instead.

Coulomb's Law

Coulomb's Law

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

F = electrostatic force between the charges (N)
Q_1, Q_2 = the two point charges (C)
r = separation between the charges (m)
ε_0 = permittivity of free space = 8.85×10^{-12} F m^{-1}

Key Points

- Like charges **repel**; unlike charges **attract**.
- The force obeys an **inverse-square law**: double the distance, quarter the force.
- Applies strictly to **point charges**, and to uniform spheres (treat as point charge at centre).
- Compare with gravity: same inverse-square form but gravity is **always attractive**.

Comparison: Coulomb's Law vs Newton's Law of Gravitation

Coulomb's Law	Newton's Law
$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	$F = \frac{G m_1 m_2}{r^2}$
Can be attractive or repulsive	Always attractive
Acts between charges	Acts between masses
Much stronger force	Much weaker force

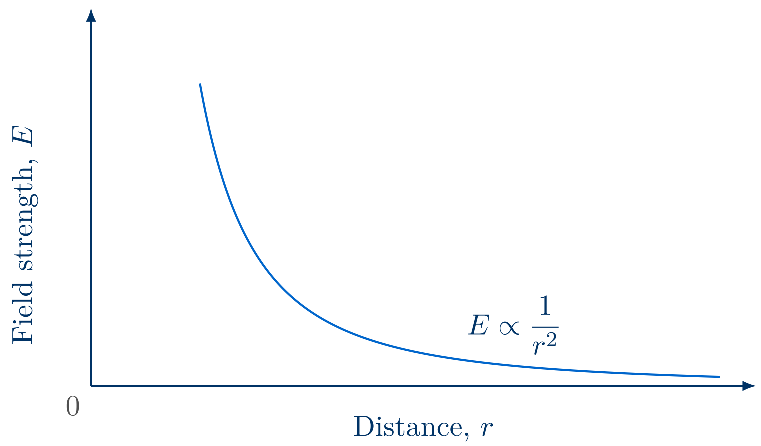
Electric Field of a Point Charge

Electric Field Strength due to a Point Charge

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

E = electric field strength at distance r (N C^{-1})
Q = point charge creating the field (C)
r = distance from the centre of Q (m)
ε_0 = 8.85×10^{-12} F m^{-1}

Variation of E with distance r



Comparing E and g fields

- Both follow an inverse-square law with distance.
- $E = Q/(4\pi\varepsilon_0 r^2)$ parallels $g = GM/r^2$.
- $F = qE$ parallels $F = mg$.
- Unlike gravitational fields, electric fields can point inward or outward depending on the sign of Q.

Electric Potential

Definition of Electric Potential V

The **electric potential** at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point.

$$V = \frac{W}{q} \qquad \text{units: J C}^{-1} \equiv \text{V}$$

- $V = 0$ at infinity (by convention).
- Around a positive charge: $V > 0$ (work must be done *against* repulsion).

- Around a negative charge: $V < 0$ (work is done *by* the field as the test charge moves in).

Electric Potential due to a Point Charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

V = electric potential at distance r (V)
Q = point charge (C)
r = distance from the charge (m)

Relationship Between E and V

Field Strength from Potential Gradient

$$E = -\frac{\Delta V}{\Delta r}$$

The electric field strength equals the **negative gradient** of the potential–distance graph. The area under an E–r graph gives the change in potential ΔV.

Electric Potential Energy

Potential Energy of Two Point Charges

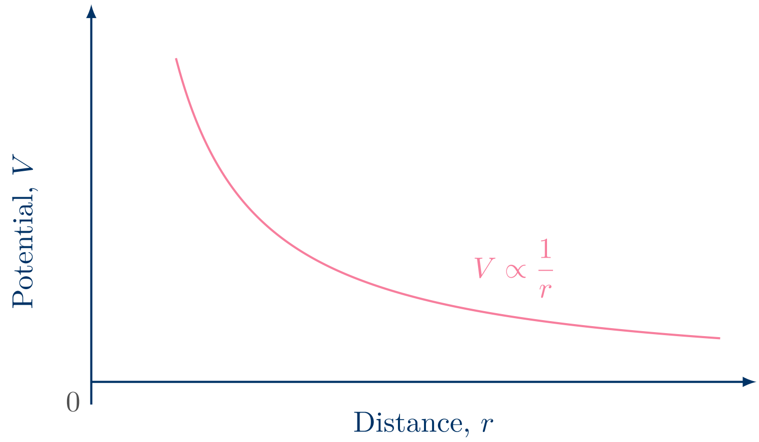
$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}$$

E_P = electric potential energy (J)
Q, q = the two point charges (C)
r = separation (m)
Note: $E_P = qV$, where V is the potential due to charge Q at the location of q.

Signs of Potential Energy

- Like charges ($Qq > 0$): $E_P > 0$ — energy must be supplied to bring them together.
- Unlike charges ($Qq < 0$): $E_P < 0$ — energy is released as they come together.
- $E_P = 0$ at infinite separation.

Graphs of V and E against r for a positive point charge



Common Mistake

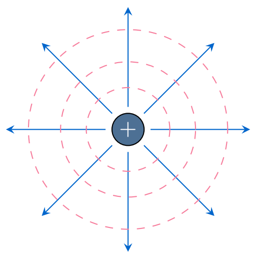
Note the difference in distance dependence: $E \propto 1/r^2$ but $V \propto 1/r$. Students often mix these up. Remember: potential falls off more *slowly* than field strength with distance.

Equipotential Surfaces

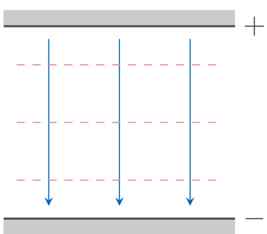
Equipotentials

An **equipotential surface** is a surface on which the electric potential is the same at every point.

- No work is done moving a charge *along* an equipotential.
- Equipotentials are always **perpendicular** to field lines.
- Around a point charge: equipotentials are concentric spheres.
- Between parallel plates: equipotentials are parallel planes equally spaced (for uniform field).



Point charge



Parallel plates

Formula Summary Sheet

Formula	Quantity	Units
$E = \dfrac{F}{q}$	Electric field strength (definition)	N C^{-1}
$F = qE$	Force on a charge in a field	N
$E = \dfrac{\Delta V}{\Delta d}$	Field between parallel plates (uniform)	V m^{-1}
$F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	Coulomb's Law	N
$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$	Field due to a point charge	N C^{-1}
$V = \dfrac{Q}{4\pi\varepsilon_0 r}$	Potential due to a point charge	V
$E = -\dfrac{\Delta V}{\Delta r}$	Field from potential gradient	V m^{-1}
$E_P = \dfrac{Qq}{4\pi\varepsilon_0 r}$	Electric potential energy	J
$E_P = qV$	Potential energy of charge in field	J

Constants: $\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}, $\dfrac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}$ N m^{2}C^{-2}, $e = 1.60\times10^{-19}$ C

Note: $E \propto 1/r^2$ (inverse-square) but $V \propto 1/r$ (inverse).

Worked Examples

Example 1 — Force Between Two Charges

Question: Calculate the electrostatic force between two charges of $+3.0\ \mu$C and $-5.0\ \mu$C separated by 0.12 m.

Solution

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{3.0\times10^{-6}\times5.0\times10^{-6}}{4\pi\times8.85\times10^{-12}\times(0.12)^2}$$

$$F = \frac{1.5\times10^{-11}}{1.61\times10^{-12}} = \mathbf{9.3}\ \text{N}$$

The force is **attractive** (unlike charges).

Example 2 — Field Strength and Potential at a Distance

Question: A point charge $Q = +4.0\ \mu$C. Calculate (a) the electric field strength and (b) the electric potential at a distance of 0.30 m.

Solution

(a) $E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.30)^2}$

$$E = \frac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} \times (0.09)^{-1}$$

$$E = \frac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} = \mathbf{4.0 \times 10^5} \text{ N C}^{-1}$$

(b) $V = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.30} = \mathbf{1.2 \times 10^5} \text{ V}$

Example 3 — Uniform Field Between Plates

Question: Two parallel plates are separated by 4.0 mm with a potential difference of 240 V. Calculate (a) the field strength and (b) the force on an electron between the plates.

Solution

(a) $E = \dfrac{\Delta V}{\Delta d} = \dfrac{240}{4.0 \times 10^{-3}} = \mathbf{6.0 \times 10^4} \text{ V m}^{-1}$

(b) $F = qE = 1.6 \times 10^{-19} \times 6.0 \times 10^4 = \mathbf{9.6 \times 10^{-15}} \text{ N}$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electric field strength and state its SI units.

[2 marks]

Q2. State Coulomb's Law and identify one similarity and one difference compared with Newton's Law of Gravitation.

[3 marks]

Q3. Define electric potential at a point. Explain why the potential around an isolated positive charge is positive, but around an isolated negative charge is negative.

[3 marks]

Q4. Two parallel plates are 6.0 mm apart. The electric field strength between them is 5.0×10^4 V m^{-1}. Calculate the potential difference between the plates.

[2 marks]

Q5. A point charge produces an electric field of 3.6×10^5 N C^{-1} at a distance of 0.10 m. Calculate the magnitude of the charge.

[3 marks]

Section B — Longer Structured Questions

Q6. Two point charges $Q_1 = +6.0$ μC and $Q_2 = +6.0$ μC are placed 0.20 m apart.

(a) Calculate the force between the two charges.

[2 marks]

(b) Calculate the electric field strength at the midpoint between the two charges. Explain your reasoning.

[3 marks]

(c) Calculate the electric potential at the midpoint between the two charges.

[3 marks]

Q7. An electron (mass 9.11×10^{-31} kg, charge -1.6×10^{-19} C) enters horizontally between two parallel plates. The plates are 30 mm apart and have a potential difference of 150 V. The electron enters midway between the plates with horizontal speed 4.0×10^{7} m s^{-1}.

(a) Calculate the electric field strength between the plates.

[1 mark]

(b) Calculate the vertical acceleration of the electron.

[2 marks]

(c) The plates are 60 mm long. Calculate the vertical deflection of the electron as it exits the plates.

[3 marks]

(d) State and explain whether the electron hits one of the plates before exiting.

[2 marks]

Mark Scheme and Answers

Q1. Electric field strength is the force per unit positive charge acting on a small stationary test charge placed at that point [1]; units: N C^{-1} or V m^{-1} [1].

Q2. The force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation [1]. Similarity: both follow an inverse-square law [1]. Difference: gravity is always attractive; electrostatic force can be attractive or repulsive [1].

Q3. Electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point [1]. Around a positive charge: the test charge is repelled, so work must be done *on* it to bring it in from infinity — the potential is positive [1]. Around a negative charge: the test charge is attracted, so work is done *by* the field — the potential is negative [1].

Q4. $\Delta V = E \times \Delta d = 5.0 \times 10^4 \times 6.0 \times 10^{-3} = \mathbf{300}$ V [2].

Q5. $E = Q/(4\pi\varepsilon_0 r^2)$ [1]; $Q = E \times 4\pi\varepsilon_0 r^2 = 3.6 \times 10^5 \times 4\pi \times 8.85 \times 10^{-12} \times (0.10)^2$ [1] $= \mathbf{4.0 \times 10^{-7}}$ C [1].

Q6(a). $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \dfrac{(6.0 \times 10^{-6})^2}{4\pi \times 8.85 \times 10^{-12} \times (0.20)^2}$ [1] $= \mathbf{8.1}$ N [1].

Q6(b). By symmetry, the two equal charges produce equal and opposite field contributions at the midpoint [1]; the fields cancel, so $E = \mathbf{0}$ at the midpoint [2].

Q6(c). $V_1 = V_2 = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{6.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.10}$ [1] $= 5.4 \times 10^5$ V; total $V = V_1 + V_2 = \mathbf{1.08 \times 10^6}$ V [2].

Q7(a). $E = \Delta V/\Delta d = 150/(30 \times 10^{-3}) = \mathbf{5.0 \times 10^3}$ V $\mathbf{m^{-1}}$ [1].

Q7(b). $F = qE = 1.6 \times 10^{-19} \times 5.0 \times 10^3 = 8.0 \times 10^{-16}$ N [1]; $a = F/m = 8.0 \times 10^{-16}/(9.11 \times 10^{-31}) = \mathbf{8.8 \times 10^{14}}$ m $\mathbf{s^{-2}}$ [1].

Q7(c). Time in field: $t = L/v = 0.060/(4.0 \times 10^7) = 1.5 \times 10^{-9}$ s [1]; vertical deflection: $y = \frac{1}{2}at^2 = \frac{1}{2} \times 8.8 \times 10^{14} \times (1.5 \times 10^{-9})^2$ [1] $= \mathbf{9.9 \times 10^{-4}}$ m $\approx \mathbf{1.0}$ mm [1].

Q7(d). The electron enters midway so has 15 mm to the nearest plate [1]; deflection of ≈ 1 mm $\ll 15$ mm, so the electron **does not** hit a plate [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define electric field strength as force per unit positive charge; use $E = F/q$	
☐	Represent electric fields using field lines for point charges and parallel plates	
☐	Use $E = \Delta V/\Delta d$ for a uniform field between parallel plates	
☐	Describe the parabolic motion of a charge in a uniform electric field	
☐	State and apply Coulomb's Law $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$	
☐	Use $E = Q/(4\pi\varepsilon_0 r^2)$ for the field due to a point charge	
☐	Define electric potential; explain why sign depends on sign of source charge	
☐	Use $V = Q/(4\pi\varepsilon_0 r)$ for potential due to a point charge	
☐	Apply $E = -\Delta V/\Delta r$ to relate field strength and potential gradient	
☐	Use $E_P = Qq/(4\pi\varepsilon_0 r)$ for potential energy of two charges	
☐	Sketch and interpret equipotential diagrams for point charges and parallel plates	
☐	Compare electric and gravitational fields (similarities and differences)	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

The key to electric fields is seeing the deep parallel with gravitational fields: same inverse-square laws, same potential formalism — but with charge replacing mass, and the crucial difference that charge can be positive or negative.

CIE International A Level Physics

Topic 19

Capacitance

Revision Booklet

This booklet covers:

- Capacitors and Capacitance
- Capacitors in Series and Parallel
- Energy Stored in a Capacitor
- Capacitor Discharge
- The Time Constant

Core Concepts and Definitions

Capacitance

The **capacitance** of a conductor is the charge stored per unit potential difference across it.

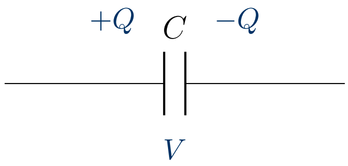
$$C = \frac{Q}{V} \qquad \text{units: F (farads)} \equiv \text{C V}^{-1}$$

- 1 F is a very large unit; practical capacitors are typically μF, nF or pF.
- Capacitance depends on the **geometry** of the conductor and the medium between the plates, not on Q or V individually.
- For an **isolated spherical conductor** of radius R: $C = 4\pi\varepsilon_0 R$.

The Parallel Plate Capacitor

Two conducting plates of area A separated by distance d:

- Charging the capacitor stores charge $+Q$ on one plate and $-Q$ on the other.
- The electric field between the plates is uniform: $E = V/d$.
- Capacitance increases with larger plate area A and smaller separation d.



Capacitors in Series and Parallel

Capacitors in Parallel

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$$

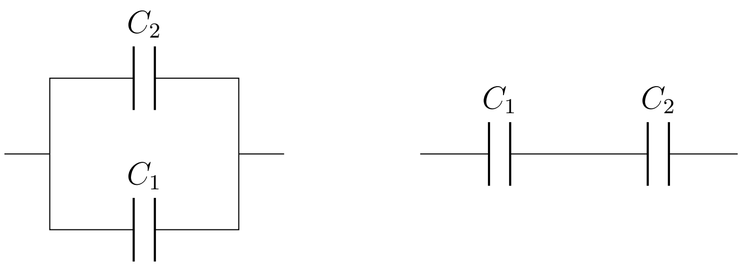
Each capacitor has the **same voltage**; charges add: $Q_{\text{total}} = Q_1 + Q_2 + \cdots$

Capacitors in Series

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Each capacitor has the **same charge**; voltages add: $V_{\text{total}} = V_1 + V_2 + \cdots$

Circuit diagrams



Parallel: $C_T = C_1 + C_2$ **Series:** $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

Derivation of Series Formula

For capacitors in series, each carries the same charge Q. The total voltage is:

$$V_{\text{total}} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Since $C_{\text{total}} = Q/V_{\text{total}}$, dividing through by Q gives $\dfrac{1}{C_{\text{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Common Mistake

Capacitors in series combine like **resistors in parallel** (reciprocal rule), and capacitors in parallel combine like **resistors in series** (direct sum). This is the opposite to resistors — don't mix them up.

Energy Stored in a Capacitor

Energy Stored

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

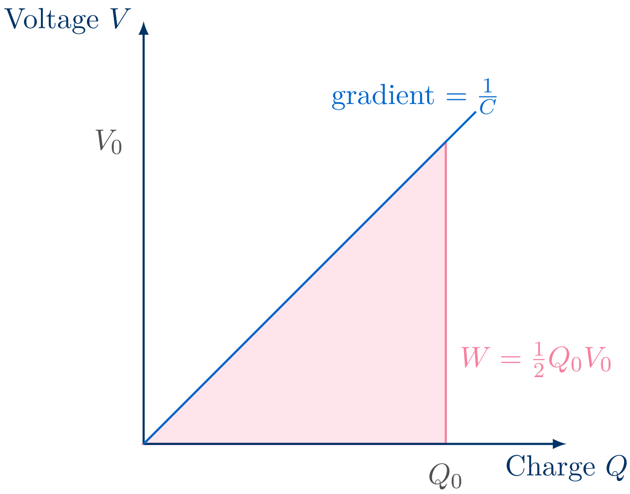
W = energy stored (J)
Q = charge stored (C)
V = potential difference across capacitor (V)
C = capacitance (F)

Energy from the V–Q Graph

The energy stored equals the **area under the V–Q graph** (a straight line through the origin with gradient $1/C$):

$$W = \text{area of triangle} = \tfrac{1}{2} \times Q \times V = \tfrac{1}{2}QV$$

V–Q graph for a capacitor



Discharging a Capacitor

Capacitor Discharge Through a Resistor

When a charged capacitor discharges through a resistor R, the charge, voltage and current all decay **exponentially** with time:

$$Q = Q_0\, e^{-t/RC} \qquad V = V_0\, e^{-t/RC} \qquad I = I_0\, e^{-t/RC}$$

where $I_0 = V_0/R = Q_0/RC$ is the initial current.

Time Constant

$$\tau = RC$$

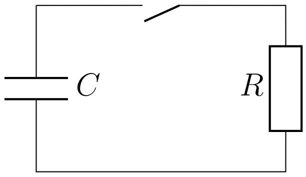
τ = time constant (s)
R = resistance of discharge path (Ω)
C = capacitance (F)

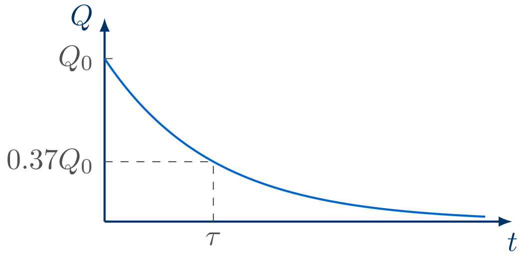
After one time constant ($t = \tau$), the charge/voltage/current has fallen to $e^{-1} \approx$ **37%** of its initial value.

Key Values During Discharge

- $t = \tau$: $Q = 0.37\, Q_0$ (37%)
- $t = 2\tau$: $Q = 0.135\, Q_0$ (13.5%)
- $t = 5\tau$: $Q \approx 0.007\, Q_0$ — capacitor considered fully discharged.
- A larger τ means slower discharge; a smaller τ means faster discharge.

Discharge circuit and Q–t graph





Analysing Discharge Graphs

- A **linear** $\ln Q$ vs t graph confirms exponential decay; gradient $= -1/RC$.
- The time constant τ can be read directly as the time for Q to fall to $0.37\,Q_0$.
- Doubling R or C doubles τ and halves the rate of discharge.

Linearising the discharge: $\ln Q$ against t

Taking logarithms of $Q = Q_0\,e^{-t/RC}$:

$$\ln Q = \ln Q_0 - \frac{1}{RC}\,t$$

This is of the form $y = mx + c$, so a graph of $\ln Q$ against t gives:

- gradient $= -\dfrac{1}{RC} = -\dfrac{1}{\tau}$
- y-intercept $= \ln Q_0$

Formula Summary Sheet

Formula	Quantity	Units
$C = Q/V$	Capacitance (definition)	F
$C_T = C_1 + C_2 + \cdots$	Capacitors in parallel	F
$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$	Capacitors in series	F
$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$	Energy stored	J
$\tau = RC$	Time constant	s
$Q = Q_0\, e^{-t/RC}$	Charge during discharge	C
$V = V_0\, e^{-t/RC}$	Voltage during discharge	V
$I = I_0\, e^{-t/RC}$	Current during discharge	A
$\ln Q = \ln Q_0 - \frac{t}{RC}$	Linearised discharge	—

Constants and values: $e^{-1} \approx 0.368$; after 1τ: 37% remains; after 5τ: $<$ 1% remains

Units check: $[\tau] = [\Omega][\text{F}] = [\text{V A}^{-1}][\text{C V}^{-1}] = [\text{C A}^{-1}] = \text{s}$

Worked Examples

Example 1 — Capacitors in Series and Parallel

Question: Three capacitors of 2.0 μF, 3.0 μF and 6.0 μF are connected (a) in parallel and (b) in series. Find the total capacitance in each case.

Solution

(a) Parallel:

$$C_T = 2.0 + 3.0 + 6.0 = \mathbf{11.0}\ \mu\text{F}$$

(b) Series:

$$\frac{1}{C_T} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3+2+1}{6.0} = \frac{6}{6.0} = 1.0\ \mu\text{F}^{-1}$$

$$C_T = \mathbf{1.0}\ \mu\text{F}$$

Example 2 — Energy Stored

Question: A 470 μF capacitor is charged to 12 V. Calculate (a) the charge stored and (b) the energy stored.

Solution

(a) $Q = CV = 470 \times 10^{-6} \times 12 = \mathbf{5.64 \times 10^{-3}}$ C
(b) $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 470 \times 10^{-6} \times 12^2 = \mathbf{3.38 \times 10^{-2}}$ J

Example 3 — Capacitor Discharge

Question: A 220 μF capacitor charged to 9.0 V discharges through a 47 kΩ resistor. Calculate (a) the time constant and (b) the voltage after 5.0 s.

Solution

(a) $\tau = RC = 47 \times 10^3 \times 220 \times 10^{-6} = \mathbf{10.3}$ s
(b) $V = V_0\, e^{-t/RC} = 9.0 \times e^{-5.0/10.3} = 9.0 \times e^{-0.485} = 9.0 \times 0.616 = \mathbf{5.5}$ V

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define capacitance and state its SI unit.

[2 marks]

Q2. A capacitor stores 360 μC of charge when connected to a 12 V supply. Calculate its capacitance.

[2 marks]

Q3. Two capacitors of 4.0 μF and 12 μF are connected in series across a 6.0 V supply.

(a) Calculate the total capacitance. *[2 marks]*

(b) Calculate the total charge stored. *[1 mark]*

(c) Calculate the voltage across the 4.0 μF capacitor. *[2 marks]*

Q4. Explain, with reference to the V–Q graph, why the energy stored in a capacitor is $W = \frac{1}{2}QV$ and not $W = QV$.

[2 marks]

Q5. Define the time constant for a capacitor–resistor discharge circuit and state what fraction of the initial charge remains after two time constants.

[3 marks]

Section B — Longer Structured Questions

Q6. A 100 μF capacitor is charged to 20 V and then discharged through a 25 kΩ resistor.

(a) Calculate the initial charge stored on the capacitor.

[1 mark]

(b) Calculate the initial discharge current.

[2 marks]

(c) Calculate the time constant for the discharge.

[1 mark]

(d) Calculate the charge remaining after 4.0 s.

[2 marks]

(e) The student plots a graph of $\ln(Q/\mathrm{C})$ against t/s. State the gradient and y-intercept of this graph.

[2 marks]

Q7. A 50 μF and a 200 μF capacitor are connected in series and charged from a 15 V supply.

(a) Calculate the combined capacitance.

[2 marks]

(b) Calculate the energy stored in the combination.

[2 marks]

(c) The two capacitors are now reconnected in parallel across the same supply. Calculate the new total energy stored and explain why it differs from part (b).

[3 marks]

Mark Scheme and Answers

Q1. Capacitance is the charge stored per unit potential difference [1]; unit: farad (F) or C V^{-1} [1].

Q2. $C = Q/V = 360 \times 10^{-6}/12 = \mathbf{30}$ μF [2].

Q3(a). $\frac{1}{C_T} = \frac{1}{4.0} + \frac{1}{12} = \frac{3+1}{12} = \frac{4}{12}$ [1]; $C_T = \mathbf{3.0}$ μF [1].

Q3(b). $Q = C_T V = 3.0 \times 10^{-6} \times 6.0 = \mathbf{1.8 \times 10^{-5}}$ C [1].

Q3(c). Same charge on each capacitor in series; $V_1 = Q/C_1 = 1.8 \times 10^{-5}/(4.0 \times 10^{-6}) =$ **4.5** V [2].

Q4. The V–Q graph is a straight line through the origin [1]; the energy is the area under this graph (a triangle), which is $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}QV$ — not QV because the voltage builds up gradually from 0 to V as charge is stored [1].

Q5. The time constant is the product RC [1]; it is the time taken for the charge (or voltage or current) to fall to $1/e \approx 37\%$ of its initial value [1]; after 2τ: $e^{-2} \approx \mathbf{13.5}\%$ remains [1].

Q6(a). $Q_0 = CV_0 = 100 \times 10^{-6} \times 20 = \mathbf{2.0 \times 10^{-3}}$ C [1].

Q6(b). $I_0 = V_0/R = 20/(25 \times 10^3)$ [1] $= \mathbf{8.0 \times 10^{-4}}$ A [1].

Q6(c). $\tau = RC = 25 \times 10^3 \times 100 \times 10^{-6} = \mathbf{2.5}$ s [1].

Q6(d). $Q = Q_0\, e^{-t/RC} = 2.0 \times 10^{-3} \times e^{-4.0/2.5}$ [1] $= 2.0 \times 10^{-3} \times e^{-1.6} = 2.0 \times 10^{-3} \times 0.202 = \mathbf{4.0 \times 10^{-4}}$ C [1].

Q6(e). Gradient $= -1/RC = -1/2.5 = \mathbf{-0.40}$ $\mathbf{s^{-1}}$ [1]; y-intercept $= \ln Q_0 = \ln(2.0 \times 10^{-3}) = \mathbf{-6.2}$ [1].

Q7(a). $\dfrac{1}{C_T} = \dfrac{1}{50} + \dfrac{1}{200} = \dfrac{4+1}{200} = \dfrac{5}{200}$ [1]; $C_T = \mathbf{40}$ μF [1].

Q7(b). $W = \frac{1}{2}C_T V^2 = \frac{1}{2} \times 40 \times 10^{-6} \times 15^2$ [1] $= \mathbf{4.5 \times 10^{-3}}$ J [1].

Q7(c). $C_T = 50 + 200 = 250$ μF; $W = \frac{1}{2} \times 250 \times 10^{-6} \times 15^2$ [1] $= \mathbf{2.8 \times 10^{-2}}$ J [1]; more energy is stored in parallel because the total capacitance is greater — more charge is drawn from the supply [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define capacitance; use $C = Q/V$	
☐	Calculate combined capacitance for series and parallel combinations	
☐	Derive the series and parallel formulae from $C = Q/V$	
☐	Use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$ for energy stored	
☐	Explain why energy stored is the area under a V–Q graph	
☐	Describe the exponential decay of Q, V and I during discharge	
☐	Use $x = x_0\, e^{-t/RC}$ for discharge of charge, voltage or current	
☐	Define the time constant $\tau = RC$ and state its physical significance	
☐	Determine τ from a discharge graph (directly or via $\ln Q$ vs t)	
☐	Sketch discharge curves for Q, V and I against time	
☐	Linearise the discharge equation and interpret gradient and intercept	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

The exponential decay of a capacitor is one of the most important mathematical forms in physics. Once you can recognise it, linearise it, and extract τ from a graph, you have a powerful tool that reappears throughout the course.

CIE International A Level Physics

Topic 20

Magnetic Fields

Revision Booklet

This booklet covers:

- Magnetic Fields and Field Lines
- Force on a Current-Carrying Conductor
- Force on a Moving Charge
- Hall Effect and Velocity Selector
- Magnetic Fields due to Currents
- Electromagnetic Induction

Magnetic Fields

Magnetic Field

A **magnetic field** is a region of space in which a moving charge, or a current-carrying conductor, experiences a force.

- Magnetic fields are produced by **moving charges** (electric currents) or by **permanent magnets**.
- The field is represented by **field lines** (flux lines); the direction is the direction of the force on a north pole.
- Field lines never cross; closer lines indicate a stronger field.
- Crosses ($\times$) represent field into the page; dots ($\bullet$) represent field out of the page.

Force on a Current-Carrying Conductor

Force on a Conductor

$$F = BIL\sin\theta$$

F = force on the conductor (N)
B = magnetic flux density (T)
I = current in the conductor (A)
L = length of conductor in the field (m)
θ = angle between the conductor and the field direction

Force is **maximum** when $\theta = 90^\circ$ (conductor perpendicular to field): $F = BIL$.
Force is **zero** when $\theta = 0^\circ$ (conductor parallel to field).

Magnetic Flux Density B

Magnetic flux density is defined as the force per unit current per unit length acting on a wire placed **at right angles** to the field.

$$B = \frac{F}{IL} \qquad \text{units: T (tesla)} \equiv \text{N A}^{-1}\text{m}^{-1}$$

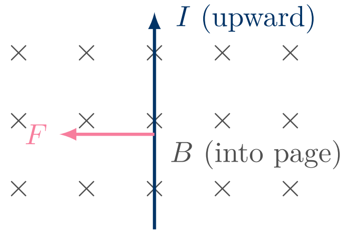
B is a vector quantity. 1 T is a strong field; Earth's field is $\approx 50\ \mu$T.

Fleming's Left-Hand Rule

For a **conventional current** in a magnetic field:

- **Thumb**: direction of force (motion) F
- **First finger**: direction of magnetic field B
- **Second finger**: direction of conventional current I

Remember: the rule gives the force on *positive* charges / conventional current. For electrons, reverse the direction.



Force on a Moving Charge

Force on a Moving Charge

$$F = BQv\sin\theta$$

F = magnetic force (N)
B = magnetic flux density (T)
Q = charge (C)
v = speed of the charge (m s^{-1})
θ = angle between velocity and field

For $\theta = 90°$: $F = BQv$, directed perpendicular to both v and B.

Circular Motion in a Magnetic Field

When a charged particle moves **perpendicular** to a uniform magnetic field, the magnetic force is always perpendicular to the velocity, so no work is done and the **speed is constant**. The particle moves in a **circle**:

$$BQv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{BQ}$$

A larger momentum or smaller B gives a larger radius.

The Hall Effect

Hall Voltage

When a current-carrying conductor is placed in a magnetic field perpendicular to the current, charge carriers experience a sideways force. They accumulate on one face until the electric force balances the magnetic force, producing the **Hall voltage**:

$$V_H = \frac{BI}{ntq}$$

V_H = Hall voltage (V)
B = magnetic flux density (T)
I = current through the conductor (A)
n = number density of charge carriers (m^{-3})
t = thickness of the conductor in the direction of B (m)
q = charge on each carrier (C)

A **Hall probe** uses this effect to measure B: since $V_H \propto B$ at constant I.

Velocity Selector

Velocity Selector

A velocity selector uses **crossed** electric and magnetic fields so that only particles with a specific speed pass through undeflected:

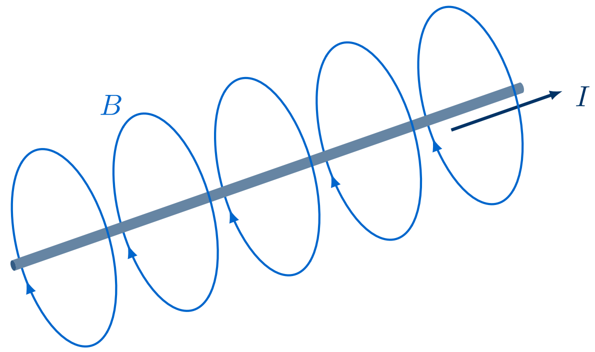
$$qE = BQv \quad \Rightarrow \quad v = \frac{E}{B}$$

- Electric force qE and magnetic force BQv act in **opposite** directions.
- Only particles where these forces balance travel in a straight line.
- Faster particles are deflected one way; slower particles the other.

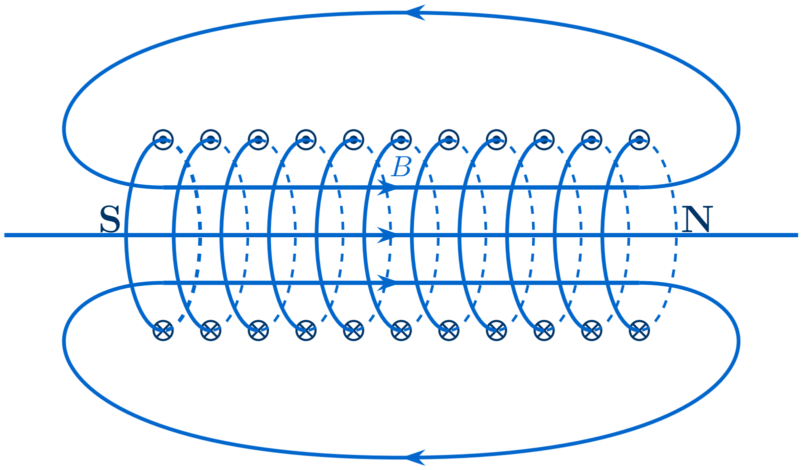
Magnetic Fields due to Currents

Field Patterns

- **Long straight wire**: concentric circles centred on the wire. Direction given by the **right-hand grip rule** — thumb points in direction of current, fingers curl in direction of field.
- **Flat circular coil**: field lines pass through the centre of the coil. Field at the centre is approximately uniform over a small region.
- **Long solenoid**: nearly uniform field inside, similar to a bar magnet outside. A **ferrous (iron) core** greatly increases the field strength.



Magnetic field around a straight wire



Magnetic field around a Solenoid

Forces Between Parallel Conductors

- **Same direction currents**: conductors **attract** each other.
- **Opposite direction currents**: conductors **repel** each other.
- Each conductor sits in the magnetic field produced by the other; the force is given by $F = BIL$.

Electromagnetic Induction

Magnetic Flux

Magnetic flux Φ is the product of the magnetic flux density and the cross-sectional area perpendicular to the field:

$$\Phi = BA\cos\theta \qquad \text{units: Wb (weber)} \equiv \text{T m}^2$$

When B is perpendicular to the area: $\Phi = BA$.

Flux linkage $N\Phi$ is the flux through a single turn multiplied by the number of turns N of the coil: units Wb-turns.

Faraday's Law and Lenz's Law

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

Faraday's Law: the induced e.m.f. is proportional to the rate of change of flux linkage.

Lenz's Law: the induced e.m.f. (and hence current) acts in a direction that **opposes** the change in flux that caused it (the minus sign above).

$\mathcal{E}$ = induced e.m.f. (V)
N = number of turns
$\Delta\Phi/\Delta t$ = rate of change of flux (Wb s^{-1} $\equiv$ V)

Factors Affecting the Induced E.M.F.

- **Rate of change** of flux: faster change ⇒ larger e.m.f.
- **Number of turns** N: more turns ⇒ larger e.m.f.
- **Strength of field** B: stronger field ⇒ larger flux change.
- **Area** of coil: larger area ⇒ more flux.

Lenz's law is a consequence of conservation of energy — the induced current creates a force that opposes the motion causing induction.

Common Mistake

An e.m.f. is induced only when the flux is **changing**. A conductor stationary in a steady field has zero induced e.m.f., even if the field is strong.

Formula Summary Sheet

Formula	Quantity	Units
$F = BIL\sin\theta$	Force on current-carrying conductor	N
$F = BQv\sin\theta$	Force on moving charge	N
$r = mv/(BQ)$	Radius of circular orbit in B field	m
$V_H = BI/(ntq)$	Hall voltage	V
$v = E/B$	Velocity selector condition	m s^{-1}
$\Phi = BA\cos\theta$	Magnetic flux	Wb
$\mathcal{E} = -\Delta(N\Phi)/\Delta t$	Faraday's / Lenz's law	V

Units: $1\ \text{T} = 1\ \text{N A}^{-1}\text{m}^{-1}$; $1\ \text{Wb} = 1\ \text{T m}^2 = 1\ \text{V s}$
Right-hand grip rule: thumb $\parallel$ current, fingers curl in direction of B field.

Worked Examples

Example 1 — Force on a Conductor

Question: A wire of length 0.15 m carries a current of 3.0 A at 60° to a uniform field of 0.25 T. Calculate the force on the wire.

Solution

$$F = BIL\sin\theta = 0.25 \times 3.0 \times 0.15 \times \sin 60^\circ$$
$$F = 0.25 \times 3.0 \times 0.15 \times 0.866 = \mathbf{9.7 \times 10^{-2}}\ \text{N}$$

Example 2 — Circular Motion of a Charged Particle

Question: A proton (mass 1.67×10^{-27} kg, charge 1.6×10^{-19} C) moves at 2.0×10^6 m s^{-1} perpendicular to a field of 0.15 T. Calculate the radius of its circular path.

Solution

$$r = \frac{mv}{BQ} = \frac{1.67 \times 10^{-27} \times 2.0 \times 10^6}{0.15 \times 1.6 \times 10^{-19}} = \frac{3.34 \times 10^{-21}}{2.40 \times 10^{-20}} = \mathbf{0.14}\ \text{m}$$

Example 3 — Induced E.M.F.

Question: A coil of 200 turns and area 50 cm^2 is in a field of 0.30 T perpendicular to the plane of the coil. The field drops to zero in 0.040 s. Calculate the induced e.m.f.

Solution

$$\Delta\Phi = B \times A = 0.30 \times 50 \times 10^{-4} = 1.5 \times 10^{-3} \text{ Wb}$$

$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t} = 200 \times \frac{1.5 \times 10^{-3}}{0.040} = \mathbf{7.5} \text{ V}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define magnetic flux density and state its SI unit.

[2 marks]

Q2. A wire of length 8.0 cm is placed perpendicular to a field of flux density 0.40 T and carries a current of 2.5 A. Calculate the force on the wire.

[2 marks]

Q3. State Faraday's Law and Lenz's Law of electromagnetic induction.

[3 marks]

Q4. Explain why the speed of a charged particle moving perpendicular to a uniform magnetic field remains constant.

[2 marks]

Q5. A Hall probe gives a voltage of 3.2 mV when a current of 50 mA passes through a slice of thickness 2.0 mm in a field B. Given $n = 8.5 \times 10^{28}$ m^{-3} and $q = 1.6 \times 10^{-19}$ C, calculate B.

[3 marks]

Section B — Longer Structured Questions

Q6. An electron (mass 9.11×10^{-31} kg, charge 1.6×10^{-19} C) enters a uniform magnetic field of 2.0×10^{-3} T perpendicular to the field with speed 5.0×10^{6} m s^{-1}.

(a) Calculate the radius of the circular path followed by the electron.

[2 marks]

(b) The field strength is doubled. State and explain the effect on the radius.

[2 marks]

(c) An electric field is now applied perpendicular to B so that the electron travels in a straight line. Calculate the electric field strength required.

[2 marks]

Q7. A rectangular coil of 80 turns and dimensions 4.0 cm × 6.0 cm is placed with its plane perpendicular to a uniform field of 0.50 T.

(a) Calculate the flux linkage through the coil.

[2 marks]

(b) The coil is rotated so that its plane becomes parallel to the field in 0.030 s. Calculate the mean induced e.m.f.

[2 marks]

(c) State and explain the direction of the induced current using Lenz's law.

[2 marks]

Mark Scheme and Answers

Q1. Magnetic flux density is the force per unit current per unit length on a wire placed at right angles to the field [1]; unit: tesla (T) or N A^{-1} m^{-1} [1].

Q2. $F = BIL \sin 90° = 0.40 \times 2.5 \times 0.080 = \mathbf{0.080}$ N [2].

Q3. Faraday's Law: the induced e.m.f. is proportional to the rate of change of flux linkage [1]; $\mathcal{E} = -\Delta(N\Phi)/\Delta t$ [1]. Lenz's Law: the induced e.m.f. acts in a direction to oppose the change in flux that caused it [1].

Q4. The magnetic force is always perpendicular to the velocity [1]; a perpendicular force does no work, so kinetic energy and hence speed remain unchanged [1].

Q5. $B = V_H ntq/I = (3.2 \times 10^{-3} \times 8.5 \times 10^{28} \times 2.0 \times 10^{-3} \times 1.6 \times 10^{-19})/(50 \times 10^{-3})$ [2] $= \mathbf{0.174}$ T [1].

Q6(a). $r = mv/(BQ) = (9.11 \times 10^{-31} \times 5.0 \times 10^6)/(2.0 \times 10^{-3} \times 1.6 \times 10^{-19})$ [1] $= \mathbf{1.4 \times 10^{-2}}$ m [1].

Q6(b). Radius halves [1]; $r = mv/BQ$, so $r \propto 1/B$; doubling B halves r [1].

Q6(c). For straight-line motion: $qE = BQv$; $E = Bv = 2.0 \times 10^{-3} \times 5.0 \times 10^6 = \mathbf{1.0 \times 10^4}$ **V m^{-1}** [2].

Q7(a). $\Phi = BA = 0.50 \times (0.040 \times 0.060) = 1.2 \times 10^{-3}$ Wb [1]; flux linkage $= N\Phi = 80 \times 1.2 \times 10^{-3} = \mathbf{9.6 \times 10^{-2}}$ Wb-turns [1].

Q7(b). $\Delta(N\Phi) = 9.6 \times 10^{-2}$ Wb-turns (falls to zero) [1]; $\mathcal{E} = 9.6 \times 10^{-2}/0.030 = \mathbf{3.2}$ V [1].

Q7(c). By Lenz's law the induced current opposes the decrease in flux [1]; the current flows in the direction that would create a field to oppose the rotation / maintain the flux through the coil [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define magnetic flux density; use $F = BIL \sin\theta$	
☐ Apply Fleming's left-hand rule to find force directions	
☐ Use $F = BQv \sin\theta$ for force on a moving charge	
☐ Derive and use $r = mv/(BQ)$ for circular orbit radius	
☐ Explain and use the Hall effect; use $V_H = BI/(ntq)$	
☐ Explain the velocity selector condition $v = E/B$	
☐ Sketch field patterns for straight wire, circular coil and solenoid	
☐ Apply the right-hand grip rule for field direction around a wire	
☐ Explain forces between parallel current-carrying conductors	
☐ Define magnetic flux $\Phi = BA \cos\theta$ and flux linkage $N\Phi$	
☐ State and apply Faraday's and Lenz's laws	
☐ Identify factors that affect the magnitude of induced e.m.f.	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Faraday's law — one equation — explains the generator, the transformer, and the electric motor. Master flux linkage and you understand how almost all electrical power is generated.

CIE International A Level Physics

Topic 21

Alternating Currents

Revision Booklet

This booklet covers:

- Characteristics of Alternating Currents
- R.M.S. Values and Power
- Half-Wave Rectification
- Full-Wave (Bridge) Rectification
- Smoothing with a Capacitor

Characteristics of Alternating Currents

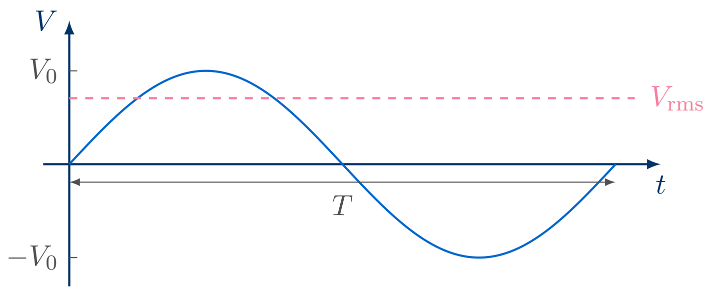
Alternating Current

An **alternating current** (a.c.) reverses direction periodically. For a sinusoidal a.c.:

$$I = I_0 \sin \omega t \qquad V = V_0 \sin \omega t$$

- I_0, V_0: **peak** (amplitude) values.
- $\omega = 2\pi f = 2\pi/T$: angular frequency (rad s^{-1}).
- T: period (s); f: frequency (Hz).
- UK mains supply: $f = 50$ Hz, $T = 20$ ms, $V_{rms} = 230$ V.

Sinusoidal a.c. voltage waveform



R.M.S. Values and Power

Root-Mean-Square (R.M.S.) Value

The **r.m.s. value** of an alternating current is defined as the value of steady direct current that would dissipate the **same power** in a purely resistive load.

R.M.S. Formulae

$$I_{rms} = \frac{I_0}{\sqrt{2}} \qquad V_{rms} = \frac{V_0}{\sqrt{2}}$$

These apply to **sinusoidal** waveforms only. For other waveforms the factor $1/\sqrt{2}$ changes.

Mean Power in a Resistive Load

$$P_{mean} = \tfrac{1}{2}P_{max} = \tfrac{1}{2}I_0^2 R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

The mean power is **half** the peak power for a sinusoidal waveform.

Why Use R.M.S.?

- The mean value of a sinusoidal current is **zero** — useless for power calculations.
- R.M.S. values allow direct use of d.c. power formulae ($P = IV$, $P = I^2R$, $P = V^2/R$).
- Meters and ratings (e.g. 230 V mains) always quote r.m.s. values.

Common Mistake

Do not use peak values in power calculations. Always convert to r.m.s. first: $V_{\text{rms}} = V_0/\sqrt{2}$. The peak mains voltage is $230\sqrt{2} \approx 325$ V, not 230 V.

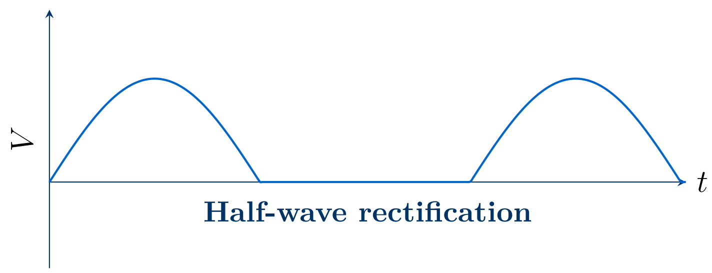
Rectification

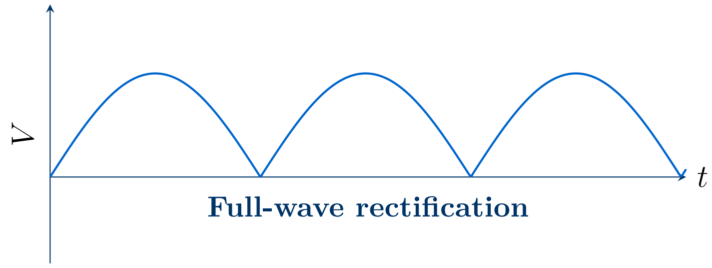
Rectification

Rectification converts alternating current into direct current flowing in one direction only.

- **Half-wave rectification**: a single diode passes only the positive (or negative) half-cycles; output is a series of pulses with gaps.
- **Full-wave rectification**: a bridge rectifier (four diodes) inverts the negative half-cycles, producing a continuous pulsating d.c. output with no gaps.

Comparison of rectified outputs



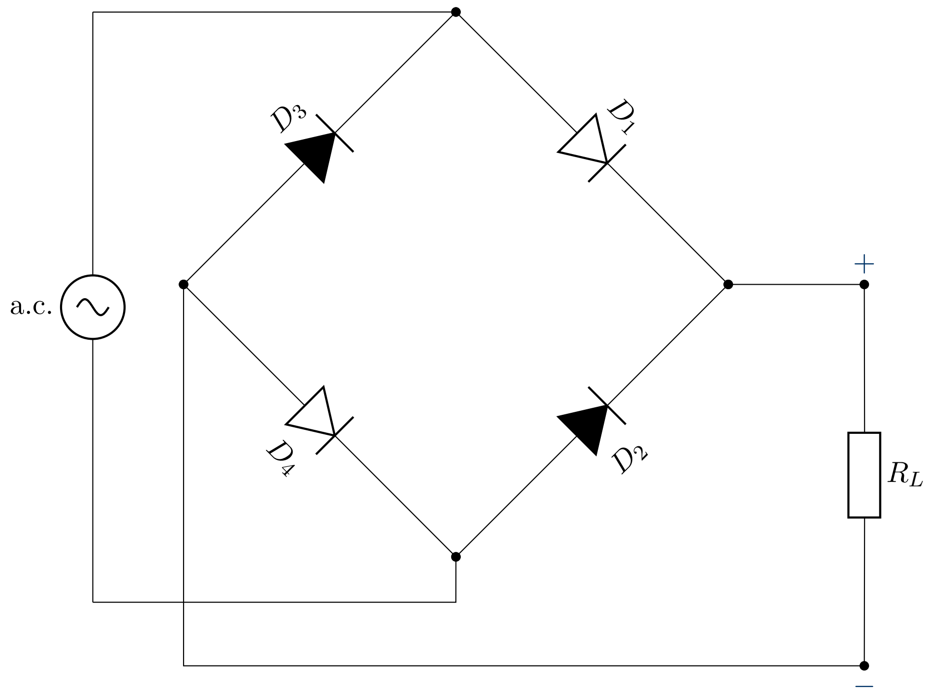


Bridge Rectifier

Bridge Rectifier

A bridge rectifier uses **four diodes** arranged in a diamond so that:

- During the **positive half-cycle**: current flows through diodes D1 and D4, through the load in the positive direction.
- During the **negative half-cycle**: current flows through diodes D2 and D3, but still through the load in the **same** direction.
- Both half-cycles contribute to the output — no wasted half-cycles.



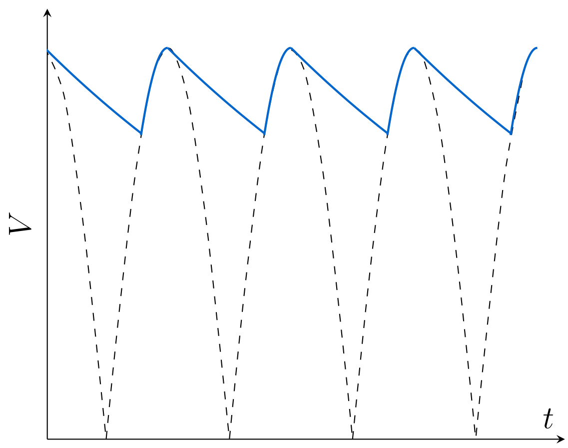
Smoothing with a Capacitor

Smoothing

A capacitor connected in **parallel with the load** reduces the ripple on a rectified output:

- The capacitor **charges up** rapidly to the peak voltage during each pulse.
- It **discharges slowly** through the load R_L between peaks, maintaining a more constant voltage.
- The output has a small residual **ripple voltage** rather than falling to zero between pulses.
- **Larger** C or **larger** R_L: longer time constant $\tau = CR_L$, less ripple.
- **Smaller** C or **smaller** R_L: faster discharge, larger ripple.

Effect of smoothing capacitor on full-wave rectified output



Common Mistake

In exam questions, always link ripple size explicitly to the time constant $\tau = CR_L$ — just saying "bigger capacitor" without explanation will not earn full marks.

Formula Summary Sheet

Formula	Quantity	Units
$I = I_0 \sin \omega t$	Sinusoidal alternating current	A
$V = V_0 \sin \omega t$	Sinusoidal alternating voltage	V
$\omega = 2\pi f = 2\pi/T$	Angular frequency	rad s^{-1}
$I_{rms} = I_0/\sqrt{2}$	R.M.S. current	A
$V_{rms} = V_0/\sqrt{2}$	R.M.S. voltage	V
$P_{mean} = \frac{1}{2}I_0^2R$	Mean power (peak values)	W
$P_{mean} = I_{rms}^2R = V_{rms}^2/R$	Mean power (r.m.s. values)	W

UK mains: $V_{rms} = 230$ V; $f = 50$ Hz; $V_0 = 230\sqrt{2} \approx 325$ V
Note: $1/\sqrt{2} \approx 0.707$; $P_{mean} = \frac{1}{2}P_{max}$ for sinusoidal waveform only.

Worked Examples

Example 1 — R.M.S. and Peak Values

Question: The mains supply has $V_{rms} = 230$ V at 50 Hz. Find (a) the peak voltage, (b) the angular frequency and (c) the mean power in a 1.2 kΩ resistor.

Solution

(a) $V_0 = V_{rms}\sqrt{2} = 230\sqrt{2} = \mathbf{325}$ V
(b) $\omega = 2\pi f = 2\pi \times 50 = \mathbf{314}$ rad s^{-1}
(c) $P = V_{rms}^2/R = 230^2/(1.2 \times 10^3) = 52900/1200 = \mathbf{44}$ W

Example 2 — Peak Power and Mean Power

Question: An a.c. supply has peak voltage $V_0 = 12$ V and is connected to a 60 Ω resistor. Calculate (a) the peak power and (b) the mean power dissipated.

Solution

(a) $P_{max} = V_0^2/R = 144/60 = \mathbf{2.4}$ W
(b) $P_{mean} = \frac{1}{2}P_{max} = \mathbf{1.2}$ W
Or equivalently: $V_{rms} = 12/\sqrt{2}$; $P = V_{rms}^2/R = 72/60 = 1.2$ W

Example 3 — Smoothing

Question: A full-wave rectifier feeds a 470 μF smoothing capacitor in parallel with a 2.2 kΩ load. Calculate the time constant and comment on the degree of smoothing for a 50 Hz supply.

Solution

$\tau = CR = 470 \times 10^{-6} \times 2200 = \mathbf{1.03}$ s
The period of the full-wave rectified signal is $T/2 = 1/(2 \times 50) = 10$ ms.
Since $\tau = 1.03$ s $\gg$ 10 ms, the capacitor discharges very little between peaks — the output is well smoothed with very small ripple.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Explain what is meant by the r.m.s. value of an alternating current and state why it is more useful than the peak value.

[3 marks]

Q2. An a.c. supply has peak voltage 340 V. Calculate (a) the r.m.s. voltage and (b) the mean power delivered to a 680 Ω resistor.

[3 marks]

Q3. Distinguish between half-wave and full-wave rectification. State the number of diodes required for each.

[3 marks]

Q4. Explain how a capacitor connected in parallel with a load resistor smooths a rectified output. State the effect of increasing the capacitance.

[3 marks]

Section B — Longer Structured Questions

Q5. The alternating voltage from a supply is given by $V = 170\sin(100\pi t)$, where V is in volts and t is in seconds.

(a) State the peak voltage and the frequency of the supply.

[2 marks]

(b) Calculate the r.m.s. voltage.

[1 mark]

(c) The supply is connected to a 500 Ω resistor. Calculate the mean power dissipated.

[2 marks]

(d) The supply is now passed through a bridge rectifier and a smoothing capacitor of 1000 μF is connected in parallel with the 500 Ω load. Calculate the time constant and comment on the effectiveness of smoothing.

[3 marks]

Q6. The graph below represents the output of a full-wave rectifier before smoothing. The peak voltage is 12 V and the supply frequency is 50 Hz.

(a) State the frequency of the rectified output.

[1 mark]

(b) On the same axes, sketch the output after connecting a large smoothing capacitor in parallel with the load. Indicate the approximate ripple voltage.

[2 marks]

(c) Explain what happens to the smoothing if the load resistance is reduced.

[2 marks]

Mark Scheme and Answers

Q1. The r.m.s. value is the equivalent steady d.c. that dissipates the same power in a resistive load [1]; it is more useful because power formulae ($P = I^2R$, $P = V^2/R$) can be applied directly [1]; the mean of a sinusoidal current is zero, so it gives no information about power [1].

Q2(a). $V_{\text{rms}} = V_0/\sqrt{2} = 340/\sqrt{2} = \mathbf{240}$ V [1]. **Q2(b).** $P = V_{\text{rms}}^2/R = 240^2/680 = 57600/680 = \mathbf{85}$ W [2].

Q3. Half-wave: uses **1 diode**; only one half of each cycle is passed; output has gaps of zero voltage [1]. Full-wave: uses **4 diodes** (bridge rectifier); both half-cycles are used; output is always positive with no gaps [2].

Q4. The capacitor charges to the peak voltage during each pulse [1]; between pulses it discharges slowly through the load, maintaining a more constant output voltage [1]; increasing C increases the time constant $\tau = CR$, so the capacitor discharges less between pulses and the ripple is smaller [1].

Q5(a). Peak voltage $V_0 = \mathbf{170}$ V [1]; $\omega = 100\pi$, so $f = \omega/2\pi = 100\pi/2\pi = \mathbf{50}$ Hz [1].

Q5(b). $V_{\text{rms}} = 170/\sqrt{2} = \mathbf{120}$ V [1].

Q5(c). $P = V_{\text{rms}}^2/R = (120)^2/500$ [1] $= 14400/500 = \mathbf{28.8}$ W [1].

Q5(d). $\tau = CR = 1000 \times 10^{-6} \times 500 = \mathbf{0.50}$ s [1]; period of full-wave output $= 1/(2 \times 50) = 10$ ms [1]; $\tau \gg T/2$ so capacitor barely discharges between peaks — very effective smoothing with tiny ripple [1].

Q6(a). The full-wave rectified output has frequency $2 \times 50 = \mathbf{100}$ Hz [1].

Q6(b). Sketch: smoothed output just below 12 V with small sawtooth ripple; ripple voltage is the small oscillation between the capacitor charge and discharge levels [2].

Q6(c). Reducing load resistance increases the discharge current [1]; the capacitor discharges faster between peaks ($\tau = CR$ decreases), so the ripple voltage increases and smoothing is less effective [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define period, frequency and peak value for an a.c. waveform	
☐ Use $x = x_0 \sin \omega t$ for sinusoidal current or voltage	
☐ Define r.m.s. value and explain why it is useful	
☐ Use $I_{\text{rms}} = I_0/\sqrt{2}$ and $V_{\text{rms}} = V_0/\sqrt{2}$	
☐ Calculate mean power using r.m.s. values: $P = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$	
☐ State that mean power is half peak power for sinusoidal a.c.	
☐ Distinguish half-wave and full-wave rectification graphically	
☐ Explain the action of a single diode for half-wave rectification	
☐ Explain the action of a bridge rectifier (four diodes)	
☐ Explain smoothing by a capacitor in terms of charge/discharge	
☐ Analyse the effect of C and R_L on the time constant and ripple	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

R.M.S. values are one of physics's most elegant ideas: a way to make a continuously varying quantity equivalent to a steady one. Once you see that $P_{\text{mean}} = \frac{1}{2}P_{\text{max}}$ comes directly from $\langle \sin^2 \rangle = \frac{1}{2}$, the whole topic falls into place.

CIE International A Level Physics

Topic 22

Quantum Physics

Revision Booklet

This booklet covers:

- The Photoelectric Effect
- Photon Energy and the Planck Relation
- Einstein's Photoelectric Equation
- Wave–Particle Duality & de Broglie Wavelength
- Energy Levels and Line Spectra

Core Concepts and Definitions

The Quantum Model of Light

Classical wave theory cannot explain certain phenomena involving light and matter. Quantum theory proposes that electromagnetic radiation is emitted and absorbed in discrete packets of energy called **photons**.

- A **photon** is a quantum of electromagnetic energy.
- The energy of a photon depends only on its **frequency**, not its intensity.
- At a fixed frequency, intensity is proportional to the **number of photons** per unit time. Increasing intensity does **not** increase the energy of individual photons.

Photon Energy — The Planck Relation

$$E = hf = \frac{hc}{\lambda}$$

E = energy of one photon (J)
h = Planck's constant = 6.63×10^{-34} J s
f = frequency of the radiation (Hz)
c = speed of light = 3.00×10^{8} m s^{-1}
λ = wavelength (m)

The Electronvolt

The **electronvolt** (eV) is a convenient unit of energy at the atomic scale.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

To convert eV → J: multiply by 1.60×10^{-19}.
To convert J → eV: divide by 1.60×10^{-19}.

The Photoelectric Effect

The Photoelectric Effect

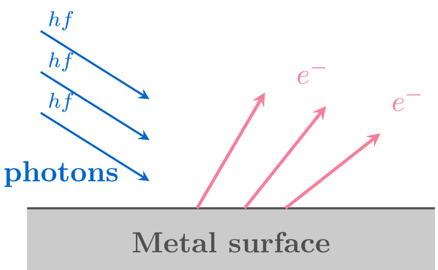
The **photoelectric effect** is the emission of electrons from a metal surface when electromagnetic radiation of sufficiently high frequency is incident on it.

- Electrons emitted are called **photoelectrons**.
- Emission occurs **instantaneously** if the frequency is above a threshold — there is no time delay.
- Below the **threshold frequency** f_0, no electrons are emitted regardless of intensity.
- Above f_0, increasing the intensity increases the **number** of photoelectrons per second, not their energy.

Why Classical Wave Theory Fails

- Wave theory predicts that any frequency of sufficient intensity should eventually eject electrons — **not observed**.
- Wave theory predicts a time delay before emission as energy builds up — **not observed** (emission is instantaneous).
- Wave theory predicts that intensity should increase the electron's kinetic energy — **not observed**.

These failures led Einstein to propose the photon model.



Einstein's Photoelectric Equation

$$hf = \phi + \tfrac{1}{2}mv_{\text{max}}^2$$

hf = energy of the incident photon (J)
ϕ = **work function** — minimum energy to remove an electron from the surface (J)
$\frac{1}{2}mv_{\text{max}}^2$ = maximum kinetic energy of emitted photoelectron (J)

The **work function** $\phi = hf_0$, where f_0 is the threshold frequency.
Maximum KE can also be written as eV_s, where V_s is the **stopping potential**.

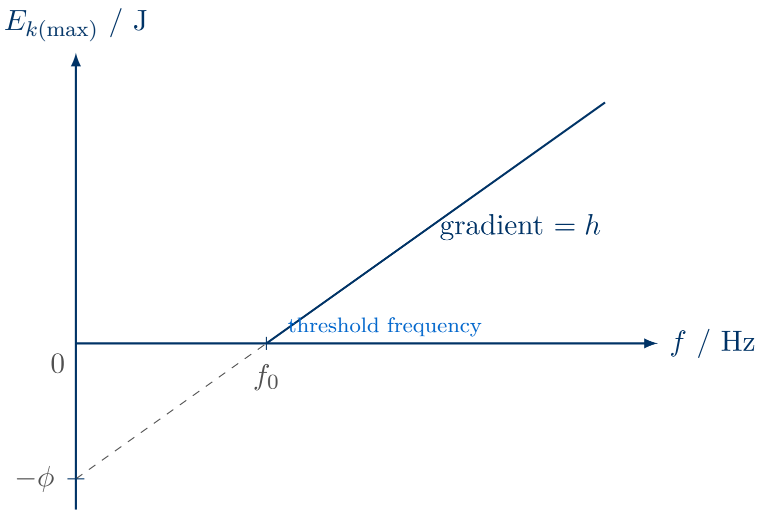
Stopping Potential V_s

The **stopping potential** is the minimum opposing potential difference required to prevent all photoelectrons from reaching the collecting electrode.

$$eV_s = \tfrac{1}{2}mv_{\text{max}}^2 = hf - \phi$$

Measuring V_s at different frequencies allows h and ϕ to be determined experimentally.

Graph of Maximum KE against Frequency



Reading the Graph

- The **gradient** of the $E_{k(\text{max})}$ vs f graph equals Planck's constant h.
- The **x-intercept** gives the threshold frequency f_0.
- The **y-intercept** (extrapolated) gives $-\phi$ (the negative of the work function).
- Different metals give **parallel lines** — same gradient (h), different intercepts (ϕ).

Wave–Particle Duality

Wave–Particle Duality

Wave–particle duality is the property of matter and radiation by which they exhibit both wave-like and particle-like behaviour depending on the experimental context.

- Light behaves as a **wave**: diffraction, interference, polarisation.
- Light behaves as **particles (photons)**: photoelectric effect.
- Electrons (and other particles) also show **wave behaviour**: electron diffraction.

de Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

λ = de Broglie wavelength (m)
h = Planck's constant (6.63×10^{-34} J s)
p = momentum of the particle (kg m s^{-1})
m = mass of the particle (kg)
v = speed of the particle (m s^{-1})

Electron Diffraction

When electrons are accelerated through a potential difference V and directed at a thin crystal or graphite film, a **diffraction pattern** of rings is observed — confirming their wave nature.

- The kinetic energy gained: $\frac{1}{2}mv^2 = eV$, giving $v = \sqrt{2eV/m}$.
- Therefore: $\lambda = \dfrac{h}{\sqrt{2meV}}$
- Increasing V increases p, decreasing $\lambda \Rightarrow$ rings become **smaller**.
- The wavelength must be comparable to the atomic spacing for diffraction to occur ($\sim 10^{-10}$ m).

Common Mistake — Confusing Photon and Particle Momentum

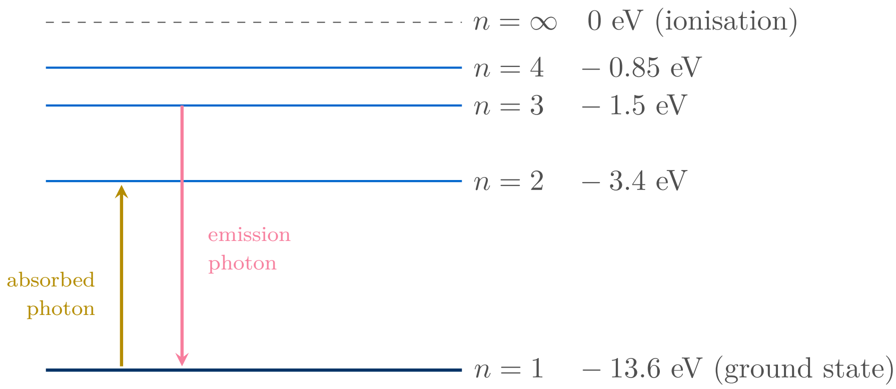
For a **photon** (massless): $p = E/c = hf/c = h/\lambda$.
For a **particle with mass**: $p = mv = h/\lambda$ (de Broglie).
Both use $\lambda = h/p$, but the relationship between energy and momentum differs. Do not use $E = hf$ for material particles.

Energy Levels and Line Spectra

Atomic Energy Levels

Electrons in an atom can only occupy discrete **energy levels**. These are quantised — only specific energies are allowed.

- The **ground state** is the lowest energy level (most stable; most negative value).
- **Excited states** have higher (less negative) energy.
- The **ionisation energy** is the energy required to remove an electron from the ground state to infinity ($E = 0$).
- Energy levels are usually expressed in electronvolts (eV) and are **negative** (bound states).



Hydrogen Energy Level Diagram (schematic)

Photon Energy from a Transition

When an electron moves between energy levels E_1 (lower) and E_2 (higher):

$$hf = E_2 - E_1$$

- **Emission**: electron falls to lower level ⇒ photon **emitted** with $hf = E_2 - E_1$.
- **Absorption**: electron promoted to higher level ⇒ photon **absorbed** with $hf = E_2 - E_1$.

Emission and Absorption Spectra

Line Spectra

- **Emission spectrum**: a series of **bright coloured lines** on a dark background. Each line corresponds to a specific photon frequency emitted during a downward electron transition. Produced by excited gases.
- **Absorption spectrum**: a **continuous spectrum** crossed by dark lines at the same frequencies as the emission lines. Cool gas in front of a broad-spectrum source absorbs specific photon energies.
- The **line positions are unique** to each element — used for identification (spectroscopy).

Why Line Spectra Prove Quantisation

Because electrons can only occupy discrete energy levels, only photons of specific energies (frequencies) can be absorbed or emitted. This produces a **line** spectrum rather than a continuous one. A continuous spectrum would imply electrons can have any energy — which they cannot.

Formula Summary Sheet

Formula	Quantity	Units
$E = hf$	Photon energy	J
$E = hc/\lambda$	Photon energy from wavelength	J
$hf = \phi + \frac{1}{2}mv_{\max}^2$	Einstein's photoelectric equation	J
$\phi = hf_0$	Work function	J
$eV_s = \frac{1}{2}mv_{\max}^2$	Stopping potential	J, V
$\lambda = h/p = h/mv$	de Broglie wavelength	m
$\lambda = h/\sqrt{2meV}$	de Broglie λ from accelerating pd	m
$hf = E_2 - E_1$	Photon from energy transition	J

Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Calculation Questions

1. **Identify** the relevant formula — is it a photon, a particle, or a transition?
2. **Convert units**: eV to J ($\times 1.60 \times 10^{-19}$); nm to m ($\times 10^{-9}$).
3. **Substitute** values carefully, showing all working.
4. **Quote** the final answer in appropriate units with correct significant figures.

Common Errors — Avoid These!

- Using the **wrong energy unit** — forgetting to convert eV to J before substituting.
- Confusing **threshold frequency** f_0 with threshold wavelength: a higher f_0 corresponds to a **shorter** threshold wavelength λ_0.
- Applying **intensity** changes to change photon energy — intensity only changes the **number** of photons.
- Using $E = hf$ for the de Broglie relation — this is for photons only; for matter use $\lambda = h/mv$.
- Forgetting that energy levels are **negative** — the transition energy is $|E_2 - E_1|$,

not just $E_2 - E_1$ as magnitudes.

- Confusing **emission** (bright lines) and **absorption** (dark lines) spectra.

Worked Examples

Example 1 — Photoelectric Effect Calculation

Question: Light of wavelength 250 nm is incident on a metal surface with a work function of 4.5 eV. Calculate the maximum kinetic energy of emitted photoelectrons and the stopping potential.

Solution

Solution:
Convert work function: $\phi = 4.5 \times 1.60 \times 10^{-19} = 7.20 \times 10^{-19}$ J
Photon energy:

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{250 \times 10^{-9}} = 7.96 \times 10^{-19} \text{ J}$$

Maximum KE:

$$E_{k(\max)} = hf - \phi = 7.96 \times 10^{-19} - 7.20 \times 10^{-19} = \mathbf{7.6 \times 10^{-20}} \text{ J}$$

Stopping potential:

$$V_s = \frac{E_{k(\max)}}{e} = \frac{7.6 \times 10^{-20}}{1.60 \times 10^{-19}} = \mathbf{0.48} \text{ V}$$

Example 2 — de Broglie Wavelength

Question: An electron is accelerated from rest through a potential difference of 3.0 kV. Calculate its de Broglie wavelength.

Solution

Solution:
Energy gained: $E_k = eV = 1.60 \times 10^{-19} \times 3000 = 4.80 \times 10^{-16}$ J
Speed: $v = \sqrt{\frac{2E_k}{m_e}} = \sqrt{\frac{2 \times 4.80 \times 10^{-16}}{9.11 \times 10^{-31}}} = 3.25 \times 10^{7}$ m s^{-1}
de Broglie wavelength:

$$\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.25 \times 10^{7}} = \mathbf{2.24 \times 10^{-11}} \text{ m}$$

This is comparable to atomic spacings ($\sim 10^{-10}$ m), confirming why electron diffraction is observed.

Example 3 — Energy Level Transition

Question: An electron in a hydrogen-like atom falls from an energy level of −1.5 eV to −3.4 eV. Find the frequency and wavelength of the emitted photon.

Solution

Solution:
Energy of emitted photon:

$$hf = E_2 - E_1 = -1.5 - (-3.4) = 1.9 \text{ eV} = 1.9 \times 1.60 \times 10^{-19} = 3.04 \times 10^{-19} \text{ J}$$

Frequency:

$$f = \frac{3.04 \times 10^{-19}}{6.63 \times 10^{-34}} = \mathbf{4.59 \times 10^{14}} \text{ Hz} \quad \text{(visible light — red/orange)}$$

Wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{4.59 \times 10^{14}} = \mathbf{654} \text{ nm}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State what is meant by the *threshold frequency* in the photoelectric effect.

[2 marks]

Q2. Explain why the photoelectric effect cannot be explained by the wave model of light. Give two reasons.

[2 marks]

Q3. Light of frequency 8.0×10^{14} Hz is incident on a metal surface with work function 2.5 eV. Show whether photoelectric emission will occur and, if so, calculate the maximum kinetic energy of the emitted electrons.

[4 marks]

Q4. Calculate the de Broglie wavelength of a proton ($m_p = 1.67 \times 10^{-27}$ kg) moving at 2.0×10^6 m s^{-1}.

[2 marks]

Q5. An electron and a proton are each accelerated from rest through the same potential difference V. Show that the de Broglie wavelength of the electron is greater than that of the proton. ($m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg)

[3 marks]

Section B — Longer Structured Questions

Q6. A student investigates the photoelectric effect using a metal surface. The graph of maximum kinetic energy against frequency is a straight line. The threshold frequency of the metal is 5.5×10^{14} Hz.

(a) Calculate the work function of the metal in joules.

[2 marks]

(b) Light of frequency 9.0×10^{14} Hz is incident on the surface. Calculate the stopping potential.

[3 marks]

(c) The intensity of the incident light is doubled whilst keeping its frequency the same. State and explain what happens to (i) the rate of electron emission, and (ii) the maximum kinetic energy of emitted electrons.

[4 marks]

Q7. The energy levels of atomic hydrogen include $E_1 = -13.6$ eV (ground state), $E_2 = -3.4$ eV, and $E_3 = -1.5$ eV.

(a) Calculate the frequency of the photon emitted when an electron transitions from $n = 3$ to $n = 1$.

[3 marks]

(b) A photon of energy 12.1 eV is incident on a hydrogen atom in the ground state. Explain whether this photon will be absorbed.

[2 marks]

(c) Describe how the line spectrum of hydrogen provides evidence for the existence of discrete energy levels in atoms.

[3 marks]

Mark Scheme and Answers

Q1. The threshold frequency is the minimum frequency of electromagnetic radiation [1] that is capable of causing photoelectric emission from the surface [1].

Q2. Any two from: wave theory predicts a time delay before emission, but emission is instantaneous [1]; wave theory predicts any frequency of sufficient intensity should cause emission, but no emission occurs below f_0 regardless of intensity [1]; wave theory predicts intensity should increase electron KE, but maximum KE depends only on frequency [1].

Q3. Photon energy: $E = hf = 6.63 \times 10^{-34} \times 8.0 \times 10^{14} = 5.30 \times 10^{-19}$ J $= 3.32$ eV [1]. Since 3.32 eV $>$ 2.5 eV, emission **will** occur [1]. Maximum KE $= 3.32 - 2.5 = 0.82$ eV $= 1.31 \times 10^{-19}$ J [2].

Q4. $\lambda = h/(m_p v) = 6.63 \times 10^{-34}/(1.67 \times 10^{-27} \times 2.0 \times 10^6) = \mathbf{1.98 \times 10^{-13}}$ m [2].

Q5. Both particles gain the same kinetic energy eV, so $\frac{1}{2}mv^2 = eV \Rightarrow p = mv = \sqrt{2meV}$ [1]; therefore $\lambda = h/\sqrt{2meV}$ [1]; since $m_e \ll m_p$, the denominator is smaller for the electron, giving a **larger** λ for the electron [1].

Q6(a). $\phi = hf_0 = 6.63 \times 10^{-34} \times 5.5 \times 10^{14} = \mathbf{3.65 \times 10^{-19}}$ J [2].

Q6(b). $E_{k(\max)} = hf - \phi = 6.63 \times 10^{-34} \times 9.0 \times 10^{14} - 3.65 \times 10^{-19} = 5.97 \times 10^{-19} - 3.65 \times 10^{-19} = 2.32 \times 10^{-19}$ J [2]; $V_s = E_{k(\max)}/e = 2.32 \times 10^{-19}/1.60 \times 10^{-19} = \mathbf{1.45}$ V [1].

Q6(c). (i) Rate of electron emission **doubles** [1]: greater intensity $\Rightarrow$ more photons per second $\Rightarrow$ more photoelectron emissions per second [1]. (ii) Maximum KE is **unchanged** [1]: each photon still has the same energy (f unchanged); intensity affects number of photons, not energy per photon [1].

Q7(a). $\Delta E = -1.5 - (-13.6) = 12.1$ eV $= 12.1 \times 1.60 \times 10^{-19} = 1.936 \times 10^{-18}$ J [1]; $f = \Delta E/h = 1.936 \times 10^{-18}/6.63 \times 10^{-34} = \mathbf{2.92 \times 10^{15}}$ Hz (ultraviolet) [2].

Q7(b). The energy difference from $n = 1$ to $n = 3$ is $-1.5 - (-13.6) = 12.1$ eV [1]; the photon energy exactly matches this transition, so it **will** be absorbed and the electron promoted to $n = 3$ [1].

Q7(c). Each line in the emission spectrum corresponds to a specific photon frequency [1]; each frequency corresponds to a specific photon energy, given by $hf = E_2 - E_1$ [1]; the existence of discrete lines (rather than a continuum) shows that electrons can only transition between fixed energy values — i.e. energy levels are quantised [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ State that electromagnetic radiation is quantised into photons	
☐ Use $E = hf$ and $E = hc/\lambda$ to calculate photon energy	
☐ Convert between joules and electronvolts	
☐ Describe the photoelectric effect and state its key observations	
☐ Explain why wave theory fails to explain the photoelectric effect	
☐ Apply Einstein's equation $hf = \phi + \frac{1}{2}mv^2_{\text{max}}$	
☐ Define work function and threshold frequency; relate via $\phi = hf_0$	
☐ Explain the significance of stopping potential and use $eV_s = E_{k(\text{max})}$	
☐ State and apply the de Broglie relation $\lambda = h/p$	
☐ Describe electron diffraction as evidence for wave–particle duality	
☐ Explain what is meant by atomic energy levels and ground/excited states	
☐ Use $hf = E_2 - E_1$ for emission and absorption transitions	
☐ Interpret emission and absorption line spectra	
☐ Explain how line spectra provide evidence for energy quantisation	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Quantum physics is genuinely strange — but the maths is manageable. Focus on understanding *why* classical physics fails, and let the photon model guide the rest. The equations follow naturally from the physics.

CIE International A Level Physics

Topic 23

Nuclear Physics

Revision Booklet

This booklet covers:

- Mass–Energy Equivalence: $E = mc^2$
- Mass Defect and Binding Energy
- Binding Energy per Nucleon and Nuclear Stability
- Nuclear Fission and Fusion
- Radioactive Decay: Activity and Decay Constant
- Half-Life and Exponential Decay

Mass–Energy Equivalence

Einstein's Mass–Energy Relation

Einstein's special theory of relativity establishes that mass and energy are equivalent. A mass m at rest has an intrinsic energy given by:

$$E = mc^2$$

- $c = 3.00 \times 10^8$ m s^{-1} (speed of light in free space)
- A small mass corresponds to an enormous amount of energy.
- In nuclear reactions, small changes in mass Δm release measurable amounts of energy.

Atomic Mass Unit

The **unified atomic mass unit** (u) is defined as one-twelfth of the mass of a carbon-12 atom.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

The energy equivalent of 1 u:

$$E = mc^2 = 1.661 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$$

So 1 u $\equiv 931.5$ MeV$/c^2$.

Nuclear Notation and Equations

A nuclide is written ${}^{A}_{Z}\text{X}$, where A is the **nucleon number** (mass number) and Z is the **proton number** (atomic number).

Nuclear equations must conserve:

- **Nucleon number** A (top numbers balance)
- **Proton number** Z (bottom numbers balance)
- **Mass–energy** (energy is released or absorbed)
- **Charge** and **momentum**

Example: ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$

Mass Defect and Binding Energy

Mass Defect

The **mass defect** Δm of a nucleus is the difference between the total mass of the separate constituent nucleons and the actual mass of the nucleus.

$$\Delta m = Z\,m_p + (A - Z)\,m_n - m_{\text{nucleus}}$$

m_p = mass of a proton = 1.6726×10^{-27} kg
m_n = mass of a neutron = 1.6749×10^{-27} kg
m_{nucleus} = actual measured mass of the nucleus

The mass defect is always **positive**: the nucleus is always less massive than its parts.

Binding Energy

The **binding energy** of a nucleus is the energy required to completely separate a nucleus into its constituent protons and neutrons (i.e. to infinity).

$$E_B = \Delta m \cdot c^2$$

Equivalently, it is the energy *released* when the nucleus is assembled from separate nucleons.

Energy Released in a Nuclear Reaction

$$E = c^2\,\Delta m$$

where Δm is the difference between the total mass of reactants and the total mass of products.
If $\Delta m > 0$ (reactants heavier than products): energy is **released**.
If $\Delta m < 0$: energy must be **supplied**.

Common Mistake — Mass Defect vs Binding Energy

Students often confuse the *sign convention*. The mass defect is always defined as a positive quantity (how much mass is "missing"). The binding energy is the energy equivalent of this missing mass. A **larger** binding energy means a **more stable** nucleus — it takes more energy to pull it apart.

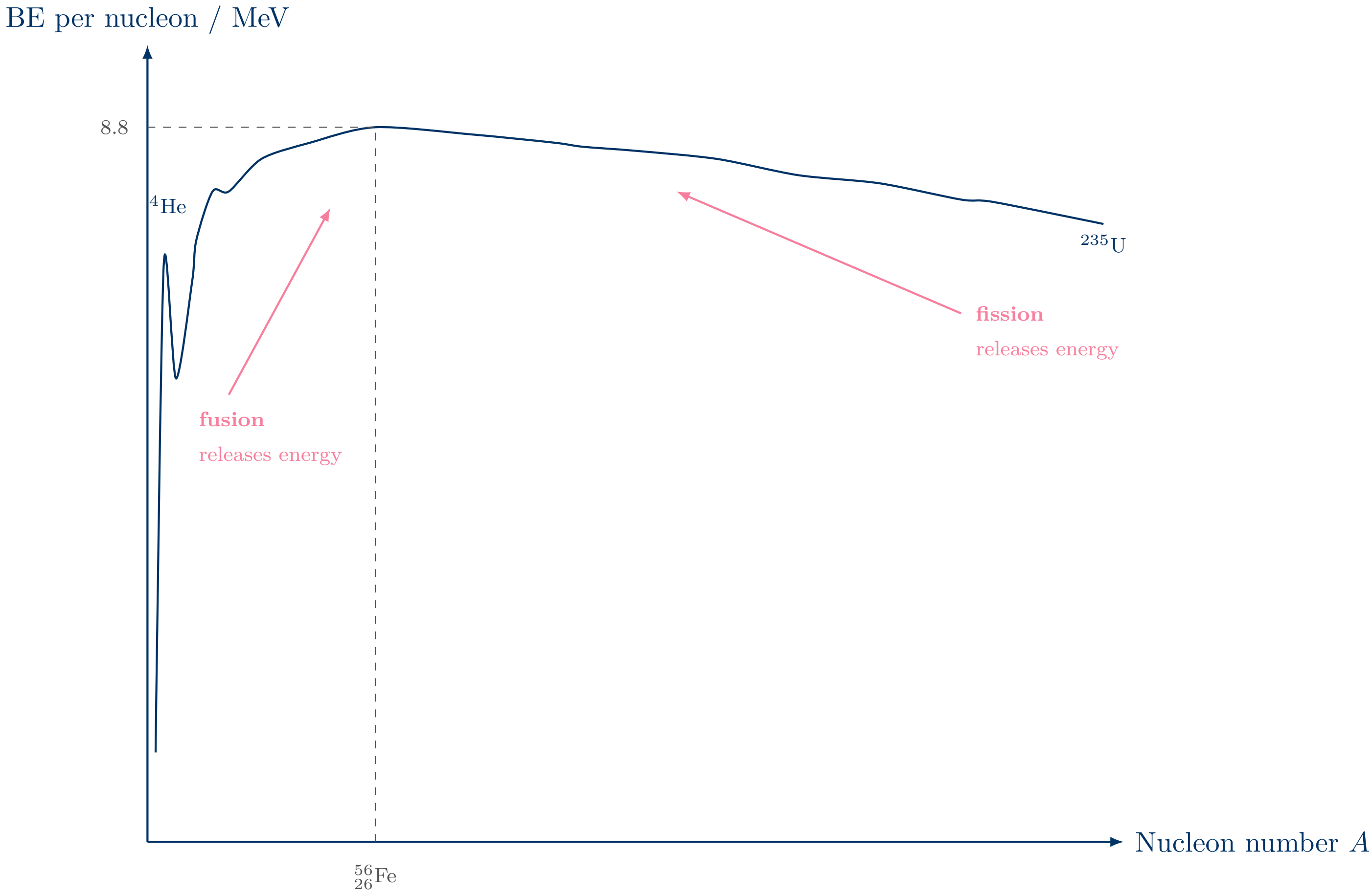
Binding Energy per Nucleon

Binding Energy per Nucleon

The **binding energy per nucleon** is the total binding energy of a nucleus divided by its nucleon number A. It is a measure of nuclear stability: the higher the value, the more stable the nucleus.

$$\text{BE per nucleon} = \frac{E_B}{A} = \frac{c^2 \Delta m}{A}$$

Variation of Binding Energy per Nucleon with Nucleon Number



Key Features of the Graph

- The curve **rises steeply** for light nuclei, peaks near ${}^{56}_{26}\text{Fe}$ at approximately 8.8 MeV per nucleon — the most stable nucleus.
- The curve **decreases gradually** for heavy nuclei ($A > 56$).
- **Fusion** of light nuclei (left of peak) moves up the curve ⇒ products are more stable ⇒ energy is released.
- **Fission** of heavy nuclei (right of peak) also moves up the curve ⇒ products are more stable ⇒ energy is released.
- ${}^{4}_{2}\text{He}$ (helium-4) lies notably *above* the curve — it is exceptionally stable for its mass number.

Nuclear Fission and Fusion

Nuclear Fission

Nuclear fission is the splitting of a large, unstable nucleus into two smaller (daughter) nuclei of roughly equal mass, accompanied by the release of neutrons and energy.

- Induced fission: a slow (thermal) neutron is absorbed by a heavy nucleus (e.g. ^{235}U), which then splits.
- The products have greater binding energy per nucleon than the original nucleus ⇒ energy is released.
- Typically releases 2–3 fast neutrons which can trigger further fissions (**chain reaction**).

Example: $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$

Nuclear Fusion

Nuclear fusion is the combining of two light nuclei to form a heavier nucleus, releasing energy.

- The product nucleus has greater binding energy per nucleon than the reactants ⇒ energy is released.
- Requires **extremely high temperatures** ($\sim 10^7$ K) to overcome electrostatic repulsion between nuclei.
- Powers stars; the basis of proposed fusion reactors.

Example: $^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$

Why These Reactions Release Energy

In both fission and fusion, the total mass of the **products** is less than the total mass of the **reactants**. This mass difference Δm is converted to kinetic energy of the products via $E = c^2\Delta m$. The reactions move nuclei *towards* the peak of the BE per nucleon curve (towards ^{56}Fe).

Radioactive Decay

Spontaneous and Random Decay

Radioactive decay is the spontaneous emission of radiation from an unstable nucleus.

- **Spontaneous**: the decay is not triggered by external conditions (temperature, pressure, chemical state); it cannot be predicted or controlled.
- **Random**: it is impossible to predict *when* any particular nucleus will decay. Each nucleus has the same probability of decaying per unit time.
- **Evidence for randomness**: fluctuations (statistical variation) in the measured count rate from a radioactive source.

Activity and Decay Constant

The **activity** A of a source is the number of nuclei that decay per unit time.

$$A = \lambda N$$

A = activity (Bq, where 1 Bq = 1 decay s^{-1})
λ = **decay constant** — the probability of decay of a nucleus per unit time (s^{-1})
N = number of undecayed nuclei present

Half-Life

The **half-life** $t_{1/2}$ is the time taken for the number of undecayed nuclei (or the activity) of a radioactive sample to fall to half its initial value.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

Half-life is constant for a given isotope — it does not depend on the number of nuclei present or external conditions.

Exponential Decay

Exponential Decay Equations

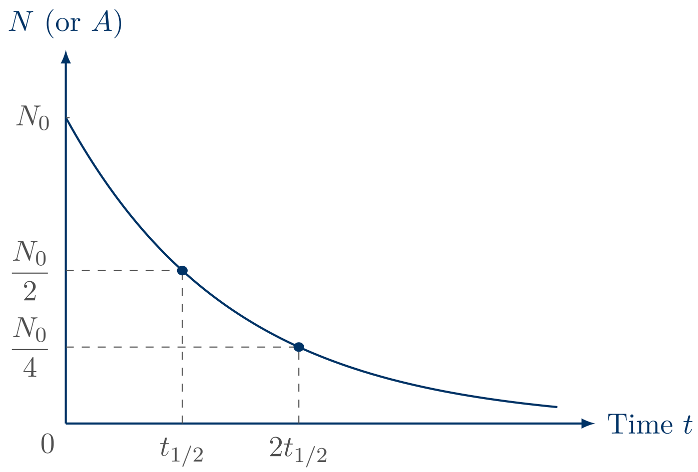
$$x = x_0\, e^{-\lambda t}$$

where x can represent:

- N = number of undecayed nuclei: $N = N_0\, e^{-\lambda t}$
- A = activity of the source: $A = A_0\, e^{-\lambda t}$
- C = received count rate: $C = C_0\, e^{-\lambda t}$

x_0 = initial value at $t = 0$
λ = decay constant (s^{-1})
t = time elapsed (s)

Graph of N against t



Linearising the Decay Equation

Taking the natural logarithm of $N = N_0 e^{-\lambda t}$:

$$\ln N = \ln N_0 - \lambda t$$

A graph of $\ln N$ (or $\ln A$) against t gives a **straight line** with:

- Gradient $= -\lambda$
- y-intercept $= \ln N_0$

This is the standard experimental method to determine λ and hence $t_{1/2}$.

Common Errors with Decay Calculations

- Using $t_{1/2}$ directly in the exponential formula — you must use λ, not $t_{1/2}$. Convert first: $\lambda = 0.693/t_{1/2}$.

- Forgetting to convert time units: if $t_{1/2}$ is in days, convert to seconds before finding λ in s^{-1}.
- Confusing **activity** $A = \lambda N$ (Bq) with **count rate** — the count rate is always less than activity due to detector efficiency and geometry.
- Applying the exponential formula to something that *increases* over time — it only applies to N, A, or count rate, which all decay.

Formula Summary Sheet

Formula	Quantity	Units
$E = mc^2$	Mass–energy equivalence	J
$E = c^2 \Delta m$	Energy from mass change	J
$\Delta m = Zm_p + (A-Z)m_n - m_{\text{nuc}}$	Mass defect	kg
$E_B = \Delta m\, c^2$	Binding energy	J
$A = \lambda N$	Activity	Bq
$\lambda = 0.693/t_{1/2}$	Decay constant from half-life	s^{-1}
$N = N_0 e^{-\lambda t}$	Number of undecayed nuclei	—
$A = A_0 e^{-\lambda t}$	Activity	Bq
$\ln N = \ln N_0 - \lambda t$	Linearised decay	—

Constants: $c = 3.00 \times 10^8$ m s^{-1}, 1 u $= 1.661 \times 10^{-27}$ kg $\equiv 931.5$ MeV, $m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Nuclear Calculations

1. **Balance the equation** — check nucleon numbers and proton numbers sum correctly on both sides.
2. **Find** Δm — total reactant mass minus total product mass; work in kg or u.
3. **Apply** $E = c^2\Delta m$ to find energy released (convert u to kg if needed, or use 1 u = 931.5 MeV).
4. For decay: **convert** $t_{1/2}$ to seconds, find $\lambda = 0.693/t_{1/2}$, then apply $x = x_0 e^{-\lambda t}$.

Common Errors — Avoid These!

- Using **atomic masses** (including electron masses) rather than nuclear masses without accounting for electron masses — take care with data provided in exam questions.
- Forgetting that Δm must be in **kg** when using $E = mc^2$ in SI units.
- Confusing **binding energy** with **ionisation energy** — binding energy refers to the nucleus.
- Stating that a higher binding energy per nucleon means **less** stable — it means **more** stable.
- Mixing up the direction of fusion and fission on the BE/nucleon graph.

Worked Examples

Example 1 — Mass Defect and Binding Energy

Question: Calculate the mass defect and binding energy of a helium-4 nucleus (^4_2He).
($m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg, $m_{^4\text{He}} = 6.6447 \times 10^{-27}$ kg)

Solution

Solution:
Mass of constituents: $2m_p + 2m_n = 2(1.6726) + 2(1.6749) = 6.6950 \times 10^{-27}$ kg
Mass defect:

$$\Delta m = 6.6950 \times 10^{-27} - 6.6447 \times 10^{-27} = \mathbf{5.03 \times 10^{-29}} \text{ kg}$$

Binding energy:

$$E_B = \Delta m\, c^2 = 5.03 \times 10^{-29} \times (3.00 \times 10^8)^2 = \mathbf{4.53 \times 10^{-12}} \text{ J } (= 28.3 \text{ MeV})$$

Binding energy per nucleon: $4.53 \times 10^{-12}/4 = 1.13 \times 10^{-12}$ J $= 7.07$ MeV per nucleon

Example 2 — Energy Released in Fission

Question: In a fission reaction, the total mass of products is 3.09×10^{-28} kg less than the total mass of reactants. Calculate the energy released in MeV.

Solution

Solution:

$$E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.09 \times 10^{-28} = 2.78 \times 10^{-11} \text{ J}$$

$$E = \frac{2.78 \times 10^{-11}}{1.60 \times 10^{-13}} = \mathbf{174} \text{ MeV}$$

Example 3 — Radioactive Decay Calculation

Question: A radioactive isotope has a half-life of 12.0 hours. A sample initially contains 8.00×10^{20} undecayed nuclei. Calculate (a) the decay constant, (b) the initial activity, (c) the number of undecayed nuclei after 30.0 hours.

Solution

Solution:
(a) Convert: $t_{1/2} = 12.0 \times 3600 = 4.32 \times 10^4$ s

$$\lambda = \frac{0.693}{4.32 \times 10^4} = \mathbf{1.60 \times 10^{-5}} \text{ s}^{-1}$$

(b) $A_0 = \lambda N_0 = 1.60 \times 10^{-5} \times 8.00 \times 10^{20} = \mathbf{1.28 \times 10^{16}}$ Bq
(c) $t = 30.0 \times 3600 = 1.08 \times 10^5$ s

$$N = N_0\, e^{-\lambda t} = 8.00 \times 10^{20} \times e^{-1.60 \times 10^{-5} \times 1.08 \times 10^5}$$

$$N = 8.00 \times 10^{20} \times e^{-1.728} = 8.00 \times 10^{20} \times 0.178 = \mathbf{1.42 \times 10^{20}}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the terms *mass defect* and *binding energy* of a nucleus.

[4 marks]

Q2. State two features of the binding energy per nucleon graph that explain why both nuclear fusion and nuclear fission can release energy.

[2 marks]

Q3. A radioactive source has an activity of 6.4×10^5 Bq and a decay constant of $2.0 \times 10^{-3}\ s^{-1}$. Calculate the number of undecayed nuclei present and the half-life of the source.

[4 marks]

Q4. Explain what is meant by saying that radioactive decay is *spontaneous* and *random*.

[2 marks]

Q5. Complete and balance the following nuclear equation, identifying the unknown particle X:

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + \text{X}$$

[2 marks]

Section B — Longer Structured Questions

Q6. The fusion reaction between deuterium and tritium is:

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$

Relevant masses: $m(^2\text{H}) = 2.01410$ u, $m(^3\text{H}) = 3.01605$ u, $m(^4\text{He}) = 4.00260$ u, $m_n = 1.00867$ u.

(a) Calculate the mass defect of this reaction in kg.

[3 marks]

(b) Calculate the energy released in this reaction in joules and in MeV.

[2 marks]

(c) Explain, with reference to the binding energy per nucleon graph, why this reaction releases energy.

[3 marks]

Q7. The isotope iodine-131 ($^{131}_{53}\mathrm{I}$) is used in medical treatment. It has a half-life of 8.04 days.

(a) Calculate the decay constant of iodine-131 in s^{-1}.

[2 marks]

(b) A patient is given a dose with an initial activity of 4.0×10^8 Bq. Calculate the activity after 24 days.

[2 marks]

(c) Sketch a graph of $\ln A$ against t for this source, labelling the y-intercept and stating the gradient in terms of λ.

[3 marks]

Mark Scheme and Answers

Q1. *Mass defect*: the difference between the total mass of the separate nucleons (protons and neutrons) and the actual mass of the nucleus [2]. *Binding energy*: the energy required to completely separate a nucleus into its constituent protons and neutrons [2].

Q2. The curve rises steeply for light nuclei — fusion of light nuclei produces a product with greater BE/nucleon, so energy is released [1]. The curve falls for heavy nuclei — fission of a heavy nucleus produces fragments with greater BE/nucleon, so energy is released [1].

Q3. $N = A/\lambda = 6.4 \times 10^5/2.0 \times 10^{-3} = \mathbf{3.2 \times 10^8}$ [2]. $t_{1/2} = 0.693/\lambda = 0.693/2.0 \times 10^{-3} = \mathbf{347}$ s [2].

Q4. *Spontaneous*: the decay is not triggered or affected by external conditions; it cannot be induced or prevented [1]. *Random*: it is impossible to predict which nucleus will decay next, or when; each nucleus has the same fixed probability of decaying per unit time [1].

Q5. Nucleon: $238 = 234 + A \Rightarrow A = 4$; proton: $92 = 90 + Z \Rightarrow Z = 2$. X is ${}^4_2$He (an alpha particle) [2].

Q6(a). $\Delta m = (2.01410+3.01605)-(4.00260+1.00867) = 5.03015-5.01127 = 0.01888$ u [1]; $= 0.01888 \times 1.661 \times 10^{-27} = \mathbf{3.14 \times 10^{-29}}$ kg [2].

Q6(b). $E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.14 \times 10^{-29} = \mathbf{2.82 \times 10^{-12}}$ J [1]; $= 2.82 \times 10^{-12}/1.60 \times 10^{-13} = \mathbf{17.6}$ MeV [1].

Q6(c). The reactants (^{2}H and ^{3}H) lie to the left of the peak of the BE/nucleon curve [1]; the product ^{4}He has a higher binding energy per nucleon than the reactants [1]; since the products are more tightly bound, mass is converted to energy and released [1].

Q7(a). $t_{1/2} = 8.04 \times 24 \times 3600 = 6.95 \times 10^5$ s; $\lambda = 0.693/6.95 \times 10^5 = \mathbf{9.97 \times 10^{-7}}$ $\mathbf{s^{-1}}$ [2].

Q7(b). 24 days $= 3 \times t_{1/2}$; $A = 4.0 \times 10^8 \times (1/2)^3 = 4.0 \times 10^8/8 = \mathbf{5.0 \times 10^7}$ Bq [2].

Q7(c). Straight line [1]; y-intercept at $\ln(4.0 \times 10^8) = 19.8$ labelled $\ln A_0$ [1]; gradient $= -\lambda$ (negative slope) [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ State and apply Einstein's mass–energy relation $E = mc^2$	
☐ Write and balance nuclear equations, conserving A and Z	
☐ Define mass defect and calculate it from nuclear masses	
☐ Define binding energy and use $E_B = \Delta m\, c^2$	
☐ Sketch the binding energy per nucleon vs nucleon number graph	
☐ Identify the most stable nucleus and the peak of the curve	
☐ Explain using the graph why fusion of light nuclei releases energy	
☐ Explain using the graph why fission of heavy nuclei releases energy	
☐ Calculate energy released in a nuclear reaction using $E = c^2 \Delta m$	
☐ Explain what is meant by spontaneous and random decay	
☐ Define activity and decay constant; use $A = \lambda N$	
☐ Define half-life and use $\lambda = 0.693/t_{1/2}$	
☐ Apply $x = x_0 e^{-\lambda t}$ to N, A, or count rate	
☐ Linearise the decay equation and interpret the graph of $\ln N$ vs t	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Nuclear physics connects the very small to the very large — from the stability of a single nucleus to the energy source of stars. Master the binding energy curve and the exponential decay equation and most of this topic follows naturally.

CIE International A Level Physics

Topic 24

Medical Physics

Revision Booklet

This booklet covers:

- Ultrasound: Generation, Detection and Imaging
- Acoustic Impedance and Reflection
- Attenuation of Ultrasound
- Production and Use of X-Rays
- Attenuation of X-Rays and CT Scanning
- PET Scanning and Annihilation

Production and Use of Ultrasound

Ultrasound

Ultrasound is sound with a frequency above the upper limit of human hearing (> 20 kHz). In medical imaging, frequencies of 1–20 MHz are typical.

Piezoelectric Transducer

A **piezoelectric crystal** exhibits two related effects:

- When a **p.d. is applied** across the crystal, it changes shape (contracts or expands). Applying an alternating p.d. at the crystal's resonant frequency causes it to **vibrate and emit ultrasound**.
- Conversely, when the crystal's shape **changes** (e.g. due to an incoming pressure wave), it **generates an e.m.f.** — it acts as a detector.

The same transducer can therefore act as both **emitter and receiver**.

A-Scan Imaging (Pulse-Echo)

- A short pulse of ultrasound is emitted into the body.
- At each **boundary between tissues** of different acoustic impedance, part of the pulse is **reflected** (echo) and part is **transmitted**.
- The time delay between emission and detection of each echo gives the **depth** of the boundary: $d = \frac{1}{2}vt$.
- The amplitude of each echo gives information about the nature of the boundary.

Acoustic Impedance and Reflection

Specific Acoustic Impedance

The **specific acoustic impedance** Z of a medium is defined as:

$$Z = \rho c$$

Z = specific acoustic impedance (kg m^{-2} s^{-1})
ρ = density of the medium (kg m^{-3})
c = speed of sound in the medium (m s^{-1})

Intensity Reflection Coefficient

The fraction of intensity reflected at a boundary between two media with impedances Z_1 and Z_2:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

- If $Z_1 = Z_2$ (matched impedances): $I_R/I_0 = 0$ — no reflection, all transmitted.
- If $Z_1 \gg Z_2$ or $Z_1 \ll Z_2$ (large mismatch): $I_R/I_0 \approx 1$ — almost all reflected.
- Air–tissue boundary: huge impedance mismatch $\Rightarrow$ almost complete reflection.

Coupling Gel

Because the acoustic impedance of air is much lower than that of tissue, a large fraction of ultrasound would be reflected at the skin–air boundary if no gel were used. A **coupling gel** (with impedance close to that of tissue) is applied between the transducer and the skin to **minimise reflection** and allow ultrasound to enter the body efficiently.

Attenuation of Ultrasound

Attenuation of Ultrasound in Matter

As ultrasound travels through a medium, its intensity decreases exponentially:

$$I = I_0 \, e^{-\mu x}$$

I = intensity at depth x (W m^{-2})
I_0 = initial intensity (W m^{-2})
μ = **absorption (attenuation) coefficient** of the medium (m^{-1})
x = distance travelled in the medium (m)

A larger μ means the medium absorbs ultrasound more strongly. Higher frequency ultrasound has a larger μ (greater attenuation) but better resolution.

Resolution vs Penetration Trade-off

- **Higher frequency**: shorter wavelength $\Rightarrow$ better resolution, but higher attenuation $\Rightarrow$ less depth penetration.
- **Lower frequency**: greater penetration but poorer resolution.
- The choice of frequency is a compromise depending on the depth of the structure being imaged.

Production and Use of X-Rays

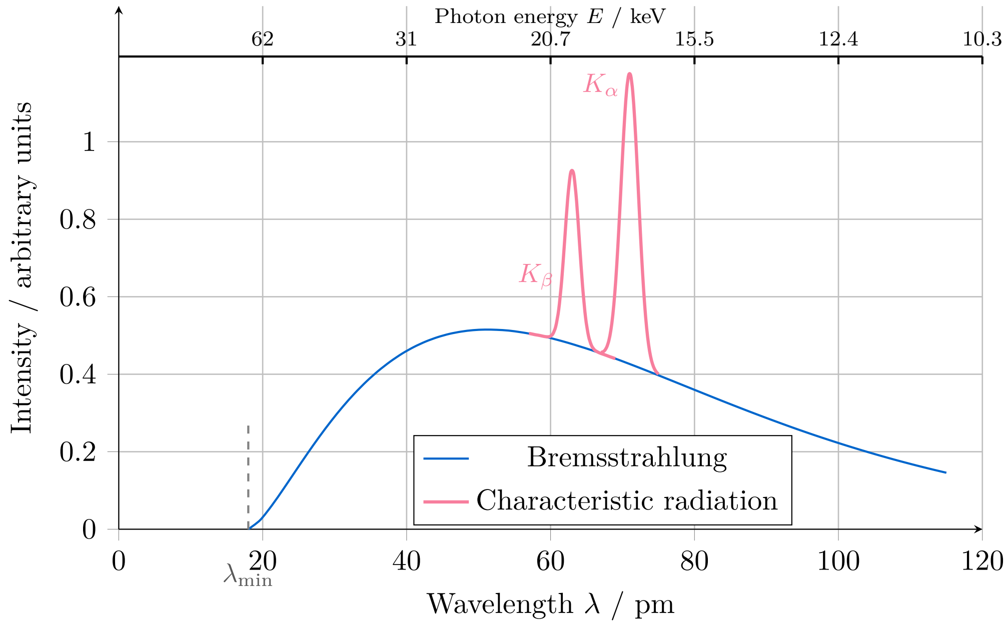
Production of X-Rays

X-rays are produced in an X-ray tube when fast-moving electrons are rapidly decelerated by a metal target (anode).

- Electrons are accelerated from a heated cathode through a high potential difference V.
- On striking the target, electrons lose kinetic energy, producing X-rays by two processes:

- **Bremsstrahlung** (braking radiation): continuous spectrum from deceleration.
- **Characteristic radiation**: discrete lines from inner-shell electron transitions in target atoms.

X-ray Emission Spectrum (Tungsten Target, 70 kV)



Minimum X-Ray Wavelength

The maximum photon energy (minimum wavelength) occurs when an electron gives all its kinetic energy to a single photon:

$$eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV}$$

Increasing the accelerating voltage V decreases λ_{min} and increases the penetrating power of the X-rays.

Contrast in X-Ray Imaging

Contrast refers to the difference in image intensity between adjacent structures, allowing them to be distinguished.

- Dense materials (e.g. bone, containing calcium) absorb X-rays strongly ⇒ appear **white** on the image.
- Soft tissues absorb less ⇒ appear **grey**.
- Air absorbs very little ⇒ appears **black**.

- **Contrast agents** (e.g. barium meal, iodine compounds) can be introduced to increase contrast for soft tissue structures such as the gut or blood vessels.

Attenuation of X-Rays in Matter

$$I = I_0\, e^{-\mu x}$$

The same exponential attenuation law applies as for ultrasound, where μ is now the **linear attenuation coefficient** for X-rays in the medium. Dense materials have larger μ.

CT Scanning (Computed Tomography)

A **CT scanner** produces a three-dimensional image of internal structures:

- Multiple X-ray images of the **same cross-sectional slice** are taken from **different angles**.
- These are combined computationally to produce a **2D image of one slice**.
- The process is **repeated along the body's axis**, producing multiple 2D slice images.
- The 2D slice images are **combined** to build a full **3D image**.

CT gives much better contrast for soft tissues than a plain X-ray, but involves a significantly higher radiation dose.

PET Scanning

Radioactive Tracers

A **tracer** is a substance containing radioactive nuclei that is introduced into the body and absorbed by the tissue under investigation. The emitted radiation is detected externally to produce an image of the tracer distribution.

- In PET scanning, a tracer that decays by β^+ **(positron) emission** is used.
- A common example is fluorine-18 labelled glucose (^{18}F-FDG), which is preferentially absorbed by metabolically active tissue (e.g. tumours).

Annihilation

Annihilation occurs when a particle meets its **antiparticle**: both are destroyed and their combined mass–energy is converted entirely into radiation.

- A positron (β^+) emitted by the tracer quickly meets an electron in the surrounding tissue.
- Both are annihilated, producing **two gamma-ray photons** travelling in **exactly opposite directions** (to conserve momentum).

- **Conservation laws**: mass–energy and momentum are both conserved in the process.

Energy of Annihilation Photons

Each photon carries energy equal to the rest-mass energy of one electron (the two particles have negligible kinetic energy at annihilation):

$$E_\gamma = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}\ \text{J} = \mathbf{0.511}\ \text{MeV}$$

Both photons have this energy (total energy released $= 2m_e c^2 = 1.02$ MeV).

How PET Produces an Image

- Detectors arranged around the patient detect the **coincident arrival** of the two gamma photons.
- Because the photons travel in opposite directions, the annihilation event must have occurred **somewhere along the line** joining the two detectors.
- By processing the **arrival times** from many such coincidences, the positions of annihilation events are reconstructed, producing a map of **tracer concentration** in the tissue.
- High tracer uptake indicates high metabolic activity — useful for identifying tumours, heart disease, and neurological conditions.

PET vs CT vs Ultrasound

- **Ultrasound**: no ionising radiation; good for soft tissue and real-time imaging (e.g. foetal scans); cannot penetrate bone or air.
- **X-ray / CT**: high contrast for bone; CT gives 3D information; ionising radiation dose (CT significantly higher than plain X-ray).
- **PET**: images *function* (metabolic activity), not just structure; requires a cyclotron to produce the short-lived tracer; relatively high radiation dose.

Formula Summary Sheet

Formula	Quantity	Units
$Z = \rho c$	Specific acoustic impedance	kg m^{-2} s^{-1}
$I_R/I_0 = (Z_1 - Z_2)^2/(Z_1 + Z_2)^2$	Intensity reflection coefficient	—
$I = I_0 e^{-\mu x}$	Attenuation (ultrasound or X-ray)	W m^{-2}
$d = \frac{1}{2}vt$	Depth from echo time	m
$\lambda_{\text{min}} = hc/eV$	Minimum X-ray wavelength	m
$E_\gamma = m_e c^2$	Energy of annihilation photon	J

Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

Exam Technique and Problem-Solving Strategy

Key Strategies

1. For **attenuation** questions: identify I_0, μ and x; substitute into $I = I_0 e^{-\mu x}$.
2. For **reflection coefficient**: identify Z_1 and Z_2; substitute directly — the formula is symmetric in Z_1 and Z_2.
3. For **echo depth**: $d = \frac{1}{2}vt$ (factor of $\frac{1}{2}$ because pulse travels to the boundary *and back*).
4. For **annihilation photon energy**: always 0.511 MeV per photon; quote this or calculate from $m_e c^2$.

Common Errors — Avoid These!

- Forgetting the **factor of 2** in $d = vt/2$ for pulse-echo depth calculations.
- Confusing the **attenuation coefficient** μ with decay constant λ from Topic 23 — both appear in exponential decay equations but refer to completely different physical processes.
- Stating that annihilation produces **one** photon — it must produce **two** travelling in opposite directions to conserve momentum.
- Confusing **CT** (multiple X-ray angles $\Rightarrow$ 3D image) with a plain X-ray (single image, 2D projection).

- Applying the reflection coefficient formula with **intensities** instead of **impedances**.

Worked Examples

Example 1 — Acoustic Impedance and Reflection

Question: The specific acoustic impedance of muscle is 1.70×10^6 kg m^{-2}s^{-1} and of bone is 7.80×10^6 kg m^{-2}s^{-1}. Calculate the intensity reflection coefficient at a muscle–bone boundary.

Solution

Solution:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2} = \frac{(1.70 \times 10^6 - 7.80 \times 10^6)^2}{(1.70 \times 10^6 + 7.80 \times 10^6)^2}$$

$$= \frac{(-6.10 \times 10^6)^2}{(9.50 \times 10^6)^2} = \frac{3.72 \times 10^{13}}{9.02 \times 10^{13}} = \mathbf{0.413}$$

About 41% of the ultrasound intensity is reflected at this boundary.

Example 2 — Ultrasound Attenuation

Question: Ultrasound with an initial intensity of 250 W m^{-2} passes through 4.0 cm of tissue with attenuation coefficient $\mu = 23$ m^{-1}. Calculate the transmitted intensity.

Solution

Solution:

$$I = I_0 e^{-\mu x} = 250 \times e^{-23 \times 0.040} = 250 \times e^{-0.92}$$

$$I = 250 \times 0.399 = \mathbf{99.7} \text{ W m}^{\mathbf{-2}}$$

Example 3 — PET Scanning Annihilation Energy

Question: In a PET scan, a positron emitted by the tracer annihilates with an electron. Calculate the energy and frequency of each gamma-ray photon produced.

Solution

Solution:
Energy of each photon:

$$E = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14} \text{ J}$$

$$= 8.20 \times 10^{-14} / 1.60 \times 10^{-13} = \mathbf{0.511} \text{ MeV}$$

Frequency:

$$f = \frac{E}{h} = \frac{8.20 \times 10^{-14}}{6.63 \times 10^{-34}} = \mathbf{1.24 \times 10^{20}} \text{ Hz}$$

(This lies in the gamma-ray region of the electromagnetic spectrum.)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Describe how a piezoelectric transducer can act as both an emitter and a detector of ultrasound.

[4 marks]

Q2. The specific acoustic impedance of soft tissue is 1.63×10^6 kg m^{-2}s^{-1} and of air is 430 kg m^{-2}s^{-1}. Show that almost all ultrasound is reflected at an air–tissue boundary.

[3 marks]

Q3. An ultrasound pulse is emitted and an echo is received 85 μs later. The speed of ultrasound in tissue is 1500 m s^{-1}. Calculate the depth of the reflecting boundary.

[2 marks]

Q4. An X-ray beam of initial intensity I_0 passes through 8.0 cm of tissue with linear attenuation coefficient $\mu = 12$ m^{-1}. Calculate the ratio I/I_0.

[2 marks]

Q5. State **two** conservation laws that apply during electron–positron annihilation in PET scanning.

[2 marks]

Section B — Longer Structured Questions

Q6. A medical ultrasound system uses a piezoelectric transducer operating at 5.0 MHz.

(a) Explain why a coupling gel is applied between the transducer and the patient's skin.

[3 marks]

(b) The attenuation coefficient of soft tissue at this frequency is 40 m^{-1}. Calculate the depth at which the intensity has fallen to 5.0% of its initial value.

[3 marks]

(c) Suggest why a lower frequency might be chosen when imaging deep structures, and state one disadvantage.

[2 marks]

Q7. PET scanning uses a tracer that emits positrons.

(a) Explain what happens when a positron emitted by the tracer encounters an electron in the body tissue.

[3 marks]

(b) Explain why two detectors placed on opposite sides of the patient must detect photons simultaneously for the event to be recorded.

[2 marks]

(c) Calculate the wavelength of the gamma-ray photons produced in the annihilation.

[2 marks]

Mark Scheme and Answers

Q1. Emitter: an alternating p.d. at the resonant frequency is applied across the crystal [1]; the crystal vibrates at that frequency [1]; emitting ultrasound waves. Detector: incoming pressure wave causes the crystal to change shape [1]; this generates an e.m.f. which is detected as an electrical signal [1].

Q2. $I_R/I_0 = (1.63 \times 10^6 - 430)^2/(1.63 \times 10^6 + 430)^2$ [1] $\approx (1.63 \times 10^6)^2/(1.63 \times 10^6)^2$ [1] ≈ 0.9995 (i.e. ≈ 1, almost total reflection) [1].

Q3. $d = \frac{1}{2}vt = \frac{1}{2} \times 1500 \times 85 \times 10^{-6} = \mathbf{6.4 \times 10^{-2}}$ m (= 6.4 cm) [2].

Q4. $I/I_0 = e^{-\mu x} = e^{-12\times0.080} = e^{-0.96} = \mathbf{0.383}$ [2].

Q5. Any two of: conservation of mass–energy [1]; conservation of momentum [1]; conservation of charge [1].

Q6(a). The acoustic impedance of air is much lower than that of tissue [1]; this large mismatch means nearly all ultrasound would be reflected at the skin–air interface [1]; the gel has impedance close to that of tissue, minimising reflection and allowing ultrasound to enter the body [1].

Q6(b). $I/I_0 = 0.050$, so $e^{-40x} = 0.050$ [1]; $-40x = \ln(0.050) = -3.00$; $x = 3.00/40 = \mathbf{0.075}$ m (= 7.5 cm) [2].

Q6(c). Lower frequency has a smaller attenuation coefficient, so ultrasound penetrates more deeply [1]; disadvantage: lower frequency has a longer wavelength, giving **poorer spatial resolution** [1].

Q7(a). The positron meets an electron [1]; both are annihilated (annihilation) [1]; two gamma-ray photons are produced travelling in exactly opposite directions [1].

Q7(b). The two photons travel in exactly opposite directions (to conserve momentum) [1]; simultaneous detection at opposite detectors confirms the annihilation occurred on the line joining the two detectors [1].

Q7(c). $E = m_e c^2 = 8.20 \times 10^{-14}$ J; $\lambda = hc/E = (6.63 \times 10^{-34} \times 3.00 \times 10^{8})/8.20 \times 10^{-14} = \mathbf{2.42 \times 10^{-12}}$ m [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Explain the piezoelectric effect in both directions (emitter and detector)	
☐	Describe how pulse-echo ultrasound produces diagnostic information	
☐	Define specific acoustic impedance using $Z = \rho c$	
☐	Use the intensity reflection coefficient formula	
☐	Explain the purpose of coupling gel	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of ultrasound	
☐	Explain the resolution vs penetration trade-off for ultrasound frequency	
☐	Explain how X-rays are produced (bremsstrahlung and characteristic)	
☐	Use $\lambda_{\text{min}} = hc/eV$ for X-ray tube calculations	
☐	Explain contrast in X-ray imaging	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of X-rays	
☐	Describe how CT scanning builds a 3D image from multiple 2D slices	
☐	Explain what a radioactive tracer is and why a β^+ emitter is used in PET	
☐	Describe electron–positron annihilation and apply conservation laws	
☐	Calculate the energy of annihilation photons using $E = m_e c^2$	
☐	Explain how coincidence detection produces a PET image	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Medical physics is where fundamental physics saves lives. The same exponential attenuation, the same $E = mc^2$, the same wave properties — seen in a new and important context. Make sure you can explain *why* each technique works, not just apply the formulas.

CIE International A Level Physics

Topic 25

Astronomy and Cosmology

Revision Booklet

This booklet covers:

- Luminosity and Radiant Flux Intensity
- Standard Candles and Distance Measurement
- Wien's Displacement Law and Stellar Temperature
- The Stefan–Boltzmann Law and Stellar Radii
- Redshift and the Expanding Universe
- Hubble's Law and the Big Bang

Luminosity and Radiant Flux Intensity

Luminosity

The **luminosity** L of a star is the total power of electromagnetic radiation emitted by the star in all directions.

$$L \quad \text{units: W (watts)}$$

Luminosity depends on the star's **surface temperature** and **surface area** — not on its distance from us.

Radiant Flux Intensity

The **radiant flux intensity** F (sometimes called apparent brightness) is the power of radiation received per unit area at a detector (e.g. a telescope on Earth).

$$F \quad \text{units: W m}^{-2}$$

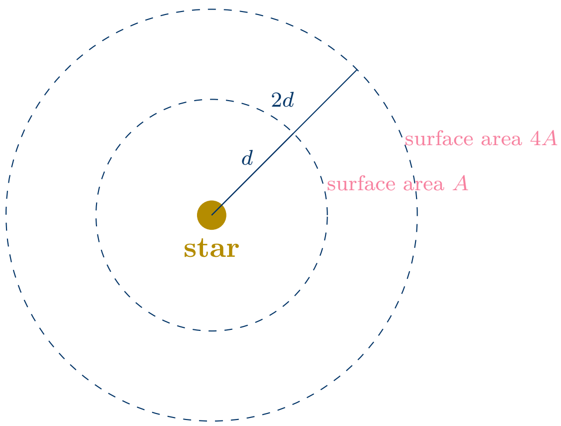
F depends on both the luminosity of the source and its distance d.

Inverse Square Law for Radiant Flux

Assuming the star radiates uniformly in all directions and there is no absorption by the intervening medium, the radiation spreads over a sphere of area $4\pi d^2$:

$$F = \frac{L}{4\pi d^2}$$

F = radiant flux intensity at distance d (W m^{-2})
L = luminosity of the star (W)
d = distance from the star (m)



$F \propto 1/d^2$: double the distance, quarter the flux

Standard Candles

Standard Candle

A **standard candle** is an astronomical object whose **luminosity is known** (or can be determined independently of distance). By measuring the radiant flux intensity F received from the object and using $F = L/(4\pi d^2)$, its distance d can be calculated:

$$d = \sqrt{\frac{L}{4\pi F}}$$

Standard Candles Used in Practice

- **Cepheid variable stars**: pulsating stars whose *period of variation* is related to their luminosity (period–luminosity relation). Measure the period $\Rightarrow$ know $L \Rightarrow$ measure $F \Rightarrow$ find d.
- **Type Ia supernovae**: thermonuclear explosions of white dwarf stars that all reach approximately the same peak luminosity. Visible across enormous distances (billions of light-years), making them useful for measuring distances to distant galaxies.

Assumptions and Limitations

The inverse square law assumes:

- No absorption of radiation between source and detector (no dust or gas in the way).
- The source radiates **isotropically** (equally in all directions).

Dust absorption causes stars to appear **dimmer** than expected, which would lead to an **overestimate** of distance if uncorrected.

Wien's Displacement Law and Stellar Temperature

Black-Body Radiation

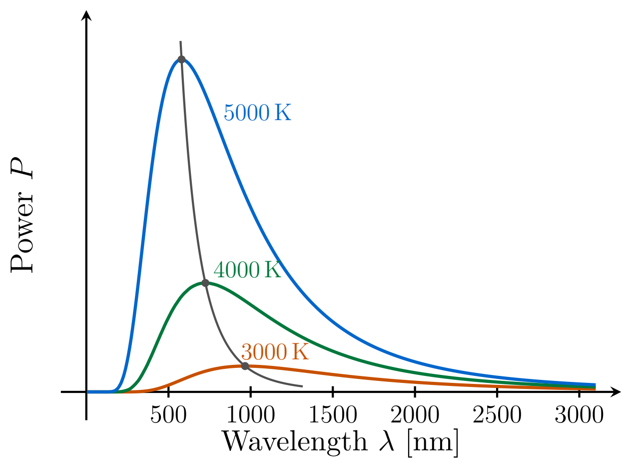
Stars approximate **black bodies** — objects that absorb all incident radiation and emit a characteristic continuous spectrum that depends only on temperature. The peak wavelength of this spectrum shifts with temperature.

Wien's Displacement Law

$$\lambda_{\max} \propto \frac{1}{T} \qquad \text{equivalently} \qquad \lambda_{\max} T = b$$

$\lambda_{\max}$ = wavelength at peak intensity of the spectrum (m)
T = surface temperature of the star (K)
b = Wien's constant = 2.90×10^{-3} m K

A **hotter** star has its peak at a **shorter** wavelength (bluer colour). A **cooler** star peaks at a **longer** wavelength (redder colour).



The Stefan–Boltzmann Law and Stellar Radii

Stefan–Boltzmann Law

For a black-body sphere (approximating a star) of radius r and surface temperature T:

$$L = 4\pi r^2 \sigma T^4$$

L = luminosity (W)
r = radius of the star (m)
σ = Stefan–Boltzmann constant = 5.67×10^{-8} W m^{-2}K^{-4}
T = surface temperature (K)

Estimating Stellar Radius

By combining Wien's law and the Stefan–Boltzmann law:

1. Measure λ_{max} from the star's spectrum ⇒ use Wien's law to find T.
2. Measure F (radiant flux intensity) and find d (e.g. via a standard candle or parallax).
3. Find L from $L = 4\pi d^2 F$.
4. Rearrange Stefan–Boltzmann: $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Using Ratios

Exam questions often ask you to *compare* two stars rather than calculate absolute values. In that case, form a ratio to cancel constants:

$$\frac{L_1}{L_2} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4}$$

This avoids large numbers and reduces the risk of errors.

Redshift and the Expanding Universe

Redshift

When a source of light moves **away** from an observer, the observed wavelength is **longer** (shifted towards the red end of the spectrum) than the wavelength emitted. This is the **Doppler effect** applied to light.
The lines in the emission or absorption spectrum of a distant galaxy are observed at **longer wavelengths** than those of the same element measured in the laboratory.

Redshift Formula

For a source moving at speed $v \ll c$ relative to an observer:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

z = redshift (dimensionless)
$\Delta\lambda$ = shift in wavelength (= observed λ − emitted λ) (m)
λ = emitted (rest-frame) wavelength (m)
Δf = shift in frequency (Hz)
v = recession speed of the source (m s^{-1})

Redshift as Evidence for Expansion

- Observations of distant galaxies show that their spectral lines are **all redshifted**.
- The greater the distance of a galaxy, the greater its redshift.
- This indicates that distant galaxies are moving **away** from us, and the further away they are, the faster they recede.
- This is consistent with the **Universe expanding**: it is space itself that is stretching, carrying galaxies apart, rather than the galaxies moving through space.

Hubble's Law and the Big Bang Theory

Hubble's Law

$$v \approx H_0\, d$$

v = recession speed of a galaxy (m s^{-1})
H_0 = Hubble constant (s^{-1}, though often quoted in km s^{-1} Mpc^{-1})
d = distance of the galaxy from Earth (m)

In CIE examinations, SI units are used: H_0 in s^{-1} and d in metres.

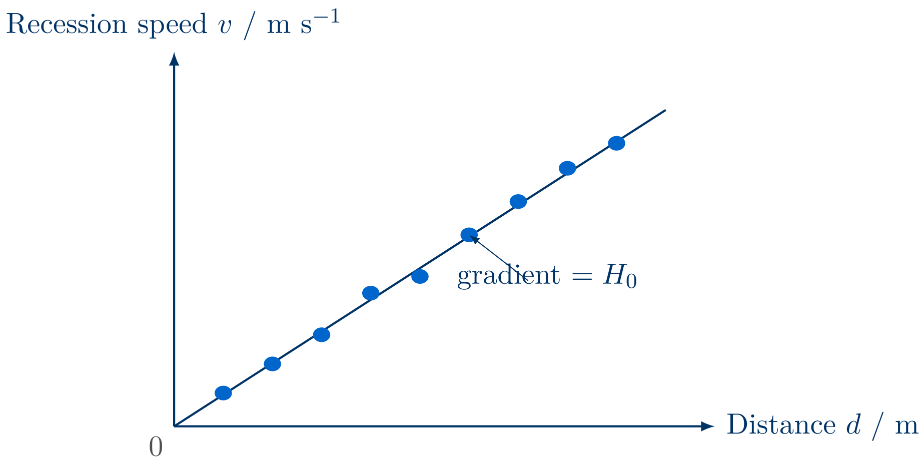
Hubble's Law and the Big Bang Theory

- Hubble's law states that the recession speed of a galaxy is proportional to its distance.
- If all galaxies are currently moving apart, then in the past they must have been **closer together**.
- Extrapolating back in time, all matter was once concentrated in an extremely hot, dense state — the **Big Bang**.
- An estimate of the **age of the Universe** can be obtained from:

$$t \approx \frac{1}{H_0}$$

(This assumes a constant rate of expansion, which is a simplification.)

Hubble Plot: Recession Speed vs Distance



Determining H_0 from a Graph

A graph of recession speed v against distance d gives a straight line through the origin. The **gradient** of this line is the Hubble constant H_0. In practice, there is significant scatter due to the difficulty of measuring distances to distant galaxies accurately.

Formula Summary Sheet

Formula	Quantity	Units
$F = L/(4\pi d^2)$	Inverse square law / flux	W m^{-2}
$\lambda_{max}T = b$	Wien's displacement law	m K
$L = 4\pi r^2\sigma T^4$	Stefan–Boltzmann law	W
$\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$	Redshift formula	—
$v \approx H_0 d$	Hubble's law	m s^{-1}, m
$t \approx 1/H_0$	Age of Universe estimate	s

Constants: $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4}, $b = 2.90 \times 10^{-3}$ m K, $c = 3.00 \times 10^{8}$ m s^{-1}

Exam Technique and Problem-Solving Strategy

Step-by-Step for Stellar Radius Problems

1. Find T: measure λ_{max} from spectrum $\Rightarrow T = b/\lambda_{max}$.
2. Find L: use standard candle or parallax to get d; then $L = 4\pi d^2 F$.
3. Find r: rearrange Stefan–Boltzmann: $r = \sqrt{L/(4\pi\sigma T^4)}$.

Common Errors — Avoid These!

- Forgetting the 4π in both $F = L/4\pi d^2$ and $L = 4\pi r^2\sigma T^4$.
- Confusing **luminosity** (intrinsic power, independent of distance) with **flux intensity** (observed brightness, depends on distance).
- Using $\Delta\lambda = \lambda_{obs} - \lambda_{emitted}$ but getting the **sign wrong**: for a receding source, $\Delta\lambda > 0$ (observed wavelength is longer).
- Applying $v = H_0 d$ with d in Mpc and H_0 in km s^{-1} Mpc^{-1} — in CIE exams **always convert to SI** (metres and s^{-1}).
- Stating that the Big Bang means galaxies are moving **through space** — more accurately, **space itself is expanding**.

Worked Examples

Example 1 — Stellar Radius from Wien and Stefan–Boltzmann

Question: A star has a peak emission wavelength of 480 nm and a luminosity of 5.2×10^{26} W. Estimate its radius. ($b = 2.90 \times 10^{-3}$ m K, $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4})

Solution

Solution:
Step 1 — Surface temperature:

$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3}}{480 \times 10^{-9}} = 6040 \text{ K}$$

Step 2 — Rearrange Stefan–Boltzmann:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times (6040)^4}}$$

$$T^4 = (6.04 \times 10^3)^4 = 1.33 \times 10^{15} \text{ K}^4$$

$$r = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times 1.33 \times 10^{15}}} = \sqrt{\frac{5.2 \times 10^{26}}{9.47 \times 10^{8}}} = \sqrt{5.49 \times 10^{17}} = \mathbf{7.4 \times 10^{8}} \text{ m}$$

Example 2 — Distance Using Flux and Luminosity

Question: A type Ia supernova has a peak luminosity of 2.0×10^{36} W. It is observed with a radiant flux intensity of 3.5×10^{-14} W m^{-2}. Calculate its distance.

Solution

Solution:

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{2.0 \times 10^{36}}{4\pi \times 3.5 \times 10^{-14}}}$$

$$d = \sqrt{\frac{2.0 \times 10^{36}}{4.40 \times 10^{-13}}} = \sqrt{4.55 \times 10^{48}} = \mathbf{2.1 \times 10^{24}} \text{ m}$$

Example 3 — Recession Speed and Age of Universe

Question: A galaxy shows a spectral line at 656 nm that is observed at 689 nm. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}. Calculate (a) the recession speed of the galaxy, (b) its distance, and (c) an estimate of the age of the Universe.

Solution

Solution:
(a) $\dfrac{\Delta\lambda}{\lambda} = \dfrac{689 - 656}{656} = \dfrac{33}{656} = 0.0503$
$v = 0.0503 \times c = 0.0503 \times 3.00 \times 10^{8} = \mathbf{1.51 \times 10^{7}}$ m s^{-1}

(b) $d = v/H_0 = 1.51 \times 10^7/2.2 \times 10^{-18} = \mathbf{6.9 \times 10^{24}}$ m
(c) $t \approx 1/H_0 = 1/(2.2 \times 10^{-18}) = \mathbf{4.5 \times 10^{17}}$ s ($\approx$ 14 billion years)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) luminosity and (b) radiant flux intensity. State the relationship between them and the distance d to the source.

[4 marks]

Q2. The Sun has a surface temperature of 5800 K. Calculate the peak wavelength of its emission spectrum and state what colour this corresponds to.

[2 marks]

Q3. Explain what is meant by a *standard candle* and describe one example of a standard candle used in astronomy.

[3 marks]

Q4. A hydrogen spectral line has a rest wavelength of 434 nm. In the spectrum of a distant galaxy it is observed at 461 nm. Calculate the recession speed of the galaxy.

[3 marks]

Q5. Explain how observations of redshift from distant galaxies lead to the conclusion that the Universe is expanding.

[3 marks]

Section B — Longer Structured Questions

Q6. Star A has surface temperature 12 000 K and radius 3.5×10^9 m. Star B has the same luminosity as Star A but a surface temperature of 4500 K.

(a) Calculate the luminosity of Star A.

[2 marks]

(b) Calculate the radius of Star B.

[3 marks]

(c) State which star would appear bluer and explain why.

[2 marks]

Q7. A galaxy is observed to have a recession speed of 4.8×10^6 m s^{-1}. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}.

(a) Calculate the distance of the galaxy.

[2 marks]

(b) A spectral line in the galaxy's spectrum has an emitted wavelength of 589 nm. Calculate the observed wavelength.

[2 marks]

(c) Explain how Hubble's law provides evidence for the Big Bang theory.

[3 marks]

Mark Scheme and Answers

Q1. (a) Luminosity: the total power of electromagnetic radiation emitted by a star (in all directions) [2]. (b) Radiant flux intensity: the power of radiation received per unit area at the observer's location [1]; $F = L/(4\pi d^2)$ [1].

Q2. $\lambda_{\text{max}} = b/T = 2.90 \times 10^{-3}/5800 = \mathbf{500}$ nm [1]; this corresponds to green light (near the centre of the visible spectrum) [1].

Q3. A standard candle is an astronomical object of known luminosity [1]; example: type Ia supernova (all reach the same peak luminosity) [1]; or Cepheid variable (period of brightness variation gives luminosity via period–luminosity relation) [1].

Q4. $\Delta\lambda = 461 - 434 = 27$ nm [1]; $v = c\Delta\lambda/\lambda = 3.00 \times 10^8 \times 27/434$ [1] $= \mathbf{1.87 \times 10^7}$ m s^{-1} [1].

Q5. Spectral lines from distant galaxies are observed at longer wavelengths (redshifted) than expected [1]; this indicates the galaxies are moving away from us [1]; the greater the distance, the greater the redshift/recession speed — consistent with all space expanding uniformly [1].

Q6(a). $L = 4\pi r^2 \sigma T^4 = 4\pi \times (3.5 \times 10^9)^2 \times 5.67 \times 10^{-8} \times (12000)^4$ [1]; $= 4\pi \times 1.225 \times 10^{19} \times 5.67 \times 10^{-8} \times 2.07 \times 10^{16} = \mathbf{1.80 \times 10^{29}}$ W [1].

Q6(b). Same L; $r_B^2 T_B^4 = r_A^2 T_A^4$ [1]; $r_B = r_A(T_A/T_B)^2 = 3.5 \times 10^9 \times (12000/4500)^2 = 3.5 \times 10^9 \times 7.11$ [1] $= \mathbf{2.49 \times 10^{10}}$ m [1].

Q6(c). Star A is bluer [1]; it has a higher surface temperature, so by Wien's law its peak wavelength is shorter (towards the blue end of the spectrum) [1].

Q7(a). $d = v/H_0 = 4.8 \times 10^6 / 2.2 \times 10^{-18} = \mathbf{2.18 \times 10^{24}}$ m [2].

Q7(b). $\Delta\lambda = \lambda v/c = 589 \times 10^{-9} \times 4.8 \times 10^6 / 3.00 \times 10^8 = 9.42 \times 10^{-9}$ m [1]; observed $\lambda = 589 + 9.4 = \mathbf{598}$ nm [1].

Q7(c). Hubble's law shows recession speed is proportional to distance — the further away a galaxy, the faster it recedes [1]; this means all galaxies are moving apart from one another, implying the Universe is expanding [1]; extrapolating back in time, all matter must have originated from a single point — the Big Bang [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define luminosity and radiant flux intensity	
☐	Use the inverse square law $F = L/(4\pi d^2)$	
☐	Define a standard candle and explain how it is used to find distance	
☐	Give examples of standard candles (Cepheid variables, Type Ia supernovae)	
☐	State and apply Wien's displacement law $\lambda_{\max}T = b$	
☐	State and apply the Stefan–Boltzmann law $L = 4\pi r^2\sigma T^4$	
☐	Combine Wien and Stefan–Boltzmann to estimate stellar radius	
☐	Explain what redshift is and how it is observed in galaxy spectra	
☐	Use $\Delta\lambda/\lambda \approx v/c$ to find recession speeds	
☐	Explain how redshift provides evidence for an expanding Universe	
☐	State and apply Hubble's law $v \approx H_0 d$ (SI units)	
☐	Use $t \approx 1/H_0$ as an estimate for the age of the Universe	
☐	Explain how Hubble's law leads to the Big Bang theory	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Cosmology asks the biggest questions in physics — and answers them with the same tools you've used all year. A few formulas, careful unit conversions, and the ability to interpret a graph are all you need to explore the scale of the Universe itself.