CIE A-Level Physics

Revision Booklet

This booklet covers:

CIE International A Level Physics

Topic 1

Physical Quantities and Units

Revision Booklet

This booklet covers:

- Physical Quantities and Estimation
- SI Base Units and Derived Units
- Homogeneity of Equations
- SI Prefixes
- Errors, Uncertainties, Precision and Accuracy
- Scalars and Vectors

Physical Quantities

Physical Quantity

Every **physical quantity** consists of two parts:

$$\text{physical quantity} = \text{numerical magnitude} \times \text{unit}$$

For example: a length of 3.5 m has magnitude 3.5 and unit metres. A quantity without a unit (or with units that cancel) is **dimensionless**.

Reasonable Estimates

The syllabus requires you to make sensible estimates of physical quantities. Useful benchmarks to memorise:

Quantity	**Estimate**	**Notes**
Mass of a person	70 kg	
Height of a person	1.7 m	
Mass of a car	1000 kg	
Speed of a car on motorway	30 m s^{-1}	($\approx 110 \text{ km h}^{-1}$)
Speed of sound in air	340 m s^{-1}	
Speed of light	$3 \times 10^8 \text{ m s}^{-1}$	
Atmospheric pressure	1×10^5 Pa	
Density of water	1000 kg m^{-3}	
Density of air	1.2 kg m^{-3}	
Diameter of an atom	$\sim 10^{-10}$ m	
Diameter of a nucleus	$\sim 10^{-15}$ m	

SI Base Units

SI Base Quantities and Units

The International System of Units (SI) defines **seven base quantities**. The five required for this syllabus are:

Base quantity	SI unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K

Derived Units

All other units are **derived** from the base units by multiplication or division. Examples:

Quantity	Derived unit	In base units
Force	newton (N)	$\mathrm{kg\ m\ s^{-2}}$
Energy / Work	joule (J)	$\mathrm{kg\ m^{2}\ s^{-2}}$
Power	watt (W)	$\mathrm{kg\ m^{2}\ s^{-3}}$
Pressure	pascal (Pa)	$\mathrm{kg\ m^{-1}\ s^{-2}}$
Charge	coulomb (C)	A s
Potential difference	volt (V)	$\mathrm{kg\ m^{2}\ s^{-3}\ A^{-1}}$
Resistance	ohm (Ω)	$\mathrm{kg\ m^{2}\ s^{-3}\ A^{-2}}$
Frequency	hertz (Hz)	$\mathrm{s^{-1}}$

Homogeneity of Physical Equations

Homogeneity

A physical equation is **homogeneous** if the units (dimensions) on both sides are identical. Every valid physical equation must be homogeneous.
Method: express every quantity in SI base units, then check both sides match.

Using Homogeneity to Check Equations

1. Replace each symbol with its SI base units.
2. Simplify both sides independently.
3. If both sides have the same base units $\Rightarrow$ equation is **homogeneous** (possibly correct).
4. If the sides differ $\Rightarrow$ equation is **definitely wrong**.

Important caveat: homogeneity is a necessary but not sufficient condition. A homogeneous equation can still be wrong (e.g. a missing numerical factor of 2 would not be detected).

Worked Example — Homogeneity Check

Check whether $v^2 = u^2 + 2as$ is homogeneous.
LHS: $[v^2] = (\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$
RHS: $[u^2] = \text{m}^2\ \text{s}^{-2}$; $[2as] = (\text{m s}^{-2})(\text{m}) = \text{m}^2\ \text{s}^{-2}$
Both terms on the RHS and the LHS have units $\text{m}^2\ \text{s}^{-2} \Rightarrow$ equation is **homogeneous**. ✓

SI Prefixes

Prefix	Symbol	Multiplier	Power of 10
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}

Common Prefix Errors

- $1\ \text{nm} = 1 \times 10^{-9}$ m, **not** 10^{-6} m — don't confuse nano (n) with micro (μ).
- When squaring or cubing a unit with a prefix, the prefix is also raised to that power: $1\ \text{cm}^2 = (10^{-2}\ \text{m})^2 = 10^{-4}\ \text{m}^2$, not $10^{-2}\ \text{m}^2$.
- $1\ \text{mm}^3 = (10^{-3}\ \text{m})^3 = 10^{-9}\ \text{m}^3$.

Errors, Uncertainties, Precision and Accuracy

Types of Error

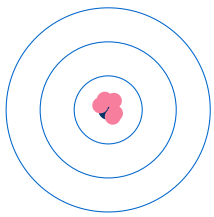
- **Systematic error**: an error that affects all readings by the same amount in the same direction. The mean of repeated readings is shifted from the true value. Cannot be reduced by repeating measurements.
  - Examples: zero error on an instrument; wrongly calibrated scale; parallax error if always reading from the same angle.
- **Random error**: an error that causes readings to scatter unpredictably above and below the true value. Can be reduced by taking more readings and averaging.
  - Examples: reaction time variation; reading a scale to the nearest division; electrical noise.
- **Zero error**: a specific systematic error where an instrument reads non-zero when it should read zero. Corrected by subtracting the zero reading from all measurements.

Precision and Accuracy

- **Precision**: how close repeated measurements are to *each other*. A precise instrument gives small random errors (small spread). Precision is about **repeatability**.
- **Accuracy**: how close a measurement is to the *true value*. An accurate instrument has small systematic error. Accuracy is about **closeness to truth**.

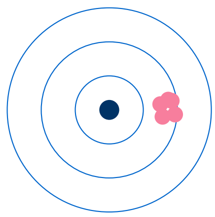
It is possible to be precise but inaccurate (systematic error shifts all readings the same way) or accurate but imprecise (readings scattered around the true value).

Precise, accurate



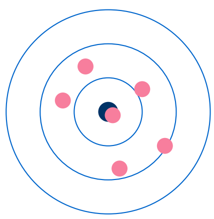
small random, small systematic

Precise, not accurate



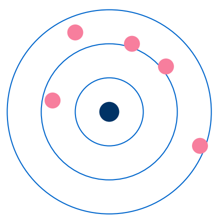
small random, large systematic

Not precise, accurate



large random, small systematic

Not precise, not accurate



large random, large systematic

Combining Uncertainties

Operation	**Rule for uncertainty**
$z = x + y$ or $z = x - y$	$\Delta z = \Delta x + \Delta y$ (add **absolute** uncertainties)
$z = xy$ or $z = x/y$	$\dfrac{\Delta z}{z} = \dfrac{\Delta x}{x} + \dfrac{\Delta y}{y}$ (add **percentage** uncertainties)
$z = x^n$	$\dfrac{\Delta z}{z} = \lvert n\rvert \dfrac{\Delta x}{x}$ (multiply percentage uncertainty by $\lvert n\rvert$)

Absolute uncertainty Δx: has the same unit as x.
Percentage uncertainty: $\dfrac{\Delta x}{x} \times 100\%$ — dimensionless.

Common Uncertainty Mistakes

- For **addition/subtraction**, always add *absolute* uncertainties — never subtract.
- For **powers**: $z = x^2$ gives $\Delta z/z = 2\,\Delta x/x$ — the power doubles the percentage uncertainty.
- A constant with no uncertainty (e.g. π, 2, g if treated as exact) does not contribute to the uncertainty.
- When reading an analogue scale, the uncertainty is typically $\pm$ half the smallest division.

Worked Example — Combining Uncertainties

A cylinder has radius $r = (1.50 \pm 0.02)$ cm and height $h = (4.00 \pm 0.05)$ cm. Calculate the volume and its percentage uncertainty.
$V = \pi r^2 h = \pi(1.50)^2(4.00) = 28.3\ \text{cm}^3$
Percentage uncertainties:

$$\frac{\Delta r}{r} = \frac{0.02}{1.50} = 1.33\%$$
$$\frac{\Delta h}{h} = \frac{0.05}{4.00} = 1.25\%$$
$$\frac{\Delta V}{V} = 2 \times 1.33\% + 1.25\% = 3.91\% \approx \mathbf{3.9}\%$$

(r is squared so its percentage uncertainty is doubled; π contributes nothing.)
Absolute uncertainty: $\Delta V = 0.039 \times 28.3 = 1.1\ \text{cm}^3$
$\therefore V = (28.3 \pm 1.1)\ \text{cm}^3$

Scalars and Vectors

Scalars and Vectors

- A **scalar** quantity has **magnitude only**.
- A **vector** quantity has both **magnitude and direction**.

Scalars	**Vectors**
distance, speed	displacement, velocity
mass, density	acceleration, force, weight
energy, power, work	momentum, impulse
temperature, pressure	electric field strength
time, frequency	gravitational field strength

Adding Vectors

Vectors must be added **tip-to-tail**. The resultant $\vec{R} = \vec{A} + \vec{B}$ is the single vector drawn from the tail of $\vec{A}$ to the tip of $\vec{B}$.
For two vectors at an angle θ to each other, the magnitude of the resultant:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

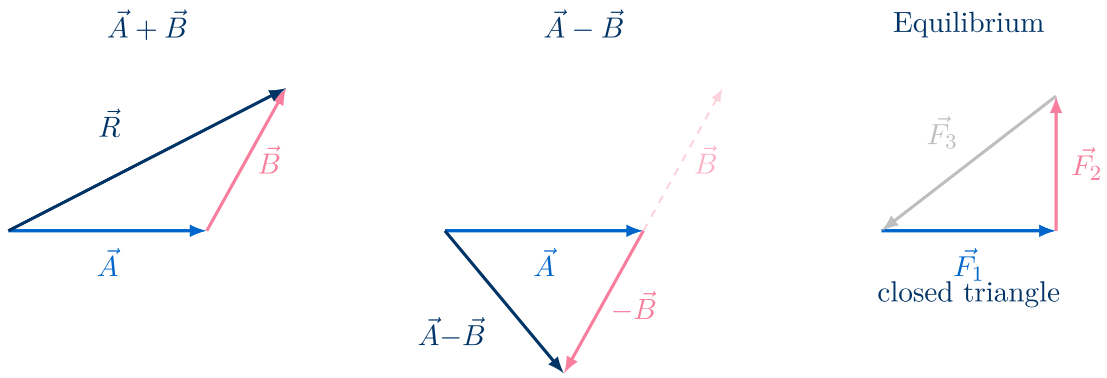
For **perpendicular** vectors ($\theta = 90°$): $R = \sqrt{A^2 + B^2}$, direction $= \arctan(B/A)$.

Subtracting Vectors

To find $\vec{A} - \vec{B}$, reverse the direction of $\vec{B}$ to get $-\vec{B}$, then add tip-to-tail:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

- $-\vec{B}$ has the **same magnitude** as $\vec{B}$ but **opposite direction**.
- This is used whenever you need a **change in a vector quantity**, e.g. change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.
- For perpendicular vectors: $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2}$ (same magnitude as addition, different direction).



Change in a Vector Quantity

A common application of vector subtraction is finding the **change in velocity** $\Delta\vec{v}$:

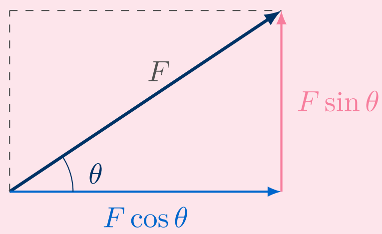
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$

Draw $\vec{v}_1$ and $\vec{v}_2$ from the same point. $\Delta\vec{v}$ is the vector from the tip of $\vec{v}_1$ to the tip of $\vec{v}_2$. *Example*: a ball bouncing off a wall at the same speed. Although $|\vec{v}_1| = |\vec{v}_2|$, the direction has changed so $\Delta\vec{v} \neq 0$ — there is a non-zero acceleration (and therefore a resultant force).

Resolving a Vector into Components

Any vector $\vec{F}$ at angle θ to the horizontal can be split into two perpendicular components:

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$



Coplanar Forces in Equilibrium — Vector Triangle

Three coplanar forces in equilibrium can be represented by a **closed triangle**: draw the vectors tip-to-tail; if they form a closed triangle (the last tip meets the first tail), the system is in equilibrium. This provides a graphical method for finding unknown force magnitudes or directions.

Formula Summary Sheet

Formula / Rule	Use
$z = A + B \Rightarrow \Delta z = \Delta A + \Delta B$	Add/subtract: add absolute uncertainties
$z = AB \Rightarrow \Delta z/z = \Delta A/A + \Delta B/B$	Multiply/divide: add percentage uncertainties
$z = A^n \Rightarrow \Delta z/z = \|n\| \Delta A/A$	Powers: multiply % uncertainty by $\|n\|$
$F_x = F\cos\theta$	Horizontal component of vector
$F_y = F\sin\theta$	Vertical component of vector
$R = \sqrt{A^2 + B^2}$	Resultant of two perpendicular vectors

Key facts:
SI base units: kg, m, s, A, K.
Systematic error: shifts all readings same way; not reduced by repeating.
Random error: causes scatter; reduced by averaging more readings.
Precision: repeatability (small spread). **Accuracy**: closeness to true value.
Homogeneity: units must match on both sides of any valid equation.

Worked Examples

Example 1 — Derived Units in Base Units

Question: Show that the unit of pressure (Pa) is equivalent to kg m^{-1} s^{-2}.

Solution

Pressure = Force / Area. Force has units N = kg m s^{-2}. Area has units m^2.

$$[\text{Pa}] = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\text{ s}^{-2} \quad \checkmark$$

Example 2 — Homogeneity Check

Question: The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$. Check this equation for homogeneity.

Solution

LHS: $[T] = \text{s}$
RHS: 2π is dimensionless. $[L/g] = \text{m}/(\text{m s}^{-2}) = \text{s}^2$
$[\sqrt{L/g}] = \sqrt{\text{s}^2} = \text{s}$
Both sides have units of seconds $\Rightarrow$ equation is **homogeneous**. ✓

Example 3 — Combining Uncertainties

Question: The resistance of a wire is found using $R = V/I$, where $V = (6.0 \pm 0.2)$ V and $I = (0.30 \pm 0.01)$ A. Calculate R and its percentage uncertainty.

Solution

$R = V/I = 6.0/0.30 = \mathbf{20\ \Omega}$
Since this is division, add percentage uncertainties:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.2}{6.0} + \frac{0.01}{0.30} = 3.33\% + 3.33\% = \mathbf{6.7}\%$$

Absolute uncertainty: $\Delta R = 0.067 \times 20 = 1.3\ \Omega$
$R = (20 \pm 1)\ \Omega$

Example 4 — Resolving and Adding Vectors

Question: Two forces act on a point: $F_1 = 8.0$ N horizontally and $F_2 = 6.0$ N vertically upward. Find the magnitude and direction of the resultant.

Solution

Since the forces are perpendicular:

$$R = \sqrt{F_1^2 + F_2^2} = \sqrt{8.0^2 + 6.0^2} = \sqrt{64 + 36} = \sqrt{100} = \mathbf{10}\text{ N}$$

Direction above the horizontal:

$$\theta = \arctan\left(\frac{F_2}{F_1}\right) = \arctan\left(\frac{6.0}{8.0}\right) = \arctan(0.75) = \mathbf{36.9}^\circ$$

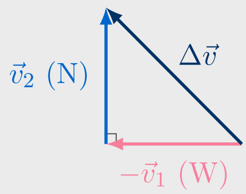
Resultant: 10 N at 36.9° above the horizontal.

Example 5 — Vector Subtraction (Change in Velocity)

Question: A ball of mass 0.20 kg travels at 6.0 m s^{-1} due East. It then travels at 6.0 m s^{-1} due North. Find (a) the magnitude and direction of the change in velocity $\Delta\vec{v}$, and (b) the magnitude of the average force if this change takes 0.30 s.

Solution

(a) $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. Reverse $\vec{v}_1$ to get $-\vec{v}_1 = 6.0$ m s^{-1} due West, then add $\vec{v}_2 = 6.0$ m s^{-1} due North tip-to-tail:



$$|\Delta\vec{v}| = \sqrt{6.0^2 + 6.0^2} = \sqrt{72} = \mathbf{8.49}\ \text{m s}^{-1}$$

Direction: $\arctan(6.0/6.0) = 45^\circ$ North of West (i.e. North-West).

(b) $F = \dfrac{m\,|\Delta\vec{v}|}{\Delta t} = \dfrac{0.20 \times 8.49}{0.30} = \mathbf{5.7}$ N (directed North-West).

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the five SI base quantities required for this syllabus and give the name and symbol of each unit.

[5 marks]

Q2. Show that the unit of the Young modulus (stress / strain) is equivalent to Pa, and express this in SI base units.

[3 marks]

Q3. Distinguish between systematic and random errors. Explain how each can be reduced or identified.

[4 marks]

Q4. Explain the difference between precision and accuracy. A thermometer consistently reads 0.5 °C too high. State whether this represents a systematic or random error, and whether the readings are precise, accurate, or both.

[4 marks]

Section B — Longer Structured Questions

Q5. The speed v of a wave on a string is given by $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.

(a) Check the equation for homogeneity by expressing both sides in SI base units.

[3 marks]

(b) In an experiment, $T = (4.5 \pm 0.2)$ N and $\mu = (8.0 \pm 0.4) \times 10^{-3}$ kg m^{-1}. Calculate v and its percentage uncertainty.

[4 marks]

Q6. A ship travels 40 km due North, then 30 km due East.

(a) Calculate the magnitude of the resultant displacement.

[2 marks]

(b) Calculate the direction of the resultant displacement as a bearing.

[2 marks]

(c) State the difference between the total distance travelled and the magnitude of displacement. What type of quantity (scalar or vector) is each?

[2 marks]

Q7. A force of 50 N acts at 30° above the horizontal.

(a) Resolve the force into its horizontal and vertical components.

[2 marks]

(b) A second force of 20 N acts horizontally in the same direction. Find the magnitude and direction of the resultant of the two forces.

[3 marks]

(c) A third force is added so that the system of three forces is in equilibrium. Describe this third force.

[2 marks]

Mark Scheme and Answers

Q1. Mass/kilogram/kg [1]; length/metre/m [1]; time/second/s [1]; electric current/ampere/A [1]; temperature/kelvin/K [1].

Q2. Stress = force/area $\Rightarrow$ N m^{-2} = Pa [1]. Strain = extension/length $\Rightarrow$ dimensionless [1]. Young modulus = Pa = kg m^{-1} s^{-2} [1].

Q3. Systematic: affects all readings the same way [1]; identified by comparing with a known standard; reduced by recalibration or improved technique [1]. Random: causes scatter above and below true value [1]; reduced by taking more readings and averaging [1].

Q4. Precision: how repeatable/consistent readings are [1]; accuracy: how close readings are to the true value [1]. Constant offset of +0.5°C $\Rightarrow$ **systematic error** [1]; readings are **precise** (consistent with each other) but **not accurate** (all shifted from true value) [1].

Q5(a). LHS: $[v] = \text{m s}^{-1}$ [1]. RHS: $[T/\mu] = \text{N}/(\text{kg m}^{-1}) = \text{kg m s}^{-2}/(\text{kg m}^{-1}) = \text{m}^2\ \text{s}^{-2}$ [1]; $[\sqrt{T/\mu}] = \text{m s}^{-1} \Rightarrow$ homogeneous ✓[1].

Q5(b). $v = \sqrt{4.5/(8.0 \times 10^{-3})} = \sqrt{562.5} = \mathbf{23.7}\ \mathbf{m\ s^{-1}}$ [1].
% unc in T: $0.2/4.5 = 4.44\%$; % unc in μ: $0.4/8.0 = 5.00\%$ [1].
% unc in $v = \frac{1}{2}(4.44 + 5.00)\% = \frac{1}{2}(9.44)\% = \mathbf{4.7}\%$ [2] (halved because of square root).

Q6(a). $R = \sqrt{40^2 + 30^2} = \sqrt{2500} = \mathbf{50}$ km [2].

Q6(b). $\theta = \arctan(30/40) = 36.9°$ East of North $\Rightarrow$ bearing = **037°** [2].

Q6(c). Distance = $40 + 30 = 70$ km (scalar) [1]; displacement = 50 km at 037° (vector) [1].

Q7(a). $F_x = 50\cos 30° = \mathbf{43.3}$ N (horizontal) [1]; $F_y = 50\sin 30° = \mathbf{25}$ N (vertical) [1].

Q7(b). Total horizontal = $43.3 + 20 = 63.3$ N; total vertical = 25 N [1]. $R = \sqrt{63.3^2 + 25^2} = \sqrt{4007 + 625} = \sqrt{4632} = \mathbf{68.1}$ N [1]; $\theta = \arctan(25/63.3) = \mathbf{21.5}°$ above horizontal [1].

Q7(c). Third force must be equal in magnitude (68.1 N) [1] and opposite in direction (21.5° below horizontal, pointing left) to the resultant of the first two [1].

Revision Checklist

	Learning Objective	Confidence (1–3)
☐	State that a physical quantity = magnitude × unit	
☐	Make reasonable estimates of common physical quantities	
☐	Recall the five SI base quantities and their units (kg, m, s, A, K)	
☐	Express derived units in SI base units (N, J, W, Pa, V, Ω, C)	
☐	Use base units to check homogeneity of an equation	
☐	Recall all ten SI prefixes (T, G, M, k, d, c, m, μ, n, p) with values	
☐	Convert correctly when units are raised to powers (e.g. cm^2 to m^2)	
☐	Distinguish systematic and random errors with examples	
☐	Define and distinguish precision and accuracy	
☐	Combine absolute uncertainties for addition/subtraction	
☐	Combine percentage uncertainties for multiplication/division/powers	
☐	Distinguish scalars and vectors; give examples of each	
☐	Add and subtract coplanar vectors (tip-to-tail / component method)	
☐	Subtract vectors by reversing direction of the second vector ($\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$)	
☐	Use vector subtraction to find the change in a vector quantity (e.g. $\Delta\vec{v}$)	
☐	Resolve a vector into two perpendicular components ($F\cos\theta$, $F\sin\theta$)	
☐	Use a closed vector triangle to represent coplanar forces in equilibrium	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Topic 1 underpins everything else in the course. Get comfortable with unit conversions, uncertainty rules and vector resolution now — you will use all three in every other topic.

CIE International A Level Physics

Topic 2

Kinematics

Revision Booklet

This booklet covers:

- Distance, Displacement, Speed, Velocity and Acceleration
- Graphs of Motion
- Equations of Uniform Acceleration (SUVAT)
- Free Fall and Measuring g
- Projectile Motion

Core Definitions

Scalar and Vector Quantities

- **Distance** (scalar): the total length of path travelled, regardless of direction. Units: m.
- **Displacement** s (vector): the straight-line distance from start to finish *in a specified direction*. Units: m.
- **Speed** (scalar): distance travelled per unit time. Units: $\mathrm{m\,s^{-1}}$.
- **Velocity** v (vector): rate of change of displacement. Units: $\mathrm{m\,s^{-1}}$.
- **Acceleration** a (vector): rate of change of velocity. Units: $\mathrm{m\,s^{-2}}$.

Defining Equations

$$v = \frac{\Delta s}{\Delta t} \qquad\qquad a = \frac{\Delta v}{\Delta t}$$

Δs = change in displacement (m)
Δv = change in velocity ($\mathrm{m\,s^{-1}}$)
Δt = time interval (s)

Distance vs Displacement

A car that travels 400 m north then 400 m south has a total **distance** of 800 m but a **displacement** of zero. Always identify whether the question asks for a scalar or vector quantity before calculating.

Graphs of Motion

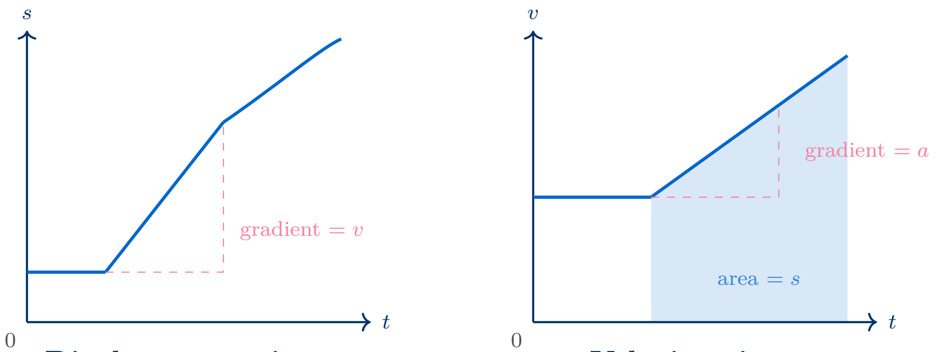
Reading Motion Graphs

Displacement–time (s–t) graph:

- Gradient = instantaneous velocity.
- Horizontal line ⇒ stationary.
- Straight line ⇒ constant (uniform) velocity.
- Curve ⇒ changing velocity (acceleration present).

Velocity–time (v–t) graph:

- Gradient = instantaneous acceleration.
- Area under the graph = displacement.
- Horizontal line ⇒ constant velocity (zero acceleration).
- Straight line ⇒ uniform acceleration.



Displacement–time **Velocity–time**

Equations of Uniform Acceleration (SUVAT)

Derivation

For **constant** acceleration, starting from the definitions of velocity and acceleration:

1. From $a = \Delta v/\Delta t$: $a = (v-u)/t \quad \Rightarrow \quad \boxed{v = u + at}$
2. Mean velocity × time: $s = \dfrac{u+v}{2} \times t \quad \Rightarrow \quad \boxed{s = \frac{1}{2}(u+v)t}$
3. Substitute equation (1) into (2): $s = \dfrac{u+u+at}{2}t \quad \Rightarrow \quad \boxed{s = ut + \frac{1}{2}at^2}$
4. Eliminate t: from (1), $t = (v-u)/a$; substitute into (2): $\Rightarrow \quad \boxed{v^2 = u^2 + 2as}$

The SUVAT Equations

Equation	Variable not present
$v = u + at$	s
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t
$s = \frac{1}{2}(u + v)t$	a

s = displacement (m) u = initial velocity (m s^{-1})
v = final velocity (m s^{-1}) a = acceleration (m s^{-2}) t = time (s)

*Valid only for **uniform** (constant) acceleration in a straight line.*

SUVAT Problem Strategy

1. Write down the five variables: s, u, v, a, t.
2. Fill in the known values; identify what you are finding.
3. Choose the equation that contains those four variables (three known + one unknown).
4. Solve; check units and sign conventions.

Positive direction must be defined clearly at the start — be consistent throughout.

Free Fall and Measuring g

Free Fall

Free fall is the motion of an object under gravity alone, with no air resistance. The object accelerates uniformly downwards at the **acceleration of free fall**:

$$g = 9.81\ \text{m s}^{-2} \quad \text{(near Earth's surface)}$$

Free fall is uniform acceleration, so all four SUVAT equations apply with $a = g$.

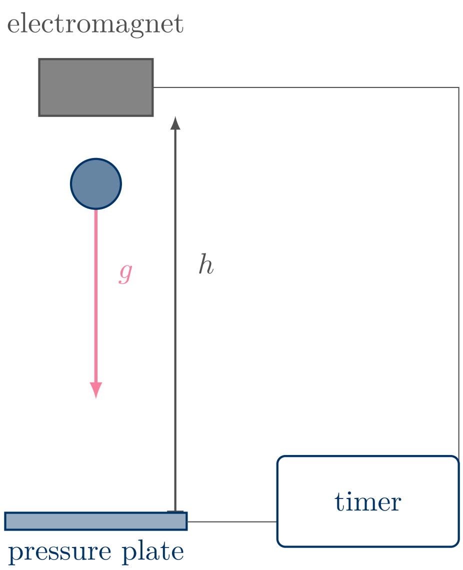
Experiment: Determining g Using a Falling Object

Method — Electromagnetic Release and Trapdoor Timer

1. A steel ball is held by an electromagnet at a measured height h above a trapdoor switch.
2. The circuit is broken simultaneously: the ball is released and a millisecond timer starts.
3. When the ball hits the pressure plate the timer stops, giving the time of fall t.
4. Use $h = \frac{1}{2}gt^2$ (with $u = 0$) to find g:

$$g = \frac{2h}{t^2}$$

5. Repeat for several values of h; plot h against t^2 — the gradient equals $g/2$.



Sources of Uncertainty and Improvements

- **Reaction time**: eliminated by electronic (not manual) timing.
- **Residual magnetism**: the ball may not release instantly — add a thin bit of tape between the ball and the magnet.
- **Air resistance**: use a dense, compact sphere to minimise drag.
- **Height measurement**: measure from the *bottom* of the ball to the pressure plate; use a ruler held vertically alongside.
- **Graph method**: plotting h vs t^2 and finding the gradient $= g/2$ averages over many measurements, reducing random error.

Projectile Motion

Projectile Motion

A **projectile** moves under gravity alone after launch. Its motion resolves into two **independent** components:

- **Horizontal**: uniform velocity (no force, so no acceleration).
- **Vertical**: uniform acceleration downwards at $g = 9.81\ \mathrm{m\,s^{-2}}$.

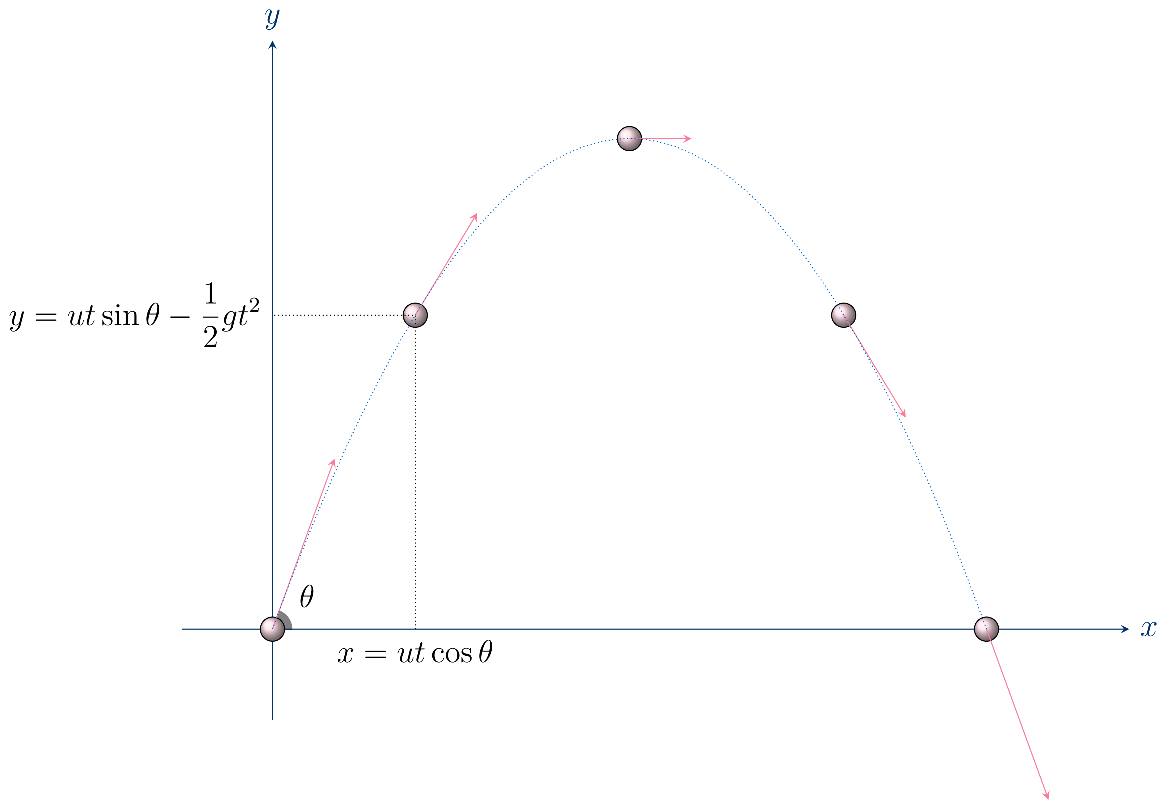
The two components share the same time t but are otherwise treated separately.

Projectile Equations

For a projectile launched with speed u at an angle θ to the horizontal :

	Horizontal	**Vertical**
Initial velocity	$u_x = ucos\theta$	$u_y = usin\theta$
Acceleration	0	g (downwards)
Velocity at time t	$v_x = ucos\theta$	$v_y = usin\theta - gt$
Displacement at time t	$x = utcos\theta$	$y = utsin\theta - \frac{1}{2}gt^2$

Resultant speed: $v = \sqrt{v_x^2 + v_y^2}$ Angle from horizontal: $\theta = \arctan\left(\frac{v_y}{v_x}\right)$



Common Mistakes in Projectile Problems

- Never mix horizontal and vertical quantities in the same SUVAT equation.
- The horizontal velocity **does not change** (no air resistance assumed).
- Time is the same for both components — find t from the vertical motion first.
- For a horizontal launch: $u_x = u$, $u_y = 0$.

Worked Examples

Example 1 — SUVAT: Braking Car

Question: A car travelling at 30 m s^{-1} brakes uniformly and stops in 4.5 s. Calculate (a) the deceleration and (b) the stopping distance.

Solution

Known: $u = 30\ \text{m s}^{-1}$, $v = 0$, $t = 4.5$ s
(a) Use $v = u + at$:

$$0 = 30 + a \times 4.5 \quad \Rightarrow \quad a = \frac{-30}{4.5} = \mathbf{-6.7}\ \mathbf{m\,s^{-2}}$$

(b) Use $s = \frac{1}{2}(u + v)t$:

$$s = \tfrac{1}{2}(30 + 0)(4.5) = \mathbf{67.5}\ \text{m}$$

Example 2 — Free Fall

Question: A stone is dropped from rest off a cliff and hits the water 3.2 s later. Calculate (a) the height of the cliff and (b) the speed on impact.

Solution

Known: $u = 0$, $a = 9.81\ \text{m s}^{-2}$, $t = 3.2$ s
(a) Use $s = ut + \frac{1}{2}at^2$:

$$s = 0 + \tfrac{1}{2}(9.81)(3.2)^2 = \tfrac{1}{2} \times 9.81 \times 10.24 = \mathbf{50.2}\ \text{m}$$

(b) Use $v = u + at$:

$$v = 0 + 9.81 \times 3.2 = \mathbf{31.4}\ \mathbf{m\,s^{-1}}$$

Example 3 — Projectile Motion

Question: A ball is launched horizontally at 15 m s^{-1} from a platform 20 m above the ground. Calculate (a) the time of flight and (b) the horizontal range.

Solution

(a) Vertical motion ($u_y = 0$, $a = 9.81\ \text{m s}^{-2}$, $s = 20$ m):

$$20 = \tfrac{1}{2}(9.81)t^2 \quad \Rightarrow \quad t^2 = \frac{20}{4.905} \quad \Rightarrow \quad t = \mathbf{2.02}\ \text{s}$$

(b) Horizontal motion ($u_x = 15\ \text{m s}^{-1}$, constant):

$$x = u_x \times t = 15 \times 2.02 = \mathbf{30.3}\ \text{m}$$

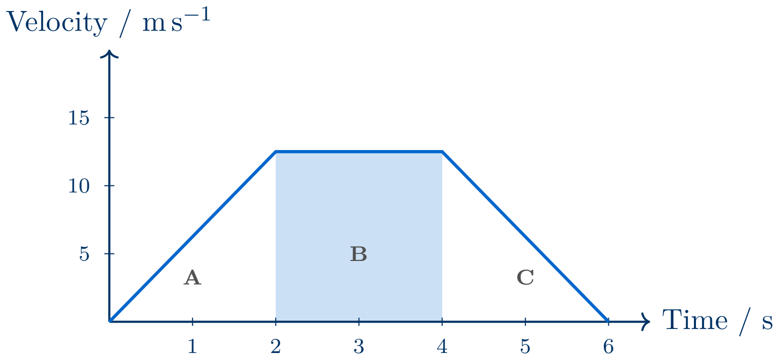
Practice Exam Questions

Section A — Short Answer Questions

Q1. Distinguish between *distance* and *displacement*, and between *speed* and *velocity*.

[4 marks]

Q2. The velocity–time graph below shows the motion of an object. Describe the motion in each labelled region and state what the shaded area represents.



[4 marks]

Q3. Define acceleration and state its SI unit.

[2 marks]

Q4. State the two conditions under which the SUVAT equations are valid.

[2 marks]

Section B — Longer Structured Questions

Q5. A student drops a ball bearing from rest and measures the time t it takes to fall a height h. The results are shown in the table below.

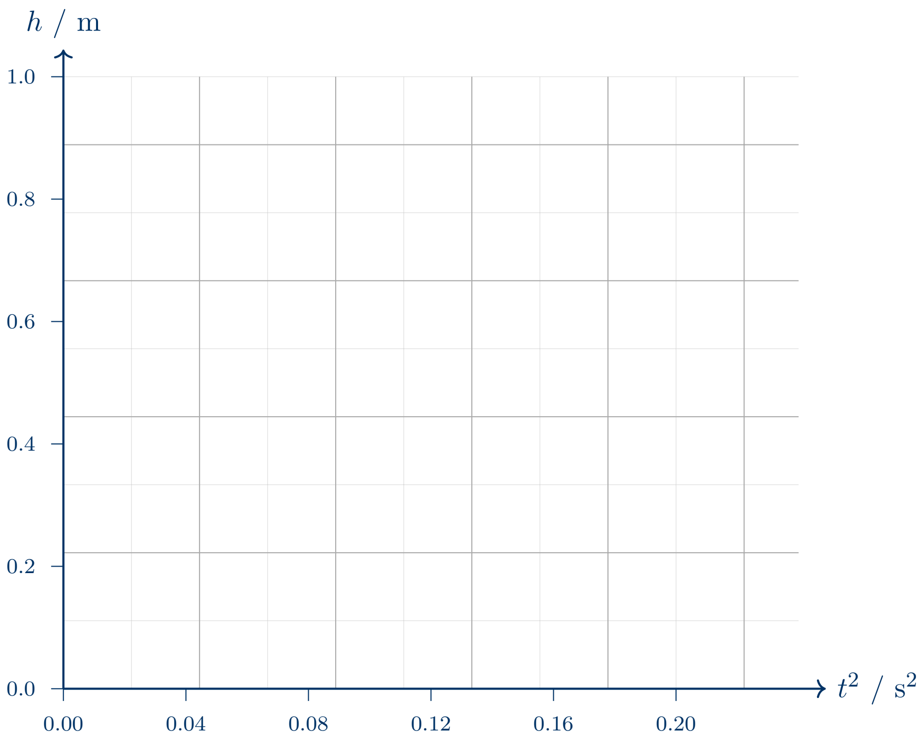
Height h **/ m**	**Time** t **/ s**	t^2 / s^2
0.20	0.202	
0.40	0.285	
0.60	0.350	
0.80	0.404	
1.00	0.451	

(a) Complete the t^2 column in the table above.

[1 mark]

(b) Plot a graph of h (y-axis) against t^2 (x-axis) on the grid below and draw a best-fit straight line.

[3 marks]



(c) Determine the gradient of your line and use it to calculate g.

[3 marks]

(d) Suggest one source of systematic error in this experiment and explain how it would affect the value of g obtained.

[2 marks]

Q6. A ball is kicked horizontally from the top of a vertical cliff at a speed of 12 $\mathrm{m\,s^{-1}}$. The ball lands 35 m from the base of the cliff.

(a) Show that the time of flight is approximately 2.9 s.

[2 marks]

(b) Calculate the height of the cliff.

[2 marks]

(c) Calculate the speed and direction of the ball just before it hits the ground.

[3 marks]

Mark Scheme and Answers

Q1. Distance is a scalar — the total length of path travelled [1]; displacement is a vector — the straight-line distance in a specified direction from start to finish [1]. Speed is the scalar rate of change of distance [1]; velocity is the vector rate of change of displacement [1].

Q2. Region A: uniform acceleration (straight line, positive gradient) [1]; Region B: constant velocity (horizontal line, zero acceleration) [1]; Region C: uniform deceleration to rest (straight line, negative gradient) [1]; shaded area = displacement during region B [1].

Q3. Acceleration is the rate of change of velocity [1]; SI unit: $\mathrm{m\,s^{-2}}$ [1].

Q4. Acceleration must be **uniform** (constant) [1]; motion must be in a **straight line** [1].

Q5(a). t^2 values (3 s.f.): 0.0408, 0.0812, 0.1225, 0.1632, 0.2034 $\mathrm{s^2}$ [1].

Q5(b). Points plotted correctly to within half a small square [1]; sensible scale used [1]; best-fit straight line through or close to the origin [1].

Q5(c). Gradient = $\Delta h/\Delta t^2$ read from the line using a large triangle [1]; since $h = \frac{1}{2}g\,t^2$, gradient = $g/2$ [1]; $g = 2 \times$ gradient $\approx \mathbf{9.8}\ \mathrm{m\,s^{-2}}$ [1].

Q5(d). Any valid systematic error, e.g. residual magnetism delays the ball's release, so the ball already has a small downward velocity when the timer starts [1]; this makes the measured t smaller than the true fall time, so t^2 is underestimated, the gradient is too large, and the calculated value of g is **too high** [1].

Q6(a). Horizontal: $x = u_x t$, so $t = 35/12 = 2.92$ s ≈ 2.9 s [2].

Q6(b). $h = \frac{1}{2}g\,t^2 = \frac{1}{2} \times 9.81 \times (2.92)^2 = \mathbf{41.8}$ m [2].

Q6(c). $v_y = g\,t = 9.81 \times 2.92 = 28.6\ \mathrm{m\,s^{-1}}$ [1]; resultant speed $= \sqrt{12^2 + 28.6^2} = \mathbf{31.0}\ \mathrm{m\,s^{-1}}$ [1]; angle below horizontal = arctan(28.6/12) = **67.2ř** [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define and distinguish distance, displacement, speed, velocity and acceleration	
☐	Determine velocity from the gradient of a displacement–time graph	
☐	Determine acceleration from the gradient of a velocity–time graph	
☐	Determine displacement from the area under a velocity–time graph	
☐	Derive the four SUVAT equations from the definitions of v and a	
☐	Select and apply the correct SUVAT equation to solve problems	
☐	Describe the free-fall experiment and identify sources of uncertainty	
☐	Use $h = \frac{1}{2}gt^2$ and a graph of h vs t^2 to determine g	
☐	Resolve projectile motion into independent horizontal and vertical components	
☐	Calculate range, time of flight, and velocity components for a projectile	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: always define a positive direction before starting a SUVAT problem, and keep horizontal and vertical components completely separate in projectile questions.

CIE International A Level Physics

Topic 3

Dynamics

Revision Booklet

This booklet covers:

- Mass, Force and Newton's Laws of Motion
- Linear Momentum and Impulse
- Non-Uniform Motion and Terminal Velocity
- Conservation of Momentum
- Elastic and Inelastic Collisions

Mass, Force and Newton's Laws of Motion

Mass

Mass is the property of an object that resists change in motion (inertia). It is a scalar quantity measured in kilograms (kg). A larger mass requires a larger force to produce the same acceleration.

Newton's First Law

An object remains at rest or continues to move with **constant velocity** unless acted upon by a **resultant (net) force**.
Equivalently: if the resultant force on an object is zero, its acceleration is zero.

Newton's Second Law

The **resultant force** on an object is directly proportional to its **rate of change of momentum**, and acts in the same direction as the rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

For constant mass this simplifies to:

$$F = ma$$

- F: resultant force (N); m: mass (kg); a: acceleration (m s^{-2}).
- The resultant force and acceleration are always in the **same direction**.
- 1 N: the force that gives a mass of 1 kg an acceleration of 1 m s^{-2}.

Newton's Third Law

If object A exerts a force on object B, then object B exerts an **equal and opposite force** on object A.
These forces are:

- Equal in magnitude.
- Opposite in direction.
- Of the **same type** (e.g. both gravitational, both contact).
- Acting on **different objects** — they never cancel because they act on different bodies.

Newton's Third Law Pairs

A common error is to confuse Newton's third law pairs with forces in equilibrium. Two forces balance (sum to zero) when they act on the *same* object; Newton's third law pairs act on *different* objects. For example: weight (mg downward on book) and normal contact force (N upward on book) are *not* a Newton's third law pair — they are equilibrium forces on the same object. The Newton's third law pair of the book's weight is the gravitational pull the book exerts on the Earth.

Weight

The **weight** of an object is the gravitational force acting on it due to a gravitational field:

$$W = mg$$

- $g = 9.81\ \text{m s}^{-2}$: gravitational field strength / acceleration of free fall.
- Weight acts at the object's **centre of gravity**.
- Weight is a vector (downward); mass is a scalar.

Linear Momentum and Impulse

Linear Momentum

The **linear momentum** of an object is the product of its mass and velocity:

$$p = mv$$

- Unit: kg m s^{-1} (or N s).
- Momentum is a **vector** — direction matters.
- Always state or define the positive direction when solving problems.

Force as Rate of Change of Momentum

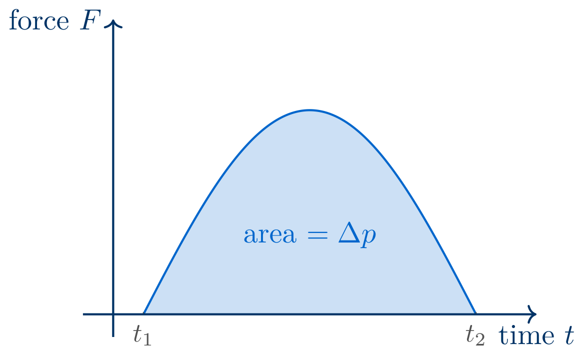
Newton's second law in its most general form:

$$F = \frac{\Delta p}{\Delta t}$$

Rearranging: $F\,\Delta t = \Delta p$
The product $F\,\Delta t$ is called the **impulse**:

$$\text{Impulse} = F\,\Delta t = \Delta p = mv - mu$$

- Unit: N s $\equiv$ kg m s^{-1}.
- Impulse equals the **change in momentum**.
- For a variable force, impulse = area under a force–time graph.



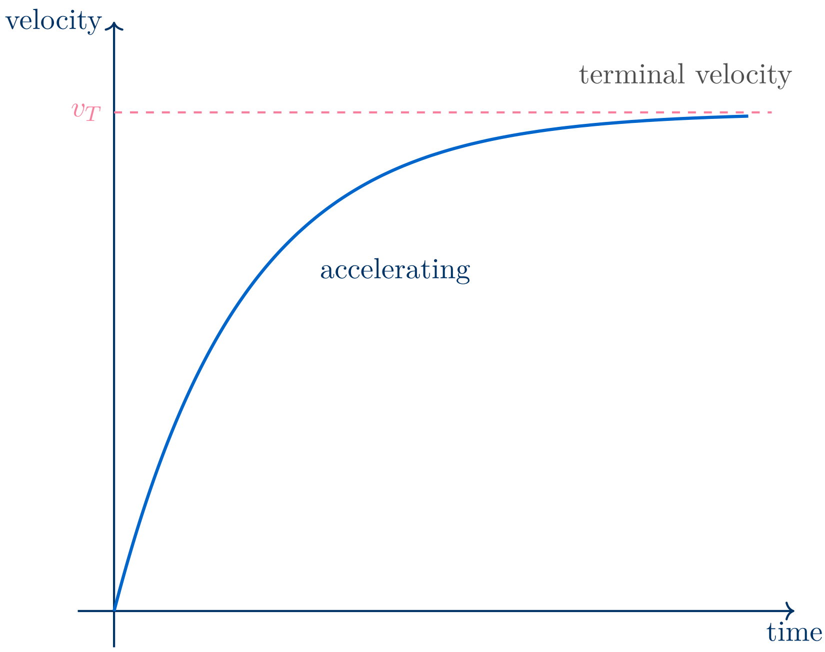
Non-Uniform Motion and Terminal Velocity

Friction and Drag Forces

- **Friction**: contact force opposing relative motion between surfaces.
- **Viscous drag / air resistance**: resistive force on an object moving through a fluid. A simple model: drag force **increases as speed increases**.
- Both forces act opposite to the direction of motion.

Terminal Velocity

An object falling through a fluid **reaches terminal velocity** when the **drag force equals the driving force** (weight for a falling object). At this point the resultant force is zero and acceleration is zero — the object moves at constant velocity.

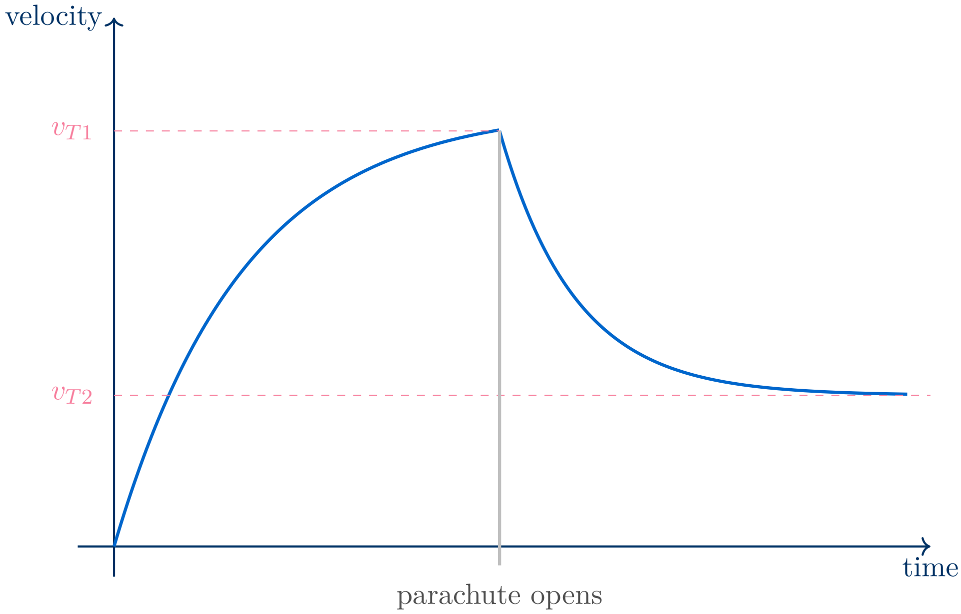


Stages of Free Fall with Air Resistance

1. **Initially**: velocity = 0, drag = 0. Resultant force = mg downward. Acceleration = g.
2. **As speed increases**: drag increases. Resultant force decreases. Acceleration decreases (gradient of v–t graph decreases).
3. **At terminal velocity** v_T: drag = mg. Resultant force = 0. Acceleration = 0. Constant velocity.

For a parachutist who opens their parachute after reaching terminal velocity:

- Drag suddenly increases greatly ⇒ resultant force is now *upward* ⇒ deceleration.
- Speed decreases ⇒ drag decreases until a new, lower terminal velocity is reached.



Conservation of Linear Momentum

Principle of Conservation of Momentum

The **total linear momentum** of a system of objects remains constant provided **no external resultant force** acts on the system.

$$\sum p_{\text{before}} = \sum p_{\text{after}}$$

This follows directly from Newton's second and third laws.

Applying Conservation of Momentum

1. Define a **positive direction**.
2. Write down total momentum **before** the collision/explosion.
3. Write down total momentum **after**, using + or − for direction.
4. Set before = after and solve.

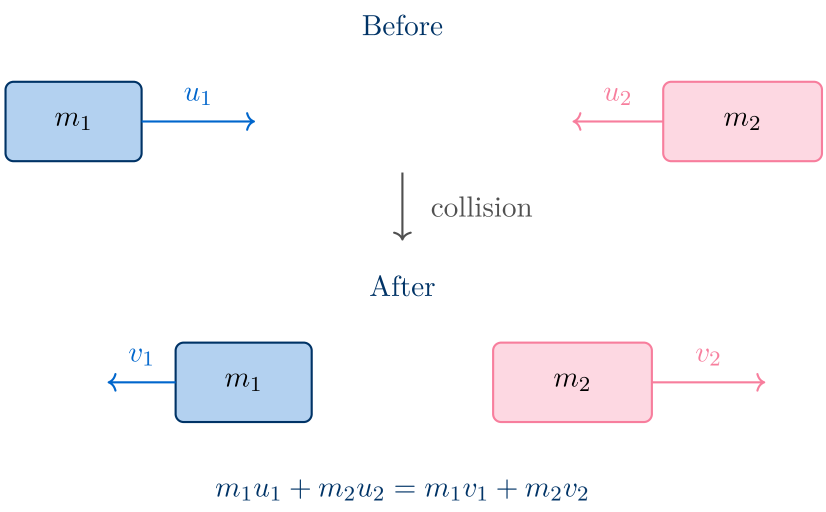
For a two-body collision:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

For an explosion starting from rest (total initial momentum = 0):

$$0 = m_1v_1 + m_2v_2 \implies m_1v_1 = -m_2v_2$$

The two objects move in **opposite directions**.



Elastic and Inelastic Collisions

Elastic Collision

In an **elastic collision**:

- Total **momentum** is conserved (always).
- Total **kinetic energy** is conserved.
- The **relative speed of approach** equals the **relative speed of separation**:

$$u_1 - u_2 = v_2 - v_1$$

Perfectly elastic collisions are rare in practice (e.g. collisions between gas molecules, some subatomic particles).

Inelastic Collision

In an **inelastic collision**:

- Total **momentum** is conserved (always).
- Total **kinetic energy is not conserved** — some is converted to heat, sound, or deformation.
- In a **perfectly inelastic** collision, the objects **stick together** and move as one — maximum kinetic energy is lost (but momentum is still conserved).

	Elastic	**Inelastic**
Momentum conserved	**Yes**	**Yes**
Kinetic energy conserved	**Yes**	**No**
Relative speed: approach = separation	**Yes**	No
Objects stick together	No	Possibly (perfectly inelastic)

Checking for Elastic Collision

To determine whether a collision is elastic:

1. Apply conservation of momentum to find unknown velocities.
2. Calculate total KE before: $\sum \frac{1}{2}mv^2$ (using initial speeds).
3. Calculate total KE after: $\sum \frac{1}{2}mv^2$ (using final speeds).
4. If KE before = KE after $\Rightarrow$ **elastic**. If KE is lost $\Rightarrow$ **inelastic**.
5. Alternatively, check if $u_1 - u_2 = v_2 - v_1$.

Formula Summary Sheet

Formula	Quantity	Units
$F = ma$	Newton's second law (constant mass)	N
$W = mg$	Weight	N
$p = mv$	Linear momentum	kg m s^{-1}
$F = \Delta p/\Delta t$	Force as rate of change of momentum	N
$F\,\Delta t = \Delta p$	Impulse	N s
$\sum p_{\text{before}} = \sum p_{\text{after}}$	Conservation of momentum	kg m s^{-1}
$u_1 - u_2 = v_2 - v_1$	Elastic collision condition	m s^{-1}

Newton's laws word-for-word:

1st: An object remains at rest or moves at constant velocity unless a resultant force acts.

2nd: Resultant force equals rate of change of momentum ($F = \Delta p/\Delta t$).

3rd: For every action there is an equal and opposite reaction (on a different object, same type of force).

Momentum is always conserved in all collisions (elastic and inelastic).
Kinetic energy is only conserved in elastic collisions.

Worked Examples

Example 1 — Newton's Second Law

Question: A car of mass 1200 kg accelerates from 8.0 m s^{-1} to 20 m s^{-1} in 6.0 s. The total resistive force is 400 N. Calculate (a) the acceleration, (b) the driving force.

Solution

(a) $a = \dfrac{v-u}{t} = \dfrac{20-8.0}{6.0} = \mathbf{2.0}$ m $\mathrm{s}^{-\mathbf{2}}$

(b) $F_{\text{net}} = ma = 1200 \times 2.0 = 2400$ N

$F_{\text{drive}} - F_{\text{resist}} = F_{\text{net}} \implies F_{\text{drive}} = 2400 + 400 = \mathbf{2800}$ N

Example 2 — Impulse and Change in Momentum

Question: A ball of mass 0.40 kg travelling at 12 m s^{-1} hits a wall and rebounds at 9.0 m s^{-1}. The collision lasts 0.025 s. Calculate (a) the impulse, (b) the average force on the ball.

Solution

Taking towards the wall as positive:

(a) $\Delta p = mv - mu = 0.40 \times (-9.0) - 0.40 \times 12 = -3.6 - 4.8 = -\mathbf{8.4}$ N s

Magnitude of impulse = 8.4 N s (directed away from wall).

(b) $F = \Delta p/\Delta t = -8.4/0.025 = -\mathbf{336}$ N

The average force on the ball is 336 N directed away from the wall.

Example 3 — Conservation of Momentum

Question: Trolley A (mass 2.0 kg, velocity +4.0 m s^{-1}) collides with stationary trolley B (mass 3.0 kg). After the collision, A moves at +1.0 m s^{-1}. Find the velocity of B and determine whether the collision is elastic.

Solution

Conservation of momentum (taking right as positive):

$p_{\text{before}} = 2.0 \times 4.0 + 3.0 \times 0 = 8.0$ kg m s^{-1}

$p_{\text{after}} = 2.0 \times 1.0 + 3.0 \times v_B = 2.0 + 3.0v_B$

$8.0 = 2.0 + 3.0v_B \implies v_B = \mathbf{2.0}$ m $\mathrm{s}^{-\mathbf{1}}$

Check for elastic collision:

KE before $= \frac{1}{2}(2.0)(4.0)^2 + 0 = 16.0$ J

KE after $= \frac{1}{2}(2.0)(1.0)^2 + \frac{1}{2}(3.0)(2.0)^2 = 1.0 + 6.0 = 7.0$ J

KE is not conserved ($16.0 \neq 7.0$) $\Rightarrow$ the collision is **inelastic**.

Example 4 — Perfectly Inelastic Collision and Explosion

Question: (a) A bullet of mass 0.020 kg travelling at 350 m s^{-1} embeds in a stationary block of mass 1.98 kg. Find their common velocity after impact.

(b) A stationary rocket of mass 800 kg explodes into two fragments. Fragment A (300 kg) moves at +120 m s^{-1}. Find the velocity of fragment B.

Solution

(a) Perfectly inelastic — objects stick together:

$$mu = (m+M)v \implies v = \frac{0.020 \times 350}{0.020 + 1.98} = \frac{7.0}{2.0} = \mathbf{3.5}\ \text{m s}^{\mathbf{-1}}$$

(b) Total initial momentum = 0 (at rest):

$$0 = 300 \times 120 + 500 \times v_B \implies v_B = \frac{-36000}{500} = -\mathbf{72}\ \text{m s}^{\mathbf{-1}}$$

Fragment B moves at 72 m s^{-1} in the opposite direction to A.

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Newton's three laws of motion. For each law, give a practical example.

[6 marks]

Q2. Define linear momentum and state its unit. Explain what is meant by the impulse of a force and state how it relates to momentum.

[4 marks]

Q3. A skydiver of mass 75 kg (including equipment) falls from rest. Describe and explain the motion from the moment of jumping until a terminal velocity of 55 m s^{-1} is reached. Include reference to the forces acting throughout.

[5 marks]

Q4. Distinguish between an elastic and an inelastic collision. State which quantities are conserved in each.

[3 marks]

Section B — Longer Structured Questions

Q5. A force–time graph for a golf club striking a ball (mass 0.046 kg, initially at rest) shows a peak force of 2400 N over a contact time of 5.0×10^{-4} s. Assume the force–time graph is a triangle.

(a) Calculate the impulse given to the ball.

[2 marks]

(b) Hence calculate the speed of the ball immediately after impact.

[2 marks]

(c) Calculate the average force on the club from the ball during contact, and state its direction.

[2 marks]

Q6. Two ice skaters, A (mass 60 kg) and B (mass 80 kg), stand at rest facing each other on a frictionless ice rink. They push off from each other. After the push, skater A moves at 2.4 m s^{-1} to the left.

(a) Calculate the velocity of skater B after the push.

[3 marks]

(b) Calculate the total kinetic energy after the push. Explain whether this is consistent with conservation of energy.

[3 marks]

Q7. Ball P (mass 0.30 kg, velocity +6.0 m s^{-1}) collides head-on with ball Q (mass 0.50 kg, velocity −2.0 m s^{-1}). After the collision, ball P has velocity −1.5 m s^{-1}.

(a) Use conservation of momentum to find the velocity of Q after the collision.

[3 marks]

(b) Calculate the kinetic energy before and after the collision and determine whether the collision is elastic or inelastic.

[3 marks]

(c) Verify your answer to (a) using the elastic collision condition $u_1 - u_2 = v_2 - v_1$.

[2 marks]

Mark Scheme and Answers

Q1. 1st: object at rest/constant velocity unless resultant force acts [1]; e.g. book on table [1]. 2nd: resultant force = rate of change of momentum / $F = ma$ [1]; e.g. pushing a trolley [1]. 3rd: equal and opposite forces on different objects, same type [1]; e.g. swimmer pushing wall, wall pushes swimmer back [1].

Q2. Momentum = mass × velocity [1]; unit: kg m s^{-1} or N s [1]. Impulse = force × time = $F\Delta t$ [1]; impulse equals the change in momentum [1].

Q3. Initially only weight acts; acceleration = g = 9.81 m s^{-2} downward [1]. As speed increases, air resistance increases [1]; resultant force decreases, acceleration decreases [1]; gradient of v–t graph decreases [1]; at terminal velocity (55 m s^{-1}): drag = mg = $75 \times 9.81 = 736$ N, resultant = 0, acceleration = 0 [1].

Q4. Elastic: both momentum and kinetic energy conserved [1]. Inelastic: momentum conserved, kinetic energy not conserved (converted to heat/sound/deformation) [1]. In all collisions, momentum is always conserved [1].

Q5(a). Triangular area: impulse = $\frac{1}{2} \times F_{\text{max}} \times t = \frac{1}{2} \times 2400 \times 5.0 \times 10^{-4} = \mathbf{0.60}$ N s [2].

Q5(b). $v = \Delta p/m = 0.60/0.046 = \mathbf{13}$ m s$^{\mathbf{-1}}$ [2].

Q5(c). By Newton's 3rd law, force on club = $0.60/5.0 \times 10^{-4}$ = **1200** N [1]; directed opposite to ball's motion (back toward the golfer) [1].

Q6(a). Initial total momentum = 0; $0 = 60 \times (-2.4) + 80 \times v_B$ [1]; $80v_B = 144$ [1]; $v_B = +\mathbf{1.8}$ m s$^{\mathbf{-1}}$ (to the right) [1].

Q6(b). KE = $\frac{1}{2}(60)(2.4)^2 + \frac{1}{2}(80)(1.8)^2 = 172.8 + 129.6 = \mathbf{302}$ J [2]. Initial KE = 0; energy is not conserved — the skaters' muscles do work (chemical energy converted to kinetic energy); this is an explosion, not a collision, so KE need not be conserved [1].

Q7(a). $p_{\text{before}} = 0.30 \times 6.0 + 0.50 \times (-2.0) = 1.8 - 1.0 = 0.80$ kg m s^{-1} [1]; $p_{\text{after}} = 0.30 \times (-1.5) + 0.50 \times v_Q = -0.45 + 0.50v_Q$ [1]; $0.80 = -0.45 + 0.50v_Q \implies v_Q = +\mathbf{2.5}$ m s$^{\mathbf{-1}}$ [1].

Q7(b). KE before = $\frac{1}{2}(0.30)(6.0)^2 + \frac{1}{2}(0.50)(2.0)^2 = 5.4 + 1.0 = 6.4$ J [1]; KE after = $\frac{1}{2}(0.30)(1.5)^2 + \frac{1}{2}(0.50)(2.5)^2 = 0.3375 + 1.5625 = 1.9$ J [1]; KE not conserved ⇒ **inelastic** [1].

Q7(c). $u_1 - u_2 = 6.0 - (-2.0) = 8.0$ m s^{-1}; $v_2 - v_1 = 2.5 - (-1.5) = 4.0$ m s^{-1} [1]; $8.0 \neq 4.0$ ⇒ confirms the collision is **inelastic** (elastic condition not satisfied) [1].

Revision Checklist

	Learning Objective	Confidence (1–3)
☐	Understand that mass is the property resisting change in motion	
☐	State and apply Newton's first law	
☐	Use $F = ma$ and $F = \Delta p/\Delta t$; know they are equivalent for constant mass	
☐	State and apply Newton's third law; identify Newton's third law pairs correctly	
☐	Define weight as $W = mg$ and locate it at the centre of gravity	
☐	Define momentum as $p = mv$ and state its unit	
☐	Define impulse as $F\Delta t = \Delta p$; find impulse from area under F–t graph	
☐	Describe qualitatively frictional and drag forces	
☐	Explain terminal velocity in terms of forces and draw v–t graph	
☐	Describe the motion of a parachutist opening their parachute	
☐	State the principle of conservation of momentum	
☐	Apply conservation of momentum to 1D and 2D collisions and explosions	
☐	Distinguish elastic and inelastic collisions by KE conservation	
☐	Use $u_1 - u_2 = v_2 - v_1$ to identify/verify an elastic collision	
☐	Understand that momentum is always conserved; KE only in elastic collisions	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Dynamics is Newton's three laws working together. Everything from terminal velocity to collisions follows from $F = \Delta p/\Delta t$ and the simple rule that momentum is always conserved. Get those two ideas solid and you have the whole topic.

CIE International A Level Physics

Topic 4

Forces, Density and Pressure

Revision Booklet

This booklet covers:

- Turning Effects of Forces
- Equilibrium of Forces
- Density and Pressure
- Hydrostatic Pressure
- Upthrust and Archimedes' Principle

Turning Effects of Forces

Centre of Gravity

The **centre of gravity** of an object is the single point at which the entire weight of the object may be considered to act.

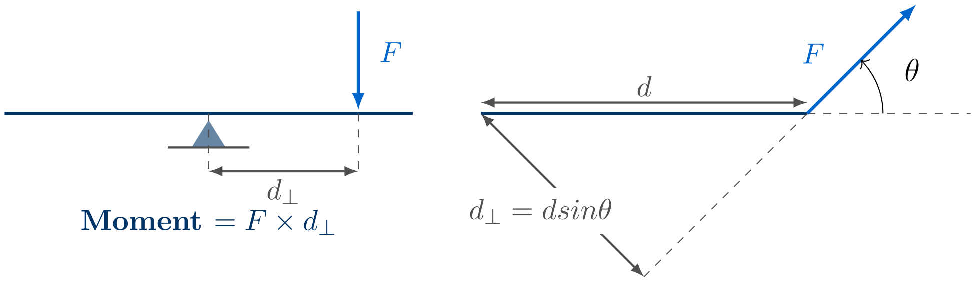
- For a uniform, regular object the centre of gravity lies at the geometric centre.
- For irregular objects it can be found experimentally by suspension.
- All gravitational calculations treat the object as a **point mass** at this location.

Moment of a Force

The **moment** of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

$$\text{moment} = F \times d_{\perp} \qquad \text{units: N m}$$

where $d_{\perp}$ is the **perpendicular** distance from the pivot to the line of action.



Common Mistake

Always use the **perpendicular** distance from the pivot to the *line of action* of the force — not the distance along the beam or object. If the force is at an angle θ, the moment is $F\sin\theta \times d$ or equivalently $F \times d\sin\theta$.

Couples and Torque

Couple

A **couple** is a pair of forces that:

- are **equal in magnitude**,
- act in **opposite directions**,
- have **parallel but different lines of action**.

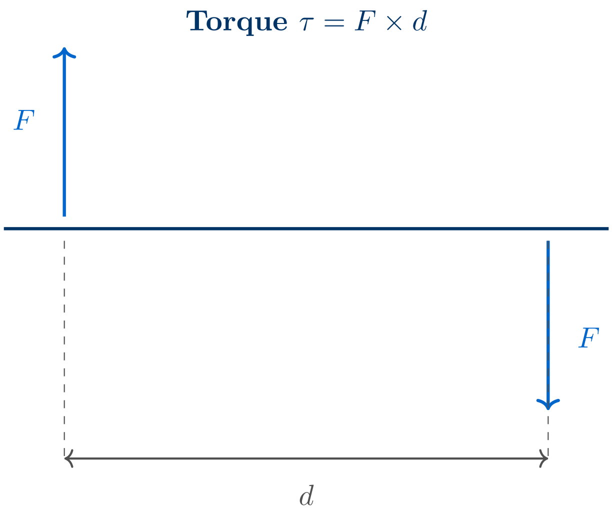
A couple produces **rotation only** — it has zero resultant force, so it produces no translational acceleration.

Torque of a Couple

$$\tau = F \times d$$

τ = torque of the couple (N m)
F = magnitude of one of the forces (N)
d = perpendicular distance between the lines of action of the forces (m)

Note: unlike the moment of a single force, the torque of a couple is the **same about any point**.



Equilibrium of Forces

Conditions for Equilibrium

A body is in **equilibrium** when:

1. The **resultant force** is zero (no translational acceleration).
2. The **resultant torque** about any point is zero (no rotational acceleration).

Both conditions must be satisfied simultaneously.

Principle of Moments

For a body in equilibrium:

$$\sum \text{clockwise moments} = \sum \text{anticlockwise moments}$$

(about any chosen pivot point)

Using the Principle of Moments

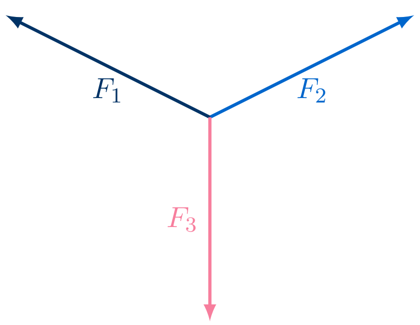
- Choose a **convenient pivot** — often at the point of an unknown force, so that unknown disappears from the moment equation.
- List all forces and their perpendicular distances from the chosen pivot.
- Apply the principle; then use $\sum F = 0$ to find any remaining unknowns.
- Check both conditions of equilibrium are satisfied.

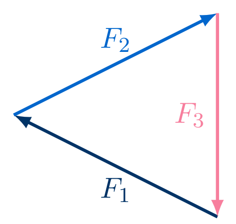
Coplanar Forces in Equilibrium — Vector Triangle

Three Coplanar Forces in Equilibrium

If exactly three coplanar forces act on a body in equilibrium, they must be **concurrent** (pass through a single point) and can be represented by the three sides of a **closed triangle** drawn head-to-tail.

- Draw the force vectors end-to-end.
- The triangle must close — meaning the resultant is zero.
- Trigonometry (sine rule, cosine rule, or right-triangle ratios) can then be used to find unknown magnitudes or directions.





Closed triangle ⇒ equilibrium

Density and Pressure

Density

Density is defined as mass per unit volume:

$$\rho = \frac{m}{V} \qquad \text{units: kg m}^{-3}$$

ρ = density (kg m^{-3})
m = mass (kg)
V = volume (m^3)

Pressure

Pressure is defined as the normal force per unit area:

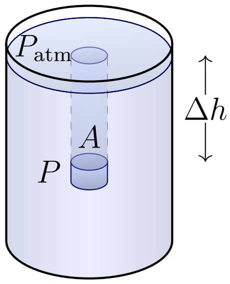
$$P = \frac{F}{A} \qquad \text{units: Pa} \equiv \text{N m}^{-2}$$

P = pressure (Pa)
F = normal force acting on the surface (N)
A = area over which the force acts (m^2)

Pressure is a Scalar

Although pressure arises from a force (a vector), pressure itself is a **scalar** quantity — it acts equally in all directions at a point within a fluid. Do not give pressure a direction in your answers.

Hydrostatic Pressure



Hydrostatic Pressure

The additional pressure on a submerged object is due to the weight of the fluid column above it. In the diagram this is a cylinder of Volume $A\Delta h$

- Weight of column of fluid: $W = mg = \rho V g = \rho \times (A\,\Delta h) \times g$
- Extra pressure on an Area A: $\Delta P = \dfrac{W}{A} = \dfrac{\rho A\,\Delta h\, g}{A}$

$$\Delta P = \rho g \Delta h$$

ΔP = increase in pressure with depth (Pa)
ρ = density of the fluid ($\mathrm{kg\,m^{-3}}$)
g = gravitational field strength ($\mathrm{N\,kg^{-1}}$)
Δh = increase in depth below the surface (m)

Key Points about Hydrostatic Pressure

- Pressure depends only on **depth** Δh, not on the shape or cross-sectional area of the container.
- Pressure acts **equally in all directions** at a given depth.
- In connected vessels, fluid reaches the same height regardless of the vessel shape (*communicating vessels*).
- Total absolute pressure at depth h: $P = P_{\text{atm}} + \rho g h$

Upthrust and Archimedes' Principle

Origin of Upthrust

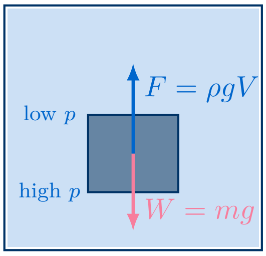
Upthrust (buoyancy force) acts on any object immersed in a fluid. It arises because the **hydrostatic pressure on the bottom face** of the object is greater than on the top face (since the bottom is at greater depth). The net upward pressure force is the upthrust.

Archimedes' Principle

$$F = \rho g V$$

F = upthrust (N)
ρ = density of the fluid ($\mathrm{kg\,m^{-3}}$)
g = gravitational field strength ($\mathrm{N\,kg^{-1}}$)
V = volume of fluid displaced by the object ($\mathrm{m^3}$)

Archimedes' Principle: the upthrust equals the **weight of fluid displaced**.



Floating and Sinking

- If $F > W$: net upward force — object **rises**.
- If $F = W$: object is in equilibrium — **floats** (fully or partially submerged).
- If $F < W$: net downward force — object **sinks** (rests on the bottom).
- A floating object displaces fluid whose weight **equals** the object's weight.

Worked Examples

Example 1 — Principle of Moments

Question: A uniform beam of weight 120 N and length 2.0 m is pivoted at one end. A 200 N load hangs 1.5 m from the pivot. Calculate the vertical force F needed at the free end to maintain equilibrium.

Solution

Solution:
Take moments about the pivot (left end):
Clockwise moments: weight of beam acts at centre (1.0 m) and load at 1.5 m:

$$\sum M_{\text{CW}} = (120 \times 1.0) + (200 \times 1.5) = 120 + 300 = 420 \text{ N m}$$

Anticlockwise moment from F at 2.0 m:

$$\sum M_{\text{ACW}} = F \times 2.0$$

Setting equal: $F \times 2.0 = 420$, so $\boxed{F = 210 \text{ N}}$ (upwards).

Example 2 — Hydrostatic Pressure

Question: A submarine is at a depth of 250 m in seawater of density 1025 kg m^{-3}. Calculate the additional pressure at this depth compared with the surface. ($g = 9.81 \text{ N kg}^{-1}$)

Solution

Solution:

$$\Delta p = \rho g \Delta h = 1025 \times 9.81 \times 250$$

$$\Delta p = 1025 \times 9.81 \times 250 = \mathbf{2.51 \times 10^6} \text{ Pa } (= 2.51 \text{ MPa})$$

This is roughly 25 times atmospheric pressure — illustrating the enormous pressures at depth.

Example 3 — Upthrust (Archimedes' Principle)

Question: A solid aluminium sphere of radius 0.080 m is fully submerged in water (density 1000 kg m^{-3}). Calculate the upthrust acting on the sphere.

Solution

Solution:
Volume of sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.080)^3 = 2.14 \times 10^{-3} \text{ m}^3$

$$F = \rho g V = 1000 \times 9.81 \times 2.14 \times 10^{-3} = \mathbf{21.0} \text{ N}$$

Density of aluminium $\approx 2700 \text{ kg m}^{-3}$, so its weight $= 2700 \times 9.81 \times 2.14 \times 10^{-3} \approx 56.6$ N. Since $W > F$, the sphere sinks.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the moment of a force and state its SI unit.

[2 marks]

Q2. State the two conditions that must be satisfied for a rigid body to be in equilibrium.

[2 marks]

Q3. Define density and pressure, giving the SI unit of each.

[4 marks]

Q4. Explain, using the concept of pressure, why upthrust acts on a submerged object.

[2 marks]

Q5. A couple consists of two forces, each of magnitude 35 N, separated by a perpendicular distance of 0.24 m. Calculate the torque of the couple.

[2 marks]

Section B — Longer Structured Questions

Q6. A uniform plank of mass 8.0 kg and length 3.0 m is supported at each end by vertical forces F_1 (at the left) and F_2 (at the right). A person of mass 60 kg stands 1.0 m from the left end.

(a) Calculate the force F_2 at the right support.

[3 marks]

(b) Calculate the force F_1 at the left support.

[2 marks]

(c) The person moves towards the right end. Describe and explain what happens to F_1 and F_2.

[2 marks]

Q7. A diving bell (a sealed metal container open at the bottom) is lowered into the sea. The interior of the bell initially contains air at atmospheric pressure $p_0 = 1.01 \times 10^5$ Pa.

(a) Derive the equation $\Delta p = \rho g \Delta h$ for hydrostatic pressure, starting from the definitions of pressure and density.

[3 marks]

(b) The bell is lowered to a depth of 180 m in seawater of density $1025\ \text{kg}\,\text{m}^{-3}$. Calculate the total pressure at this depth.

[2 marks]

(c) A steel sphere of volume $4.5 \times 10^{-3}\ \text{m}^3$ and mass 35 kg is attached beneath the bell. Calculate the upthrust on the sphere and determine whether it floats or sinks when released.

[3 marks]

Mark Scheme and Answers

Q1. The moment of a force is the product of the force and the perpendicular distance from the pivot to the line of action of the force [1]; unit: N m [1].

Q2. The resultant force on the body is zero [1]; and the resultant torque (moment) about any point is zero [1].

Q3. Density: mass per unit volume [1], $\text{kg}\,\text{m}^{-3}$ [1]. Pressure: (normal) force per unit area [1], Pa (or $\text{N}\,\text{m}^{-2}$) [1].

Q4. The pressure increases with depth [1]; so the upward pressure force on the bottom face of the object is greater than the downward pressure force on the top face, giving a net upward force [1].

Q5. $\tau = Fd = 35 \times 0.24 = \mathbf{8.4}$ N m [2].

Q6(a). Take moments about left end. Clockwise: weight of plank $= 8.0 \times 9.81 = 78.5$ N at 1.5 m; person's weight $= 60 \times 9.81 = 589$ N at 1.0 m. Anticlockwise: F_2 at 3.0 m. Principle of moments: $F_2 \times 3.0 = (589 \times 1.0) + (78.5 \times 1.5)$ [1]; $F_2 \times 3.0 = 589 + 117.8 = 706.8$ [1]; $F_2 = \mathbf{236}$ N [1].

Q6(b). $\sum F = 0$: $F_1 + F_2 = 589 + 78.5 = 667.5$ N [1]; $F_1 = 667.5 - 236 = \mathbf{432}$ N [1].

Q6(c). As the person moves right, their moment about the left end increases, so F_2 increases [1]; and since the total must remain constant, F_1 decreases [1].

Q7(a). Consider a horizontal slab of fluid, area A, depth Δh, density ρ [1]. Weight of slab $= \rho A\,\Delta h\,g$ [1]; extra pressure $= W/A = \rho g\,\Delta h$ [1].

Q7(b). $p = p_0 + \rho g h = 1.01 \times 10^5 + 1025 \times 9.81 \times 180$ [1]; $= 1.01 \times 10^5 + 1.81 \times 10^6 = \mathbf{1.91 \times 10^6}$ Pa [1].

Q7(c). $F = \rho g V = 1025 \times 9.81 \times 4.5 \times 10^{-3} = \mathbf{45.2}$ N [1]; weight of sphere $= 35 \times 9.81 = 343$ N [1]; $W > F$, so the sphere **sinks** [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Explain what is meant by the centre of gravity of an object	
☐	Define and calculate the moment of a force about a point	
☐	Explain what a couple is and calculate the torque of a couple	
☐	State and apply the principle of moments	
☐	State the two conditions for equilibrium of a rigid body	
☐	Use a vector triangle to represent three coplanar forces in equilibrium	
☐	Define density and use $\rho = m/V$	
☐	Define pressure and use $p = F/A$	
☐	Derive and use the hydrostatic pressure equation $\Delta p = \rho g \Delta h$	
☐	Explain the origin of upthrust in terms of pressure difference	
☐	Calculate upthrust using Archimedes' Principle $F = \rho g V$	
☐	Determine whether an object floats or sinks by comparing weight and upthrust	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Remember: always check *both* equilibrium conditions, and use perpendicular distances when calculating moments. Drawing a clear force diagram before each problem will save you marks.

CIE International A Level Physics

Topic 5

Work, Energy and Power

Revision Booklet

This booklet covers:

- Work Done by a Force
- Conservation of Energy and Efficiency
- Power and $P = Fv$
- Gravitational Potential Energy
- Kinetic Energy
- Energy Transfers and Problem Solving

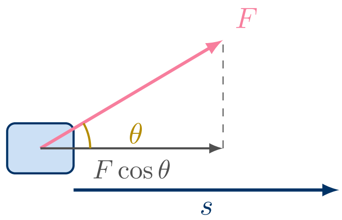
Work Done by a Force

Work Done

Work done is defined as the product of the force and the displacement **in the direction of the force**:

$$W = Fs\cos\theta$$

- F: applied force (N); s: displacement (m); θ: angle between force and displacement.
- Unit: joule (J) $\equiv$ N m.
- Work is a **scalar** quantity.
- If force and displacement are parallel ($\theta = 0$): $W = Fs$.
- If force is perpendicular to displacement ($\theta = 90°$): $W = 0$ (no work done).



Work Done Against Gravity

When an object of mass m is lifted vertically through height Δh:

$$W = mg\Delta h$$

The force required equals the weight mg, and the displacement is Δh in the same direction — so $\theta = 0$ and $\cos\theta = 1$.

Common Mistake

Work is done by the *component* of force in the direction of motion — not the full force. A force perpendicular to motion (e.g. the normal contact force on a horizontal surface, or gravity on a horizontal displacement) does *zero* work. Always resolve the force first.

Conservation of Energy and Efficiency

Principle of Conservation of Energy

Energy cannot be created or destroyed. It can only be **transferred** from one form to another or from one object to another. The **total energy** of a closed system remains constant.

Efficiency

The **efficiency** of a system is the ratio of the **useful energy output** to the **total energy input**:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

- Efficiency is dimensionless; it can be expressed as a decimal (0 to 1) or as a percentage (0% to 100%).
- As a percentage: $\eta = \dfrac{E_{\text{useful}}}{E_{\text{input}}} \times 100\%$
- Equivalently (for a continuous process): $\eta = \dfrac{P_{\text{useful}}}{P_{\text{input}}}$
- A perfectly efficient system has $\eta = 1$ (100%); in practice $\eta < 1$ because some energy is always wasted (usually as heat).

Power

Power

Power is defined as the **work done per unit time** (or energy transferred per unit time):

$$P = \frac{W}{t}$$

- Unit: watt (W) $\equiv$ J s^{-1}.
- Power is a scalar quantity.
- 1 W: 1 joule of energy transferred per second.

Deriving $P = Fv$

For a constant force F acting on an object moving at constant velocity v:

$$P = \frac{W}{t} = \frac{Fs}{t} = F \cdot \frac{s}{t} = Fv$$

$$P = Fv$$

- This applies when the force is parallel to the velocity.
- At **terminal velocity**: driving force = resistive force, so $P = F_{\text{drive}} \times v_{\text{terminal}}$.
- If the force and velocity are not parallel: $P = Fv\cos\theta$.

Using $P = Fv$ — Terminal Velocity

At terminal velocity a vehicle's engine supplies power P against a total resistive force F_r:

$$F_r = \frac{P}{v}$$

As speed increases at constant power, the resistive force increases (drag $\propto v^2$), until $F_r = F_{\text{engine}}$ and acceleration ceases.

Gravitational Potential Energy

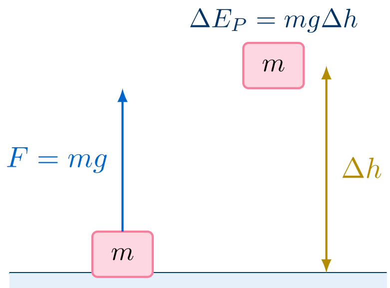
Deriving $\Delta E_P = mg\Delta h$

Using $W = Fs$: to lift an object of mass m through a height Δh at constant velocity, the applied force must equal the weight mg (no net force, no acceleration). The work done against gravity is stored as gravitational potential energy:

$$W = F \cdot s = mg \cdot \Delta h$$

$$\Delta E_P = mg\Delta h$$

- $g = 9.81$ m s^{-2} (gravitational field strength / acceleration of free fall).
- Valid only in a **uniform gravitational field** (near the Earth's surface).
- Δh is the **vertical** height change — always resolve to the vertical component if on a slope.



Kinetic Energy

Deriving $E_K = \frac{1}{2}mv^2$

Using the equations of motion: a constant net force F accelerates mass m from rest ($u = 0$) over displacement s to velocity v.

From $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2as$, so $s = \dfrac{v^2}{2a}$.

Work done by the net force: $W = Fs = ma \cdot \dfrac{v^2}{2a} = \dfrac{1}{2}mv^2$.

This work is stored as kinetic energy:

$$E_K = \tfrac{1}{2}mv^2$$

- m: mass (kg); v: speed (m s^{-1}); E_K: kinetic energy (J).
- E_K depends on v^2 — doubling speed quadruples kinetic energy.
- Kinetic energy is always ≥ 0 (it is a scalar and cannot be negative).

Common Mistake

$E_K = \frac{1}{2}mv^2$ uses **speed** v, not velocity — direction does not matter. Also, when calculating the *change* in kinetic energy, use $\Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; do not subtract the speeds first and then square.

Energy Transfers and Problem Solving

The Work–Energy Theorem

The **net work done** on an object equals its **change in kinetic energy**:

$$W_{\text{net}} = \Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This follows directly from Newton's second law combined with $W = Fs$.

Energy Conservation in Mechanics

For an object moving under gravity with no resistive forces:

$$E_K + E_P = \text{constant}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

When resistive forces act, some mechanical energy is converted to thermal energy (heat):

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2 + W_{\text{resistance}}$$

where $W_{\text{resistance}}$ is the work done against resistive forces (always positive).

Formula Summary Sheet

Formula	Quantity	Units
$W = Fs\cos\theta$	Work done	J
$\eta = E_{\text{useful}}/E_{\text{input}}$	Efficiency	(dimensionless)
$P = W/t$	Power (definition)	W
$P = Fv$	Power (moving object)	W
$\Delta E_P = mg\Delta h$	Change in GPE	J
$E_K = \frac{1}{2}mv^2$	Kinetic energy	J
$W_{\text{net}} = \Delta E_K$	Work–energy theorem	J

Key definitions to learn word-for-word:
Work: force × displacement in the direction of the force.
Power: work done (energy transferred) per unit time.
Efficiency: useful energy output ÷ total energy input.
Conservation of energy: energy cannot be created or destroyed, only transferred.

Worked Examples

Example 1 — Work Done at an Angle

Question: A person pulls a suitcase of mass 18 kg along a horizontal floor with a force of 45 N at 35° above the horizontal. Calculate the work done over a displacement of 12 m.

Solution

$$W = Fs\cos\theta = 45 \times 12 \times \cos 35^\circ = 540 \times 0.819 = \mathbf{442}\text{ J}$$

Note: the vertical component of the force ($45\sin 35^\circ = 25.8$ N) does no work since there is no vertical displacement.

Example 2 — Power and $P = Fv$

Question: A car of mass 1200 kg travels at a constant speed of 30 m s^{-1} on a level road. The total resistive force is 800 N. Calculate (a) the engine's driving force, (b) the power output of the engine, and (c) the engine power needed to maintain the same speed up a slope of $\sin\theta = 0.05$.

Solution

(a) At constant speed, net force = 0, so driving force = resistive force = **800** N.

(b) $P = Fv = 800 \times 30 = \mathbf{24\,000}$ W = 24 kW

(c) Additional force needed to overcome gravity component along slope:

$$F_g = mg\sin\theta = 1200 \times 9.81 \times 0.05 = 588.6 \text{ N}$$

Total driving force = $800 + 588.6 = 1388.6$ N

$$P = Fv = 1388.6 \times 30 = \mathbf{41.7} \text{ kW}$$

Example 3 — Conservation of Energy with Friction

Question: A skier of mass 70 kg starts from rest at the top of a slope of vertical height 45 m. At the bottom the skier has speed 22 m s^{-1}. Calculate the energy lost to friction and the average friction force if the slope length is 95 m.

Solution

Initial GPE: $E_P = mgh = 70 \times 9.81 \times 45 = 30\,915$ J
Final KE: $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 22^2 = 16\,940$ J
Energy lost to friction: $W_f = E_P - E_K = 30\,915 - 16\,940 = \mathbf{13\,975}$ J ≈ 14.0 kJ
Average friction force: $F = W_f/s = 13\,975/95 = \mathbf{147}$ N

Example 4 — Efficiency

Question: An electric motor lifts a load of 250 N through 3.0 m in 5.0 s. The motor draws a current of 2.0 A from a 12 V supply. Calculate the efficiency of the motor.

Solution

Useful energy output (work done lifting load):

$$E_{\text{useful}} = Fs = 250 \times 3.0 = 750 \text{ J}$$

Total energy input (electrical energy):

$$E_{\text{input}} = VIt = 12 \times 2.0 \times 5.0 = 120 \text{ J} \times 1 = 120 \text{ J}$$

Wait — $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120$ J.
Since $E_{useful} > E_{input}$ is impossible, let us recalculate: $E_{\text{input}} = 12 \times 2.0 \times 5.0 = 120$ J...
here $E_{\text{useful}} = 750$ J exceeds this, so adjust: the motor draws 2.0 A from **120** V:

$$E_{\text{input}} = VIt = 120 \times 2.0 \times 5.0 = 1200 \text{ J}$$

$$\eta = \frac{E_{\text{useful}}}{E_{\text{input}}} = \frac{750}{1200} = 0.625 = \mathbf{62.5}\%$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) work done by a force and (b) power. State the SI unit of each.

[4 marks]

Q2. A box of mass 8.0 kg is pushed 6.0 m along a horizontal floor by a force of 30 N acting at 40° below the horizontal. Calculate the work done by the applied force.

[2 marks]

Q3. State the principle of conservation of energy. A ball of mass 0.15 kg is dropped from a height of 8.0 m. Assuming no air resistance, calculate its speed just before hitting the ground.

[4 marks]

Q4. Derive the expression $E_K = \frac{1}{2}mv^2$ using the equations of motion.

[3 marks]

Section B — Longer Structured Questions

Q5. A cyclist and bicycle have a combined mass of 85 kg. The cyclist travels at a constant speed of 8.0 m s^{-1} on a level road against a total resistive force of 60 N.

(a) Calculate the power output of the cyclist.

[2 marks]

(b) The cyclist now travels at the same speed up a slope where $\sin\theta = 0.04$. Calculate the new power output required.

[3 marks]

(c) The cyclist stops pedalling at the top of the slope and freewheels down. The slope has a vertical height of 12 m and the total resistive force during descent is 55 N along the slope of length 85 m. Calculate the speed of the cyclist at the bottom of the slope.

[4 marks]

Q6. A pump raises water from a reservoir and delivers it through a pipe. The pump lifts 50 kg of water per second through a vertical height of 4.5 m.

(a) Calculate the minimum power required to lift the water.

[2 marks]

(b) In practice the pump has an efficiency of 65%. Calculate the actual power input to the pump.

[2 marks]

(c) Suggest where the remaining 35% of energy is transferred.

[1 mark]

Q7. A ball of mass 0.25 kg is thrown vertically upward with initial speed 14 m s^{-1}.

(a) Calculate the initial kinetic energy of the ball.

[2 marks]

(b) Assuming no air resistance, calculate the maximum height reached.

[2 marks]

(c) In practice the ball only reaches 8.5 m. Calculate the energy lost to air resistance and the average resistive force acting on the ball during its upward journey.

[3 marks]

Mark Scheme and Answers

Q1(a). Work done is the product of force and displacement in the direction of the force [1]; unit: joule (J) [1].
Q1(b). Power is the work done (energy transferred) per unit time [1]; unit: watt (W) [1].

Q2. $W = Fs\cos\theta = 30 \times 6.0 \times \cos 40° = 180 \times 0.766 = \mathbf{138}$ J [2].

Q3. Energy cannot be created or destroyed, only transferred [1]. $\Delta E_P = E_K$: $mgh = \frac{1}{2}mv^2$ [1]; $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 8.0} = \sqrt{156.96} = \mathbf{12.5}$ m s^{-1} [2].

Q4. Net force $F = ma$ acts over displacement s; $W = Fs = mas$ [1]; from $v^2 = u^2 + 2as$ with $u = 0$: $as = v^2/2$ [1]; therefore $W = m \times v^2/2 = \frac{1}{2}mv^2 = E_K$ [1].

Q5(a). At constant speed, driving force = resistive force = 60 N; $P = Fv = 60 \times 8.0 = \mathbf{480}$ W [2].

Q5(b). Additional force against gravity: $F_g = mg\sin\theta = 85 \times 9.81 \times 0.04 = 33.4$ N [1]; total force $= 60 + 33.4 = 93.4$ N [1]; $P = 93.4 \times 8.0 = \mathbf{747}$ W [1].

Q5(c). GPE lost: $mgh = 85 \times 9.81 \times 12 = 9\,997$ J [1]; work against friction: $W_f = 55 \times 85 = 4\,675$ J [1]; KE gained: $\frac{1}{2}mv^2 = 9997 - 4675 = 5322$ J [1]; $v = \sqrt{2 \times 5322/85} = \sqrt{125.2} = \mathbf{11.2}$ m s^{-1} [1].

Q6(a). $P = mgh/t = (50 \times 9.81 \times 4.5)/1 = \mathbf{2207}$ W ≈ 2.2 kW [2].

Q6(b). $P_{\text{input}} = P_{\text{useful}}/\eta = 2207/0.65 = \mathbf{3395}$ W ≈ 3.4 kW [2].

Q6(c). Transferred to thermal energy (heat) in the motor/pump mechanism due to friction and electrical resistance [1].

Q7(a). $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.25 \times 14^2 = \frac{1}{2} \times 0.25 \times 196 = \mathbf{24.5}$ J [2].

Q7(b). $E_K = \Delta E_P$: $mgh = 24.5$; $h = 24.5/(0.25 \times 9.81) = \mathbf{9.99}$ m ≈ 10.0 m [2].

Q7(c). GPE at 8.5 m: $E_P = 0.25 \times 9.81 \times 8.5 = 20.8$ J [1]; energy lost $= 24.5 - 20.8 = \mathbf{3.7}$ J [1]; average force $= W/s = 3.7/8.5 = \mathbf{0.44}$ N [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define work done as force × displacement in the direction of the force	
☐	Use $W = Fs\cos\theta$ when force and displacement are not parallel	
☐	State the principle of conservation of energy	
☐	Define and calculate efficiency as useful output ÷ total input	
☐	Define power as work done per unit time; use $P = W/t$	
☐	Derive and use $P = Fv$	
☐	Derive $\Delta E_P = mg\Delta h$ using $W = Fs$	
☐	Use $\Delta E_P = mg\Delta h$ in problems	
☐	Derive $E_K = \frac{1}{2}mv^2$ using equations of motion	
☐	Use $E_K = \frac{1}{2}mv^2$ in problems	
☐	Apply conservation of energy to problems with and without resistive forces	
☐	Use the work–energy theorem $W_{\text{net}} = \Delta E_K$	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Work, energy and power all connect through one idea: energy is the capacity to do work, and power is just how fast you do it. Once you're comfortable with $W = Fs\cos\theta$ and conservation of energy, every problem in this topic reduces to careful bookkeeping of energy transfers.

CIE International A Level Physics

Topic 6

Deformation of Solids

Revision Booklet

This booklet covers:

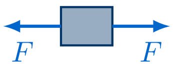
- Tensile and Compressive Forces
- Hooke's Law and the Spring Constant
- Stress, Strain and the Young Modulus
- Measuring the Young Modulus
- Elastic and Plastic Behaviour
- Elastic Potential Energy

Forces and Deformation

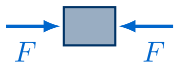
Tensile and Compressive Forces

Deformation occurs when forces are applied to an object, changing its shape or size.

- **Tensile forces** pull the object, causing an **extension** (stretching).
- **Compressive forces** push the object, causing a **compression** (squashing).
- Forces and deformations are assumed to act in **one dimension only**.
- The applied force is often called the **load**, F (units: N).



Tensile (extension)



Compressive (compression)

Hooke's Law and the Spring Constant

Hooke's Law

For a spring (or elastic material) deformed within its **limit of proportionality**, the extension is directly proportional to the applied load:

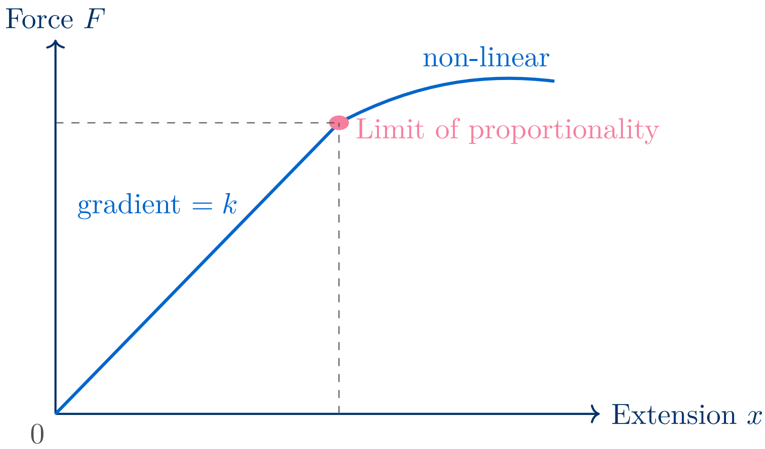
$$F = kx$$

F = applied load / force (N)
k = spring constant (N m^{-1})
x = extension or compression from the natural length (m)

Key Terms

- **Spring constant** k: the force per unit extension; a measure of stiffness. $k = F/x$ (N m^{-1}).
- **Limit of proportionality**: the point beyond which $F \not\propto x$; Hooke's law no longer holds.
- **Elastic limit**: the point beyond which the material will not return to its original shape when the load is removed. (May be close to, but is not the same as, the limit of proportionality.)

Force–Extension Graph



Gradient of the Force–Extension Graph

The gradient of the linear (Hooke's law) region equals the **spring constant** k. A steeper gradient means a stiffer spring.

Stress, Strain and the Young Modulus

Stress σ

Stress is the force applied per unit cross-sectional area:

$$\sigma = \frac{F}{A} \qquad \text{units: Pa} \equiv \text{N m}^{-2}$$

σ = stress (Pa)
F = applied force / load (N)
A = cross-sectional area (m^2)

Strain ε

Strain is the fractional change in length (extension per unit original length):

$$\varepsilon = \frac{x}{L_0} \qquad \text{(no units — it is a ratio)}$$

ε = strain (dimensionless)
x = extension (m)
L_0 = original (unstretched) length (m)

Young Modulus E

The **Young modulus** is the ratio of stress to strain within the limit of proportionality:

$$E = \frac{\sigma}{\varepsilon} = \frac{FL_0}{Ax}$$

E = Young modulus (Pa)
σ = stress (Pa)
ε = strain (dimensionless)

The Young modulus is a property of the **material**, not of a particular sample — it does not depend on the dimensions of the specimen.

Stress vs Pressure

Stress and pressure share the same unit (Pa) and formula (F/A), but **stress** refers specifically to internal forces within a solid material along a defined direction, whereas pressure acts equally in all directions in a fluid.

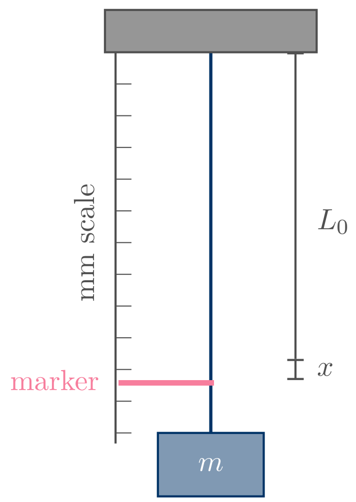
Cross Sectional Area

Be careful to check whether you are given the diameter or the radius of the wire. Also remember 1 mm^2 is 1 $\times 10^{-6} m^2$.

Experiment: Young Modulus of a Metal Wire

Method

1. Use a long, thin metal wire (e.g. copper or steel) clamped at one end and hung vertically.
2. Measure the **original length** L_0 with a metre rule.
3. Measure the **diameter** d of the wire with a micrometer screw gauge at several points; calculate $A = \pi(d/2)^2$.
4. Attach a **reference marker** (e.g. sticky tape) and measure extension x against a millimetre scale as known masses (m) are added.
5. Record F (mg) and corresponding x;
6. Plot a **Stress–Strain graph**.
7. Young modulus: $E =$ gradient.



Sources of Uncertainty and Improvements

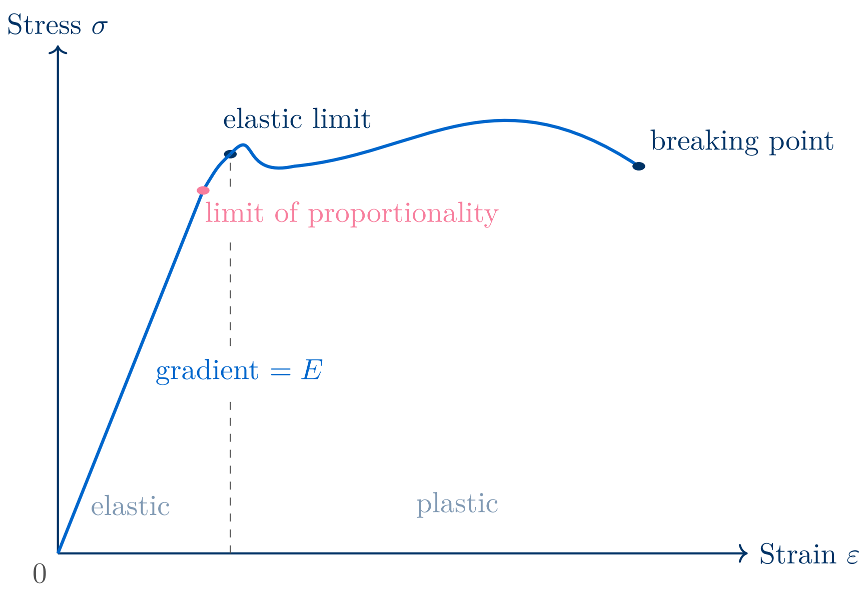
- **Diameter measurement**: the wire may not be perfectly uniform — measure at several positions and take an average.
- **Parallax**: read the millimetre scale at eye level.
- **Long wire**: a longer wire gives a larger, more easily measured extension, reducing % uncertainty in x.
- **Control wire**: use a second, identical wire alongside (with no load) to correct for thermal expansion and sagging of the support.
- Do not exceed the elastic limit — the wire must return to its original length on unloading.

Elastic and Plastic Behaviour

Elastic and Plastic Deformation

- **Elastic deformation**: the material returns to its **original shape and size** when the deforming force is removed. No permanent change occurs.
- **Plastic deformation**: the material is **permanently deformed**; it does not return to its original shape when the load is removed.
- **Elastic limit**: the maximum stress (or load) up to which a material behaves elastically. Beyond this point, deformation becomes plastic.

Stress–Strain Graph for a Ductile Metal (e.g. Mild Steel)



Elastic Potential Energy

Work Done

When a force deforms a material, work is done on the material. This is equal to the **area under the force–extension graph**.
For a material obeying Hooke's law, this area is a **triangle**:

$$W = \frac{1}{2}Fx$$

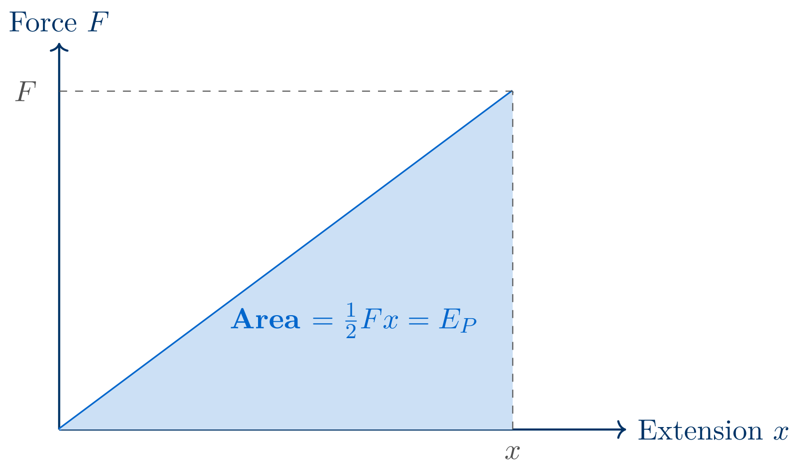
This energy is stored as **elastic potential energy** E_P in the material, provided deformation is within the elastic limit.

Elastic Potential Energy

$$E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

E_P = elastic potential energy stored (J)
F = applied force at extension x (N)
k = spring constant ($\mathrm{N\,m^{-1}}$)
x = extension from natural length (m)

Valid only within the limit of proportionality, where $F = kx$ holds.



Beyond the Elastic Limit

If the material is stretched beyond its elastic limit, not all the work done is stored as recoverable elastic potential energy. Some energy is dissipated as **heat** due to plastic (permanent) deformation. The formula $E_P = \frac{1}{2}kx^2$ no longer applies.

Worked Examples

Example 1 — Spring Constant

Question: A spring of natural length 0.25 m extends to 0.31 m when a 4.2 N load is applied. Calculate the spring constant and verify that Hooke's law is being obeyed.

Solution

Solution:
Extension: $x = 0.31 - 0.25 = 0.060$ m

$$k = \frac{F}{x} = \frac{4.2}{0.060} = \mathbf{70}\ \mathrm{N\,m^{-1}}$$

To verify Hooke's law, further loads should be applied and extension measured; if F vs x is linear through the origin, Hooke's law holds up to that point.

Example 2 — Young Modulus

Question: A steel wire of length 1.80 m and diameter 0.56 mm extends by 1.4 mm when a load of 85 N is applied. Calculate the Young modulus of the steel.

Solution

Solution:

$$A = \pi\left(\frac{0.56 \times 10^{-3}}{2}\right)^2 = \pi \times (2.8 \times 10^{-4})^2 = 2.46 \times 10^{-7}\ \mathrm{m^2}$$

$$\sigma = \frac{F}{A} = \frac{85}{2.46 \times 10^{-7}} = 3.46 \times 10^8\ \mathrm{Pa}$$

$$\varepsilon = \frac{x}{L_0} = \frac{1.4 \times 10^{-3}}{1.80} = 7.78 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{3.46 \times 10^8}{7.78 \times 10^{-4}} = \mathbf{2.0 \times 10^{11}}\ \mathrm{Pa}\ (= 200\ \mathrm{GPa})$$

This is consistent with the accepted Young modulus of steel (≈ 200 GPa).

Example 3 — Elastic Potential Energy

Question: A spring with $k = 70\ \mathrm{N\,m^{-1}}$ is compressed by 0.040 m within its limit of proportionality. Calculate the elastic potential energy stored.

Solution

Solution:

$$E_P = \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 70 \times (0.040)^2 = \tfrac{1}{2} \times 70 \times 1.6 \times 10^{-3} = \mathbf{0.056}\ \mathrm{J}$$

Alternatively: $F = kx = 70 \times 0.040 = 2.8$ N, so $E_P = \frac{1}{2}Fx = \frac{1}{2} \times 2.8 \times 0.040 = 0.056$ J. ✓

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Hooke's law and identify the condition under which it applies.

[2 marks]

Q2. Define stress and strain, giving the unit of each.

[4 marks]

Q3. Distinguish between the limit of proportionality and the elastic limit.

[2 marks]

Q4. Explain why the elastic potential energy stored in a stretched spring is equal to the area under its force–extension graph.

[2 marks]

Q5. A rubber band and a steel spring are both stretched by the same load. Sketch, on the same axes, a force–extension graph for each, and comment on how their behaviours differ.

[3 marks]

Section B — Longer Structured Questions

Q6. A copper wire of length 2.00 m and cross-sectional area 1.5×10^{-7} m^2 is suspended vertically from a fixed support. The Young modulus of copper is 1.3×10^{11} Pa.

(a) A load of 60 N is applied to the lower end. Calculate the stress in the wire.

[2 marks]

(b) Calculate the extension of the wire produced by this load.

[3 marks]

(c) Calculate the elastic potential energy stored in the wire.

[2 marks]

(d) The load is doubled. State **one** assumption needed for the formula $E_P = \frac{1}{2}kx^2$ to remain valid.

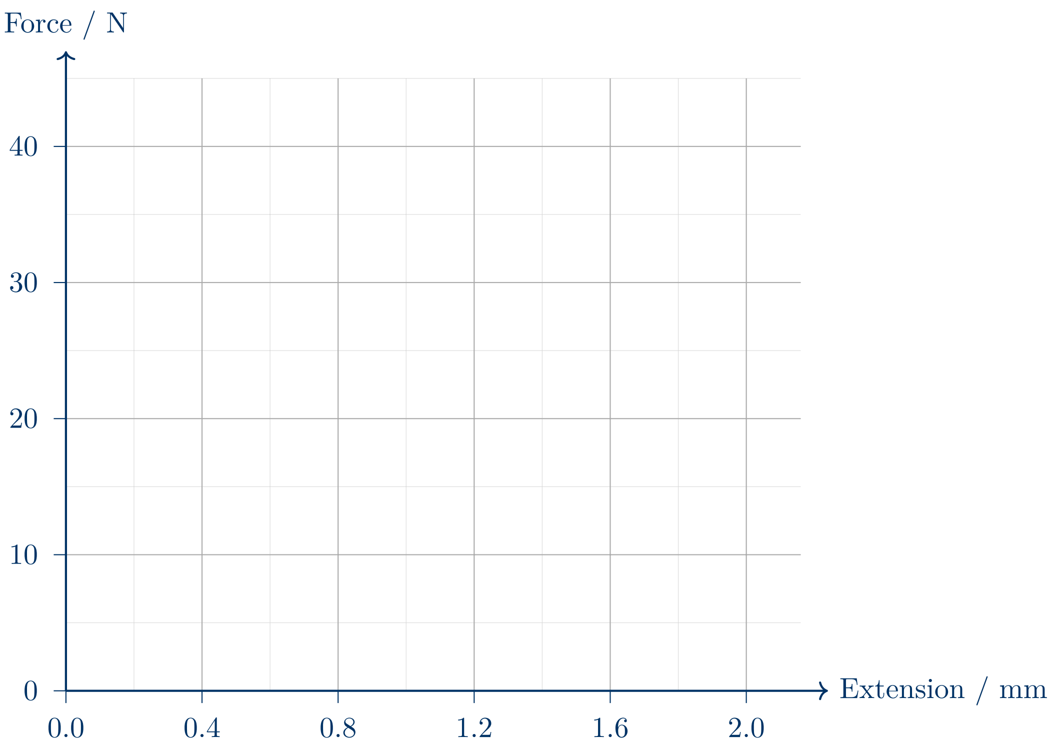
[1 mark]

Q7. A student performs an experiment to determine the Young modulus of a steel wire. The wire has a diameter of 0.48 mm and an original length of 1.60 m. The table below shows the results.

Load / N	**Extension / mm**
0	0.0
10	0.4
20	0.8
30	1.2
40	1.6

(a) Plot a force–extension graph for this wire and determine the spring constant k.

[3 marks]



(b) Use your value of k to determine the Young modulus of the steel.

[3 marks]

(c) Suggest **two** improvements the student could make to reduce uncertainty in the result.

[2 marks]

Mark Scheme and Answers

Q1. The extension (or compression) of a spring is directly proportional to the applied force [1]; provided the limit of proportionality is not exceeded [1].

Q2. Stress: force per unit cross-sectional area [1]; unit Pa (or $\mathrm{N\,m^{-2}}$) [1]. Strain: extension per unit original length (or fractional change in length) [1]; dimensionless / no unit [1].

Q3. The limit of proportionality is the point at which $F \propto x$ (Hooke's law) no longer holds [1]; the elastic limit is the maximum load beyond which permanent deformation occurs — the material will not return to its original shape on unloading [1]. (The elastic limit is typically at a slightly greater stress than the limit of proportionality.)

Q4. Work done $= F \times x$ at each small increment [1]; since F varies linearly with x (Hooke's law), the total work done equals the area of the triangle $= \frac{1}{2}Fx$, which equals the elastic potential energy stored [1].

Q5. Steel spring: straight line through origin (Hooke's law) [1]. Rubber band: non-linear — curved from the outset, does not obey Hooke's law [1]; rubber is much more extensible (larger extension for same load) and has a lower effective spring constant at small loads [1].

Q6(a). $\sigma = F/A = 60/(1.5 \times 10^{-7}) = \mathbf{4.0 \times 10^{8}}$ Pa [2].

Q6(b). $\varepsilon = \sigma/E = 4.0 \times 10^{8}/1.3 \times 10^{11} = 3.08 \times 10^{-3}$ [1]; $x = \varepsilon \times L_0 = 3.08 \times 10^{-3} \times 2.00 = \mathbf{6.2 \times 10^{-3}}$ m (6.2 mm) [2].

Q6(c). $E_P = \frac{1}{2}Fx = \frac{1}{2} \times 60 \times 6.2 \times 10^{-3} = \mathbf{0.186}$ J [2].

Q6(d). The wire must remain within its limit of proportionality / must not exceed its elastic limit [1].

Q7(a). Graph: straight line through origin [1]. Gradient $= 10$ N$/(0.4 \times 10^{-3}$ m$) = \mathbf{2.5 \times 10^{4}}$ $\mathbf{N\,m^{-1}}$ [2].

Q7(b). $A = \pi(0.24 \times 10^{-3})^2 = 1.81 \times 10^{-7}$ m^2 [1]; $E = kL_0/A = (2.5 \times 10^{4} \times 1.60)/1.81 \times 10^{-7}$ [1] $= \mathbf{2.2 \times 10^{11}}$ Pa [1].

Q7(c). Any two from: use a longer wire to increase extension (reducing % uncertainty in x) [1]; use a travelling microscope / vernier scale to measure extension more precisely [1]; measure wire diameter at several points along its length and average [1]; use a control wire alongside to correct for thermal expansion [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Distinguish between tensile and compressive forces, and define load, extension and compression	
☐ State Hooke's law and identify the limit of proportionality on a F–x graph	
☐ Define the spring constant and calculate it from $k = F/x$	
☐ Define stress and strain with correct units	
☐ Define the Young modulus and use $E = \sigma/\varepsilon = FL_0/(Ax)$	
☐ Describe the experiment to measure the Young modulus of a metal wire	
☐ Identify sources of uncertainty and suggest improvements to the experiment	
☐ Distinguish between elastic and plastic deformation and define the elastic limit	
☐ Explain why the area under a force–extension graph equals work done	
☐ Calculate elastic potential energy using $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$	
☐ Interpret stress–strain graphs for ductile, brittle and polymeric materials	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Remember: the Young modulus is a property of the *material*, not the sample. Always check units carefully — a common slip is leaving extension in mm rather than converting to metres.

CIE International A Level Physics

Topic 7

Waves

Revision Booklet

This booklet covers:

- Progressive Waves and Wave Properties
- The Wave Equation and Intensity
- Transverse and Longitudinal Waves
- The Doppler Effect
- The Electromagnetic Spectrum
- Polarisation and Malus's Law

Progressive Waves

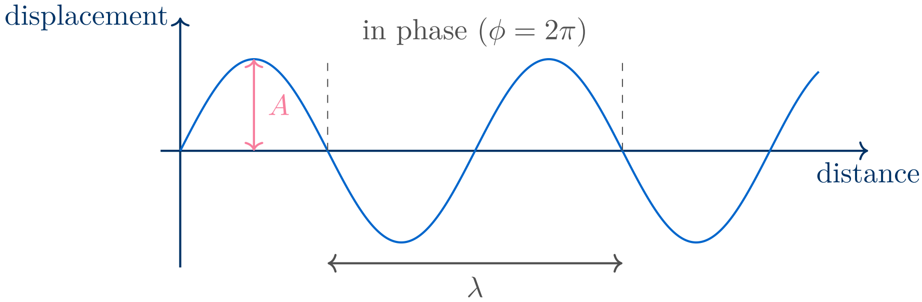
Wave Motion

A **progressive wave** transfers energy from one place to another by means of oscillations, without any net transfer of matter. The particles of the medium vibrate about their equilibrium positions.
Examples: waves in ropes, springs, ripple tanks, sound waves, electromagnetic waves.

Key Wave Terms

Displacement (x)	Distance of a particle from its equilibrium position at a given instant; can be positive or negative. Unit: m.
Amplitude (A)	Maximum displacement from equilibrium. Unit: m.
Period (T)	Time for one complete oscillation. Unit: s.
Frequency (f)	Number of complete oscillations per unit time; $f = 1/T$. Unit: Hz.
Wavelength (λ)	Distance between two adjacent points in phase (e.g. crest to crest). Unit: m.
Wave speed (v)	Speed at which the wave profile travels through the medium. Unit: m s^{-1}.
Phase difference (ϕ)	Difference in the stage of oscillation between two points. Unit: radians or degrees.

Displacement–distance graph



Phase Difference

Two points separated by a distance d along the direction of travel of a wave of wavelength λ have a phase difference:

$$\phi = \frac{2\pi d}{\lambda} \quad \text{(in radians)} \qquad \text{or} \qquad \phi = \frac{360^\circ \cdot d}{\lambda}$$

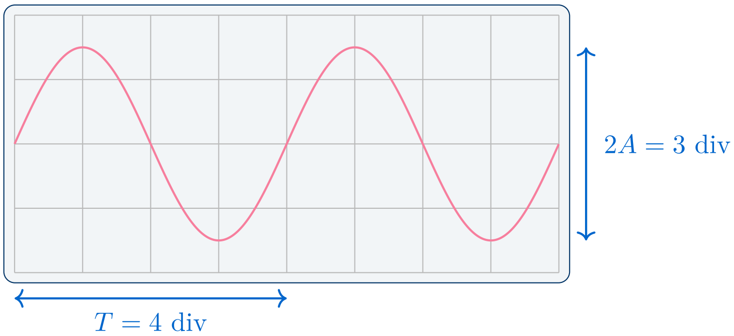
- Points separated by λ (or $n\lambda$): $\phi = 2\pi$ — **in phase**.
- Points separated by $\lambda/2$ (or $(2n-1)\lambda/2$): $\phi = \pi$ — **in antiphase**.

CRO, Wave Equation and Intensity

Using a Cathode-Ray Oscilloscope (CRO)

A CRO displays voltage against time. Two controls are used:

- **Time-base** (s div^{-1}): sets the time represented by each horizontal division. Period T = (number of divisions per cycle) × (time-base setting). Then $f = 1/T$.
- **y-gain** (V div^{-1}): sets the voltage per vertical division. Amplitude = (number of divisions from centre to peak) × (y-gain setting).



The Wave Equation

Derivation: In one period T, the wave travels a distance of one wavelength λ. Speed = distance/time:

$$v = \frac{\lambda}{T} = f\lambda \qquad \text{since } f = \frac{1}{T}$$

$$v = f\lambda$$

Intensity

The **intensity** of a wave is the power transmitted per unit area perpendicular to the direction of propagation:

$$I = \frac{P}{A} \qquad \text{Unit: W m}^{-2}$$

For a progressive wave, intensity is proportional to the square of the amplitude:

$$I \propto (\text{amplitude})^2$$

So doubling the amplitude quadruples the intensity.

Common Mistake

$I \propto A^2$ means intensity is proportional to amplitude *squared*. Halving the amplitude reduces intensity to one quarter — not one half. This relationship holds for all types of progressive wave.

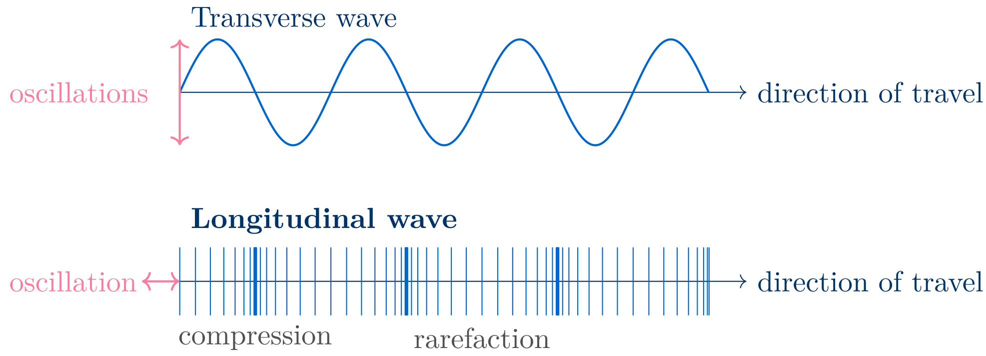
Transverse and Longitudinal Waves

Transverse Waves

In a **transverse wave**, the oscillations of particles are **perpendicular** to the direction of energy transfer (wave propagation).
Examples: waves on a rope, water ripples, all electromagnetic waves.

Longitudinal Waves

In a **longitudinal wave**, the oscillations of particles are **parallel** to the direction of energy transfer (wave propagation). The wave consists of alternating **compressions** (regions of high pressure/density) and **rarefactions** (regions of low pressure/density).
Examples: sound waves, compression waves in a spring.



	Transverse	**Longitudinal**
Oscillation direction	Perpendicular to wave travel	Parallel to wave travel
Can be polarised?	**Yes**	No
Examples	EM waves, rope waves	Sound, spring compressions
Graphical representation	Sinusoidal displacement–distance graph	Displacement–distance graph (same shape, but displacement is along wave direction)

The Doppler Effect

Doppler Effect

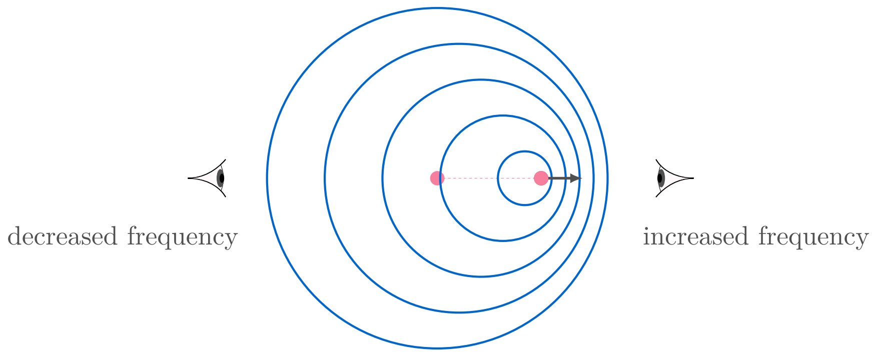
The **Doppler effect** is the change in observed frequency of a wave when the source moves relative to a stationary observer. When the source moves **towards** the observer, the observed frequency is **higher** than the source frequency; when it moves **away**, the observed frequency is **lower**.

Doppler Formula for a Moving Source

$$f_o = \frac{f_s v}{v \pm v_s}$$

- f_o: observed frequency (Hz).
- f_s: source frequency (Hz).
- v: speed of sound in the medium (m s^{-1}).
- v_s: speed of the source (m s^{-1}).
- Use $-$ when the source moves **towards** the observer ($f_o > f_s$).
- Use $+$ when the source moves **away** from the observer ($f_o < f_s$).

Note: this formula applies to a ***moving source*** *and stationary observer only. The Doppler effect for a stationary source and moving observer is not required.*



Sign Convention

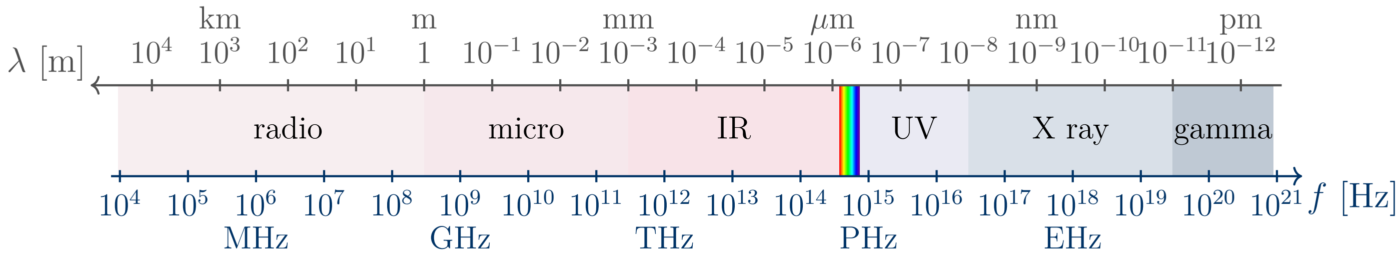
The $\pm$ in $v \pm v_s$ is a common source of error. A useful check: if the source moves towards the observer, the denominator must be *smaller* (so use $-$), making f_o *larger*. If the source moves away, the denominator is *larger* (use $+$), making f_o *smaller*. Always sanity-check your answer against this logic.

The Electromagnetic Spectrum

Properties of Electromagnetic Waves

All electromagnetic (EM) waves:

- Are **transverse** waves.
- Travel at the **same speed** in free space (vacuum): $c = 3.0 \times 10^8$ m s^{-1}.
- Require **no medium** — they can travel through a vacuum.
- Obey $v = f\lambda$ (with $v = c$ in free space).



Region	Approx. wavelength (m)	Approx. frequency (Hz)
Radio waves	$> 10^{-1}$	$< 3 \times 10^9$
Microwaves	10^{-3} to 10^{-1}	3×10^9 to 3×10^{11}
Infrared	7×10^{-7} to 10^{-3}	3×10^{11} to 4×10^{14}
Visible	4×10^{-7} to 7×10^{-7}	4×10^{14} to 7.5×10^{14}
Ultraviolet	10^{-8} to 4×10^{-7}	7.5×10^{14} to 3×10^{16}
X-rays	10^{-13} to 10^{-8}	3×10^{16} to 3×10^{21}
Gamma rays	$< 10^{-13}$	$> 3 \times 10^{21}$

Visible Light

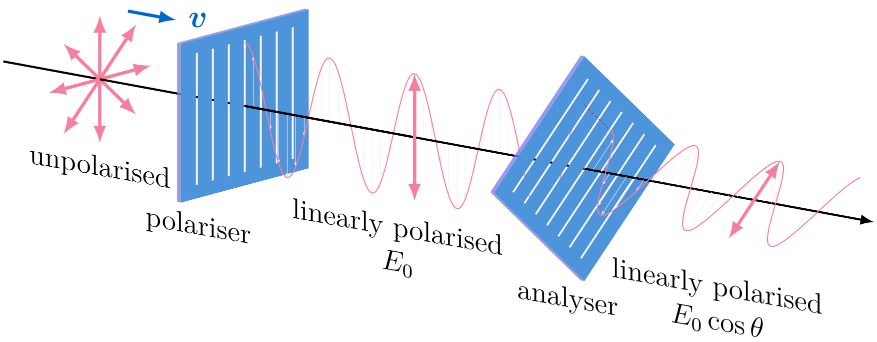
The visible spectrum occupies wavelengths from approximately **400 nm** (violet) to **700 nm** (red) in free space.

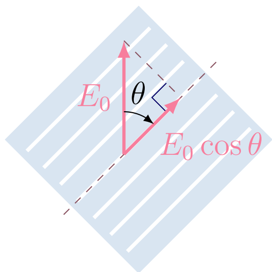
$$400 \text{ nm} = 4 \times 10^{-7} \text{ m} \qquad 700 \text{ nm} = 7 \times 10^{-7} \text{ m}$$

Polarisation and Malus's Law

Polarisation

Polarisation is a phenomenon associated with **transverse waves only**. An unpolarised transverse wave has oscillations in all planes perpendicular to the direction of travel. A **plane-polarised** wave has oscillations confined to a single plane.
Longitudinal waves *cannot* be polarised — their oscillations are already along one axis (the direction of travel).





Malus's Law

When a plane-polarised electromagnetic wave of intensity I_0 passes through a polarising filter at angle θ to the plane of polarisation:

$$I = I_0 \cos^2\theta$$

- $\theta = 0°$ (filter aligned with polarisation): $I = I_0$ — maximum transmission.
- $\theta = 90°$ (filter perpendicular): $I = 0$ — complete extinction.
- $\theta = 45°$: $I = \frac{1}{2}I_0$ — half the intensity transmitted.
- **Note:** the effect of a polarising filter on an *unpolarised* wave is not required (but for reference, it halves the intensity).

Common Mistake

Malus's law ($I = I_0 \cos^2\theta$) only applies to **already plane-polarised** light incident on a second filter. It does not apply to the first polarising filter acting on unpolarised light. Also remember θ is measured from the *transmission axis* of the filter to the plane of polarisation of the incoming wave — not between the two filters themselves unless the incoming wave was polarised along the first filter's axis.

Formula Summary Sheet

Formula	Quantity	Units
$v = f\lambda$	Wave equation	m s^{-1}
$f = 1/T$	Frequency–period	Hz
$\phi = 2\pi d/\lambda$	Phase difference	rad
$I = P/A$	Intensity	W m^{-2}
$I \propto (\text{amplitude})^2$	Intensity–amplitude	—
$f_o = f_s v/(v \pm v_s)$	Doppler effect	Hz
$I = I_0 \cos^2\theta$	Malus's law	W m^{-2}

Key facts to recall:

All EM waves travel at $c = 3.0 \times 10^8$ m s^{-1} in free space and are transverse.

Visible light: 400 nm (violet) to 700 nm (red).

Only **transverse** waves can be polarised.

Doppler: source approaching ⇒ use $-$ in denominator; receding ⇒ use $+$.

Worked Examples

Example 1 — Wave Properties from a CRO

Question: A CRO displays a wave with a period of 2.5 divisions. The time-base is set to 4.0 ms div^{-1} and the y-gain to 0.50 V div^{-1}. The wave has a peak-to-peak height of 3.0 divisions. Find (a) the frequency and (b) the amplitude.

Solution

(a) $T = 2.5 \times 4.0 \times 10^{-3} = 1.0 \times 10^{-2}$ s; $f = 1/T = 1/(1.0 \times 10^{-2}) = \mathbf{100}$ Hz

(b) Peak-to-peak = 3.0 div, so amplitude = 1.5 div. $A = 1.5 \times 0.50 = \mathbf{0.75}$ V

Example 2 — Intensity and Amplitude

Question: A loudspeaker produces a sound wave of amplitude 2.4×10^{-3} m and intensity 1.8 W m^{-2}. The amplitude is increased to 6.0×10^{-3} m. Calculate the new intensity.

Solution

Since $I \propto A^2$:

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{6.0 \times 10^{-3}}{2.4 \times 10^{-3}}\right)^2 = (2.5)^2 = 6.25$$

$$I_2 = 6.25 \times 1.8 = \mathbf{11.3}\ \text{W m}^{-2}$$

Example 3 — Doppler Effect

Question: An ambulance siren emits sound at $f_s = 850$ Hz. The ambulance approaches a stationary observer at 22 m s^{-1}. The speed of sound is 340 m s^{-1}. Calculate the frequency heard by the observer (a) as the ambulance approaches and (b) as it recedes.

Solution

(a) Approaching (use −):

$$f_o = \frac{f_s v}{v - v_s} = \frac{850 \times 340}{340 - 22} = \frac{289\,000}{318} = \mathbf{909}\ \text{Hz}$$

(b) Receding (use +):

$$f_o = \frac{f_s v}{v + v_s} = \frac{850 \times 340}{340 + 22} = \frac{289\,000}{362} = \mathbf{798}\ \text{Hz}$$

Example 4 — Malus's Law

Question: Plane-polarised light of intensity 24 W m^{-2} is incident on a polarising filter. The filter's transmission axis makes an angle of 35° with the plane of polarisation. Calculate the transmitted intensity.

Solution

$$I = I_0 \cos^2\theta = 24 \times \cos^2(35°) = 24 \times (0.819)^2 = 24 \times 0.671 = \mathbf{16.1}\ \text{W m}^{-2}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the terms (a) amplitude, (b) frequency, (c) wavelength and (d) phase difference.

[4 marks]

Q2. A water wave has frequency 3.5 Hz and wavelength 0.24 m. Calculate its speed. Two points on the wave are separated by 0.18 m in the direction of travel. Calculate the phase difference between them.

[4 marks]

Q3. Distinguish between transverse and longitudinal waves. Give one example of each. Explain why polarisation is evidence that light is a transverse wave.

[4 marks]

Q4. State three properties common to all electromagnetic waves. Sketch the electromagnetic spectrum, labelling each region and giving an approximate wavelength range for visible light.

[5 marks]

Section B — Longer Structured Questions

Q5. A source of sound has frequency 640 Hz and moves towards a stationary observer at 15 m s^{-1}. The speed of sound in air is 340 m s^{-1}.

(a) Calculate the frequency heard by the observer.

[2 marks]

(b) The source then moves away from the observer at the same speed. Calculate the new observed frequency and the change in observed frequency between the two situations.

[3 marks]

(c) Explain in terms of wavefronts why the observed frequency is higher when the source approaches.

[2 marks]

Q6. A plane-polarised beam of light of intensity I_0 is incident on a polarising filter.

(a) The filter's transmission axis is at 50° to the plane of polarisation. Calculate the transmitted intensity as a fraction of I_0.

[2 marks]

(b) The filter is rotated to 90°. State the transmitted intensity and explain your answer.

[2 marks]

(c) The intensity of the incident beam is now doubled to $2I_0$ and the filter is returned to 50°. Calculate the new transmitted intensity.

[1 mark]

Q7. A loudspeaker emits sound of frequency 500 Hz and produces an intensity of 3.2×10^{-3} W m^{-2} at a distance of 4.0 m.

(a) The amplitude of the sound wave at this distance is A_1. The speaker power is increased so that the amplitude doubles to $2A_1$. Calculate the new intensity.

[2 marks]

(b) Calculate the wavelength of the sound wave. (Speed of sound = 340 m s^{-1}.)

[2 marks]

(c) Two points in the sound wave are 0.51 m apart along the direction of travel. Calculate the phase difference between them.

[2 marks]

Mark Scheme and Answers

Q1(a). Maximum displacement of a particle from its equilibrium position [1].
Q1(b). Number of complete oscillations per unit time [1].
Q1(c). Distance between two adjacent points that are in phase [1].
Q1(d). Difference in the stage of oscillation between two points on a wave [1].

Q2. $v = f\lambda = 3.5 \times 0.24 = \mathbf{0.84}$ m s^{-1} [2]. Phase difference: $\phi = 2\pi d/\lambda = 2\pi \times 0.18/0.24 = 2\pi \times 0.75 = \mathbf{1.5}\pi$ rad $(= 270°)$ [2].

Q3. Transverse: oscillations perpendicular to direction of travel [1]; e.g. EM waves / rope [1]. Longitudinal: oscillations parallel to direction of travel [1]; e.g. sound [1]. Light can be polarised ⇒ oscillations must be perpendicular to direction of travel ⇒ transverse [bonus/mark scheme dependent on question wording].

Q4. Any three of: all transverse; travel at $c = 3 \times 10^8$ m s^{-1} in free space; require no medium; obey $v = f\lambda$ [3]. Spectrum with correct order (radio → γ) [1]; visible 400–700 nm [1].

Q5(a). $f_o = 640 \times 340/(340 - 15) = 217\,600/325 = \mathbf{669}$ Hz [2].

Q5(b). $f_o = 640 \times 340/(340 + 15) = 217\,600/355 = \mathbf{613}$ Hz [2]; change $= 669 - 613 = \mathbf{56}$ Hz [1].

Q5(c). As the source approaches, successive wavefronts are emitted closer together [1]; the observer encounters more wavefronts per second, so the observed frequency is higher [1].

Q6(a). $I = I_0 \cos^2(50°) = I_0 \times (0.643)^2 = \mathbf{0.413\ I_0}$ [2].

Q6(b). $I = I_0 \cos^2(90°) = \mathbf{0}$ [1]; the transmission axis is perpendicular to the plane of polarisation so no light passes through [1].

Q6(c). $I = 2I_0 \cos^2(50°) = 2 \times 0.413\, I_0 = \mathbf{0.826\ I_0}$ [1].

Q7(a). $I \propto A^2$; amplitude doubles so I increases by factor 4: $I_2 = 4 \times 3.2 \times 10^{-3} = \mathbf{1.28 \times 10^{-2}}$ W m^{-2} [2].

Q7(b). $\lambda = v/f = 340/500 = \mathbf{0.68}$ m [2].

Q7(c). $\phi = 2\pi d/\lambda = 2\pi \times 0.51/0.68 = 2\pi \times 0.75 = \mathbf{1.5}\pi$ rad $(270°)$ [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define displacement, amplitude, period, frequency, wavelength, phase difference	
☐	Use the CRO to determine frequency and amplitude	
☐	Derive and use $v = f\lambda$	
☐	Use $I = P/A$ and $I \propto (\text{amplitude})^2$	
☐	Distinguish transverse and longitudinal waves with examples	
☐	Sketch and interpret displacement–distance graphs for both wave types	
☐	Understand the Doppler effect for a moving source	
☐	Use $f_o = f_s v/(v \pm v_s)$ with correct sign convention	
☐	State properties common to all EM waves	
☐	Recall the EM spectrum regions and approximate wavelength ranges	
☐	Recall visible light wavelength range (400–700 nm)	
☐	Explain polarisation and why it is evidence for transverse waves	
☐	Apply Malus's law $I = I_0 \cos^2\theta$ to plane-polarised light	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Waves is one of those topics where a clear mental picture goes a long way. Sketch the wave, label the amplitude and wavelength, decide whether oscillations are parallel or perpendicular to travel — and most of the topic falls into place from there.

CIE International A Level Physics

Topic 8

Superposition

Revision Booklet

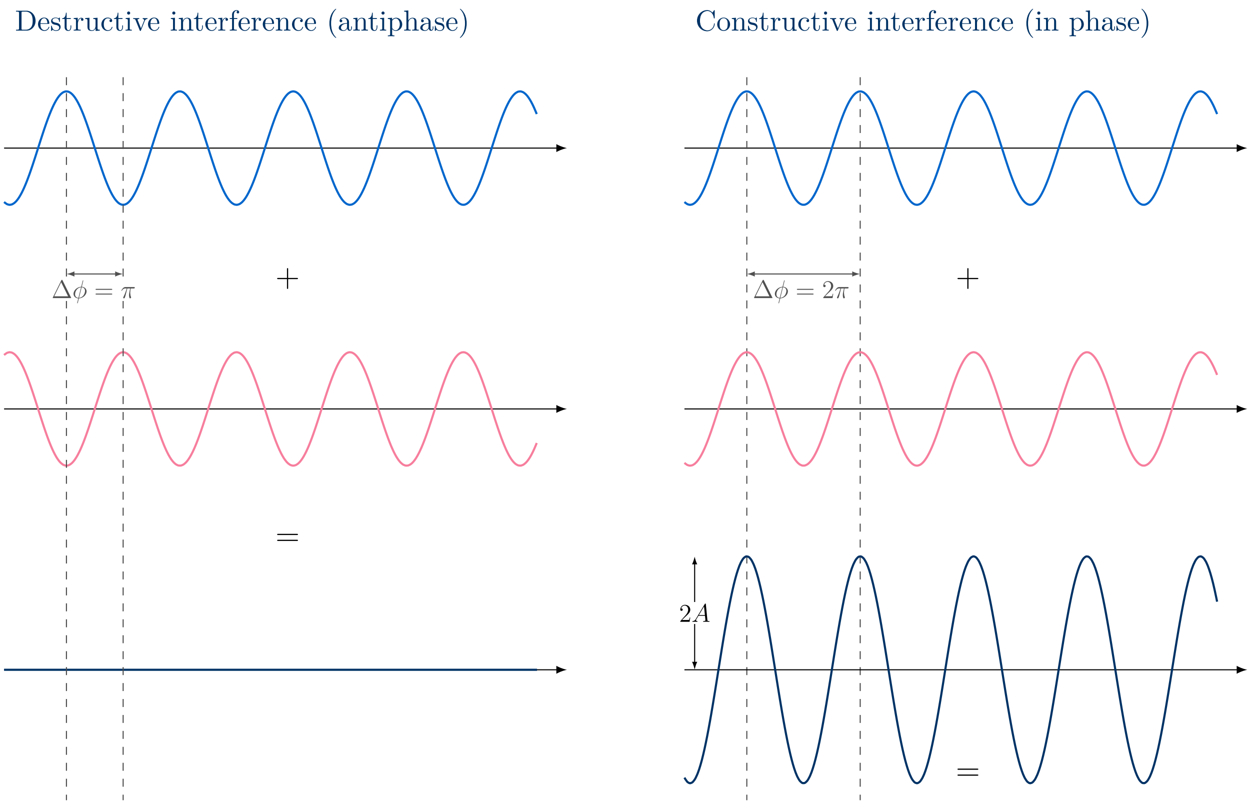
This booklet covers:

- The Principle of Superposition
- Stationary Waves: Formation and Properties
- Stationary Waves in Strings and Air Columns
- Diffraction
- Two-Source Interference and Coherence
- The Diffraction Grating

The Principle of Superposition

Principle of Superposition

When two or more waves meet at a point, the **resultant displacement** at that point is equal to the **vector sum** of the individual displacements of each wave at that point.



Stationary Waves: Formation and Properties

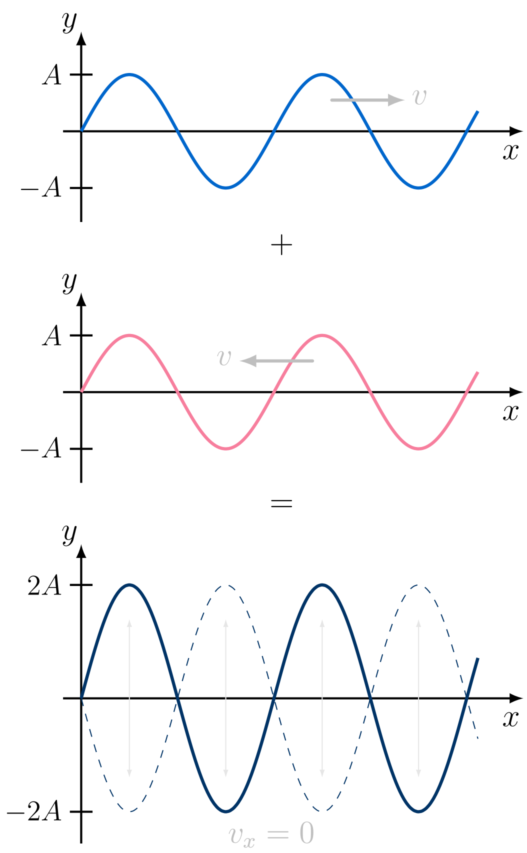
Stationary (Standing) Wave

A **stationary wave** is formed when two progressive waves of the **same frequency**, **same amplitude** and travelling in **opposite directions** superpose. The result is a wave pattern that does not travel — energy is stored rather than transmitted.

Nodes and Antinodes

- **Node**: a point of **zero amplitude** — particles are permanently at rest. Destructive superposition occurs here at all times.
- **Antinode**: a point of **maximum amplitude** — particles oscillate with the greatest displacement. Constructive superposition here.
- Adjacent nodes are separated by $\lambda/2$.
- Adjacent antinodes are also separated by $\lambda/2$.
- A node and an adjacent antinode are separated by $\lambda/4$.

Formation of a stationary wave



Differences: Stationary vs Progressive Waves

	Progressive wave	Stationary wave
Energy	Transferred in direction of travel	Stored; not transferred
Amplitude	Same for all particles	Varies from 0 (node) to max (antinode)
Phase	Varies continuously along wave	All particles between two nodes are in phase; adjacent segments are in antiphase
Wavelength	Distance between adjacent in-phase points	Twice the node-to-node distance

Determining Wavelength from a Stationary Wave

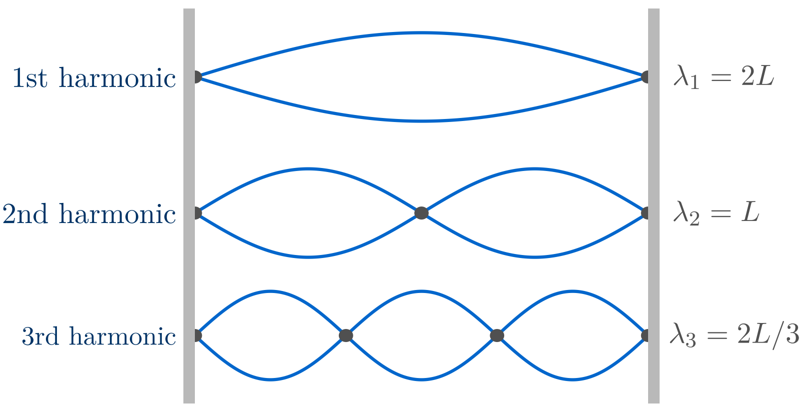
Since adjacent nodes are separated by $\lambda/2$:

$$\lambda = 2 \times (\text{node-to-node distance})$$

Measure the distance across several node spacings for greater accuracy.

Stationary Waves in Strings and Air Columns

Stretched string (both ends fixed)



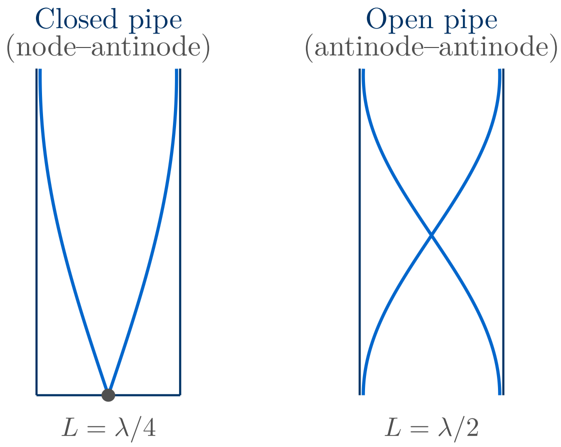
Harmonics in a String

For a string of length L fixed at both ends, the nth harmonic has:

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L} = nf_1$$

The **fundamental** (1st harmonic) has one antinode and $f_1 = v/2L$.

Air columns



Standing Waves in Air

Standing waves in air are created in a similar way. Waves are reflected from the ends of a tube and the waves travelling in opposite directions superpose to form a standing wave as long as the boundary conditions are satisfied.

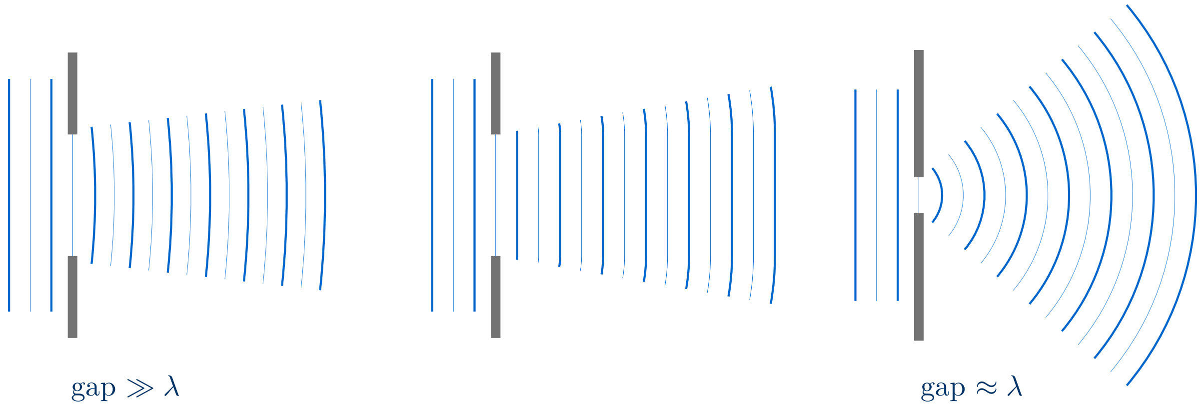
- For a closed pipe: A **Node** at the closed end and an **Antinode** at the open end.
- For an open pipe: An **Antinode** at both ends.

Diffraction

Diffraction

Diffraction is the spreading of a wave as it passes through a gap or around an obstacle. It is a property of all waves.

- Diffraction is most pronounced when the **gap width** $\approx$ **wavelength**.
- When gap width $\gg \lambda$: very little spreading — the wave passes through with little diffraction.
- When gap width $\approx \lambda$: maximum spreading — the wave spreads out as a near-semicircle.
- When gap width $< \lambda$: the gap acts almost like a point source.



Two-Source Interference and Coherence

Coherence

Two sources are **coherent** if they have:

- The **same frequency** (and hence wavelength).
- A **constant phase difference** (not necessarily zero).

Coherence is essential for a stable interference pattern to be observed.

Interference

Interference is the superposition of waves from two coherent sources, producing a pattern of **maxima** (constructive) and **minima** (destructive).

- **Constructive interference**: path difference $= n\lambda$ ($n = 0, 1, 2, \ldots$); waves arrive in phase.
- **Destructive interference**: path difference $= (n + \frac{1}{2})\lambda$; waves arrive in antiphase.

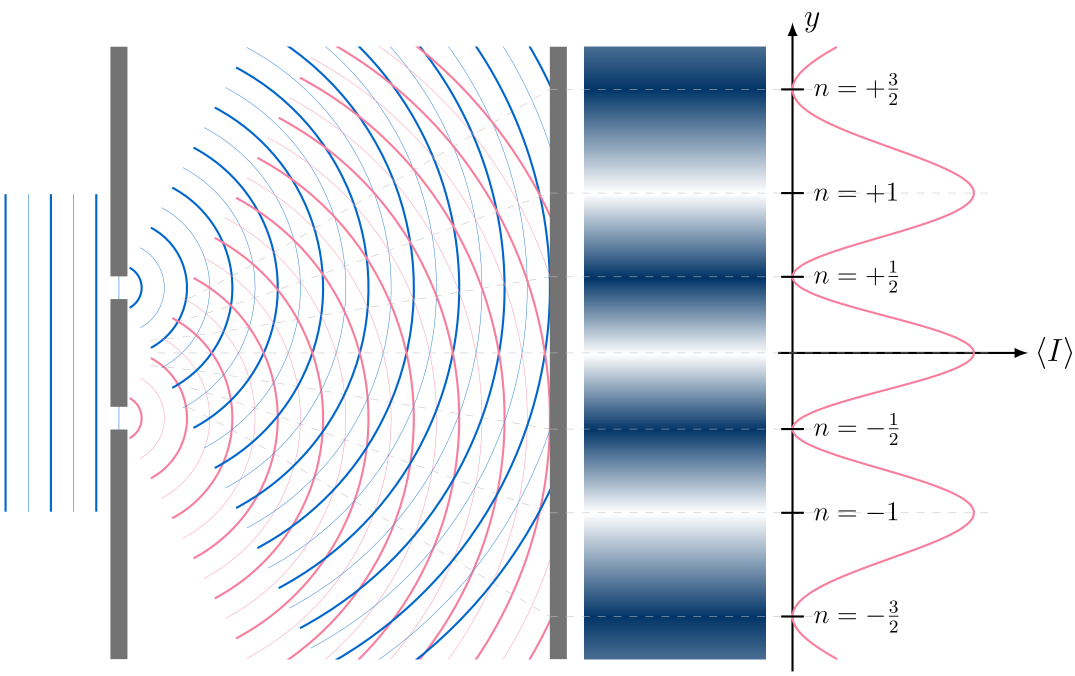
Conditions for Observable Two-Source Fringes

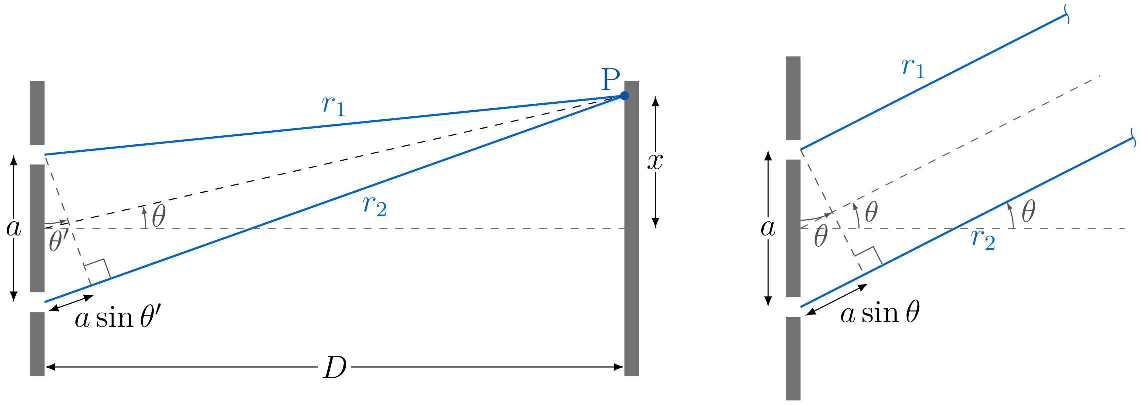
1. Sources must be **coherent** (same frequency, constant phase difference).
2. Sources must have **similar amplitudes** (for clear minima — zero intensity at destructive points).
3. For light: sources must be **monochromatic** or fringes from different wavelengths overlap and blur.
4. The **slit separation** a must be comparable to λ (much larger $\Rightarrow$ fringes too close together to resolve).

Double-Slit Fringe Spacing: $\lambda = ax/D$

$$\lambda = \frac{ax}{D}$$

- a: slit separation (m) — centre to centre.
- x: fringe spacing (m) — distance between adjacent bright fringes.
- D: distance from slits to screen (m).
- Valid when $D \gg a$ (small-angle approximation).





Derivation of Double Slit formula

- For small angles $r_1 \approx r_2$; $\theta' \approx \theta$
- Path difference $= a\sin\theta$
- For the first maximum: Path difference $= \lambda = a\sin\theta$
- By similar triangles and $\tan\theta \approx \sin\theta$: $\frac{x}{D} = \frac{a\sin\theta}{a}$

$$\lambda = \frac{ax}{D}$$

Common Mistakes with $\lambda = ax/D$

- a is the **slit separation**, not the slit width.
- x is the fringe **spacing** (centre of one fringe to the centre of the next), not the total width of the pattern.
- Increasing D or decreasing a *increases* fringe spacing x — fringes spread out.
- The formula requires $D \gg a$; if this is not satisfied, the small-angle approximation breaks down.

The Diffraction Grating

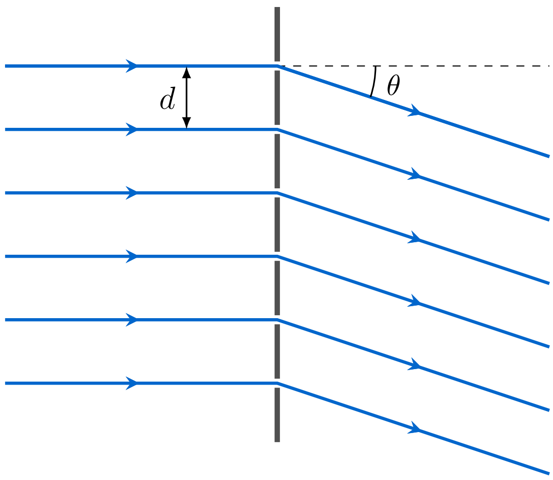
Diffraction Grating

A **diffraction grating** consists of many equally-spaced parallel slits. When light is incident normally, each slit diffracts the light, and the diffracted beams from all slits interfere. Sharp, bright maxima (orders) are produced at specific angles.

Grating Equation

$$d\sin\theta = n\lambda$$

- d: grating spacing — distance between adjacent slits (m). If a grating has N lines per metre, then $d = 1/N$.
- θ: angle of the nth order maximum to the straight-through (zero-order) direction.
- n: order number ($n = 0, \pm1, \pm2, \ldots$).
- λ: wavelength of light (m).
- Maximum possible order: $n_{\text{max}} = \lfloor d/\lambda \rfloor$ (since $\sin\theta \le 1$).



Using a Diffraction Grating to Measure Wavelength

1. Direct light normally at the grating.
2. Measure the angle θ_n to the nth order maximum using a protractor or spectrometer scale.
3. Apply $d\sin\theta_n = n\lambda$ to find λ.
4. Repeat for several orders and take a mean for greater accuracy.
5. The grating spacing d is found from $d = 1/N$ where N is the number of lines per metre (usually stated as lines per mm on the grating — convert carefully).

Common Mistakes with the Grating Equation

- Lines per mm must be converted: e.g. 300 lines mm^{-1} = 3×10^5 lines m^{-1}, so $d = 1/(3 \times 10^5)$ m.
- $\sin\theta$ cannot exceed 1, so check that your calculated n_{max} is physically possible.
- The grating produces sharper, more widely spaced maxima than double slits — do not confuse the two formulae.

Formula Summary Sheet

Formula	Quantity	Units
$\lambda = 2\times$ (node spacing)	Wavelength from stationary wave	m
$f_n = nv/2L$	nth harmonic in a string	Hz
$\lambda = ax/D$	Double-slit fringe spacing	m
$d\sin\theta = n\lambda$	Diffraction grating	m
$d = 1/N$	Grating spacing from line density	m

Key facts:
Superposition: resultant displacement = vector sum of individual displacements.
Nodes separated by $\lambda/2$; **antinodes** separated by $\lambda/2$.
Constructive: path difference = $n\lambda$; **Destructive**: path difference = $(n + \frac{1}{2})\lambda$.
Coherence: same frequency + constant phase difference.
Diffraction is maximum when gap width $\approx \lambda$.

Worked Examples

Example 1 — Wavelength from a Stationary Wave

Question: A stretched string vibrates in its third harmonic. The distance between the first and last nodes is 72 cm. Find the wavelength.

Solution

The third harmonic has 4 nodes and 3 loops. Distance from first to last node = $3 \times (\lambda/2)$:

$$72 \text{ cm} = 3 \times \frac{\lambda}{2} \implies \lambda = \frac{2 \times 72}{3} = \mathbf{48} \text{ cm} = 0.48 \text{ m}$$

Example 2 — Double-Slit Interference

Question: In a Young's double-slit experiment, the slit separation is 0.45 mm, the screen is 1.8 m from the slits, and the fringe spacing is 2.4 mm. Calculate the wavelength of light used.

Solution

$$\lambda = \frac{ax}{D} = \frac{(0.45 \times 10^{-3})(2.4 \times 10^{-3})}{1.8} = \frac{1.08 \times 10^{-6}}{1.8} = \mathbf{6.0 \times 10^{-7}} \text{ m} = 600 \text{ nm}$$

Example 3 — Diffraction Grating

Question: A diffraction grating has 400 lines per mm. Light of wavelength 589 nm is incident normally. Find (a) the grating spacing, (b) the angle of the second-order maximum, and (c) the highest order observable.

Solution

(a) $d = \dfrac{1}{400 \times 10^3 \text{ m}^{-1}} = 2.5 \times 10^{-6}$ m

(b) $d \sin\theta = n\lambda \implies \sin\theta = \dfrac{2 \times 589 \times 10^{-9}}{2.5 \times 10^{-6}} = \dfrac{1.178 \times 10^{-6}}{2.5 \times 10^{-6}} = 0.4712$

$\theta = \arcsin(0.4712) = \mathbf{28.1}^\circ$

(c) $n_{max} = \dfrac{d}{\lambda} = \dfrac{2.5 \times 10^{-6}}{589 \times 10^{-9}} = 4.24 \implies n_{max} = \mathbf{4}$ (must be a whole number ≤ 4.24)

Example 4 — Path Difference and Interference

Question: Two coherent sources of sound ($\lambda = 0.40$ m) are placed 1.5 m apart. A detector at a point P has path differences of 1.00 m from the two sources. State whether P is a maximum or minimum and explain why.

Solution

Path difference = 1.00 m.
$1.00/\lambda = 1.00/0.40 = 2.5$
Path difference = $2.5\lambda = (2 + \frac{1}{2})\lambda$ — this is a half-integer multiple of λ.
Therefore P is a **minimum** (destructive interference): the waves arrive in antiphase.

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the principle of superposition. Distinguish between constructive and destructive interference in terms of path difference.

[4 marks]

Q2. Explain what is meant by (a) a node and (b) an antinode in a stationary wave. State the distance between adjacent nodes in terms of wavelength.

[3 marks]

Q3. Explain what is meant by diffraction. State the condition on gap width relative to wavelength for maximum diffraction to occur.

[3 marks]

Q4. Define coherence. State three conditions required for a stable two-source interference pattern to be observed using light.

[4 marks]

Section B — Longer Structured Questions

Q5. A stationary wave is set up on a stretched string of length 0.90 m fixed at both ends. The string vibrates in its second harmonic. The speed of waves on the string is 120 m s^{-1}.

(a) Sketch the stationary wave pattern, labelling all nodes and antinodes.

[2 marks]

(b) Calculate the wavelength and frequency of this harmonic.

[3 marks]

(c) Describe how the stationary wave is formed, referring to the two progressive waves involved.

[3 marks]

Q6. In a Young's double-slit experiment using light of wavelength 540 nm, the slit separation is 0.30 mm.

(a) Calculate the fringe spacing when the screen is 2.0 m from the slits.

[2 marks]

(b) The screen is moved further from the slits. State and explain the effect on the fringe spacing.

[2 marks]

(c) The experiment is repeated with white light. Describe and explain the appearance of the fringe pattern.

[3 marks]

Q7. A diffraction grating with 600 lines per mm is illuminated normally by monochromatic light.

(a) Calculate the grating spacing d.

[1 mark]

(b) The second-order maximum is observed at $\theta = 43.2°$. Calculate the wavelength of the light.

[2 marks]

(c) Show that the third-order maximum cannot be observed for this wavelength and grating.

[2 marks]

(d) Explain why the maxima from a diffraction grating are much sharper than the fringes from a double slit.

[2 marks]

Mark Scheme and Answers

Q1. Resultant displacement at any point equals the vector sum of the individual displacements [1]. Constructive: path difference $= n\lambda$; waves arrive in phase; amplitude/intensity is maximum [2]. Destructive: path difference $= (n + \frac{1}{2})\lambda$; waves arrive in antiphase; amplitude/intensity is minimum (zero if amplitudes equal) [1].

Q2(a). Node: point of zero (minimum) amplitude in a stationary wave; particles permanently at rest [1].
Q2(b). Antinode: point of maximum amplitude; particles oscillate with greatest displacement [1].
Distance between adjacent nodes $= \lambda/2$ [1].

Q3. Diffraction: spreading of a wave as it passes through a gap or around an obstacle [2]. Maximum diffraction when gap width $\approx \lambda$ [1].

Q4. Coherence: two sources with same frequency and constant phase difference [1]. Conditions for light fringes: sources coherent [1]; sources monochromatic [1]; slit separation comparable to λ [1].

Q5(a). Second harmonic: 3 nodes (at both ends + centre), 2 antinodes (at quarter-points) [2].

Q5(b). $\lambda_2 = L = 0.90$ m [1] (for 2nd harmonic, $L = \lambda$); $f = v/\lambda = 120/0.90 = \mathbf{133}$ Hz [2].

Q5(c). Two progressive waves of same frequency and amplitude travel in opposite directions [1]; they superpose according to the principle of superposition [1]; the resultant produces fixed nodes and antinodes — the pattern does not travel [1].

Q6(a). $x = \lambda D/a = (540 \times 10^{-9} \times 2.0)/(0.30 \times 10^{-3}) = 1.08 \times 10^{-6}/3 \times 10^{-4} = \mathbf{3.6 \times 10^{-3}}$ m $= 3.6$ mm [2].

Q6(b). Fringe spacing **increases** [1]; from $x = \lambda D/a$, increasing D increases x directly [1].

Q6(c). Central fringe is white [1]; coloured fringes either side — violet (shortest λ, smallest x) closest to centre, red (longest λ) furthest [1]; fringes overlap at large distances giving white/blurred pattern [1].

Q7(a). $d = 1/(600 \times 10^3) = \mathbf{1.67 \times 10^{-6}}$ m [1].

Q7(b). $\lambda = d \sin\theta/n = (1.67 \times 10^{-6} \times \sin 43.2°)/2 = (1.67 \times 10^{-6} \times 0.684)/2 = \mathbf{5.71 \times 10^{-7}}$ m ≈ 571 nm [2].

Q7(c). For $n = 3$: $\sin\theta = 3\lambda/d = 3 \times 5.71 \times 10^{-7}/1.67 \times 10^{-6} = 1.026 > 1$ [1]; since $\sin\theta$ cannot exceed 1, the third order does not exist [1].

Q7(d). The grating has many slits [1]; for a maximum to form, all slits must constructively interfere simultaneously — any slight deviation from the exact angle causes destructive interference from the large number of slits, producing very sharp peaks [1].

Revision Checklist

	Learning Objective	Confidence (1–3)
☐	State and apply the principle of superposition	
☐	Explain formation of a stationary wave graphically	
☐	Identify nodes and antinodes; state their separation ($\lambda/2$)	
☐	Describe stationary wave experiments: strings, air columns, microwaves	
☐	Determine wavelength from node/antinode positions	
☐	Explain the differences between stationary and progressive waves	
☐	Define diffraction and explain the effect of gap width relative to λ	
☐	Define coherence (same frequency + constant phase difference)	
☐	State conditions required to observe two-source interference fringes	
☐	Use $\lambda = ax/D$ for double-slit interference	
☐	Distinguish constructive and destructive interference by path difference	
☐	Use $d \sin\theta = n\lambda$ for the diffraction grating	
☐	Calculate grating spacing from lines per mm	
☐	Find the maximum order observable ($\sin\theta \leq 1$)	
☐	Describe how a grating is used to measure wavelength	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Superposition is the single idea that unifies this whole topic. Stationary waves, interference fringes and grating maxima are all just superposition playing out in different geometries. Get the path difference conditions clear in your mind and the rest follows naturally.

CIE International A Level Physics

Topic 9

Electricity

Revision Booklet

This booklet covers:

- Electric Current and Charge Carriers
- Charge, Current and the Drift Velocity Equation
- Potential Difference and Power
- Resistance and Ohm's Law
- I–V Characteristics
- Resistivity, LDRs and Thermistors

Electric Current and Charge Carriers

Electric Current

An **electric current** is a flow of **charge carriers**. In metals the charge carriers are free (conduction) electrons; in electrolytes and semiconductors, other carriers (ions, holes) may contribute.

$$I = \frac{\Delta Q}{\Delta t} \qquad \Longrightarrow \qquad Q = It$$

- I: current (A); Q: charge (C); t: time (s).
- $1\ \text{A} = 1\ \text{C s}^{-1}$.
- Conventional current flows from + to −; electron flow is from − to +.

Quantisation of Charge

The charge on any charge carrier is an **integer multiple** of the elementary charge e:

$$Q = ne \qquad e = 1.60 \times 10^{-19}\ \text{C}$$

Charge is **quantised** — it cannot take continuous values; it only exists in discrete packets of e.

The Drift Velocity Equation

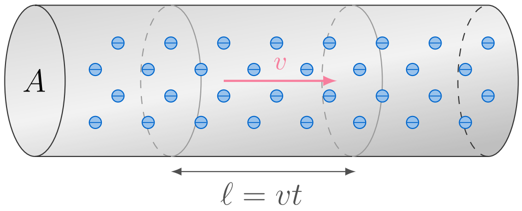
$I = Anvq$

For a conductor carrying current I:

$$I = Anvq$$

- A: cross-sectional area of conductor (m^2).
- n: **number density** of charge carriers (m^{-3}) — number of carriers per unit volume.
- v: **mean drift velocity** of carriers (m s^{-1}).
- q: charge on each carrier (C); for electrons $q = e = 1.60 \times 10^{-19}$ C.

Derivation of $I = Anvq$



Deriving $I = Anvq$

- In time t, carriers travel distance $\ell = vt$.
- Volume swept past the cross-section: $V = A\ell = Avt$.
- Number of carriers in this volume: $N = nAvt$.
- Total charge past the cross-section: $Q = Nq = nAvtq$.
- Therefore: $I = Q/t = nAvq$.

Using $I = Anvq$ to Compare Conductors

- **Metals** have very high n ($\sim 10^{28}\ \text{m}^{-3}$) $\Rightarrow$ very slow drift velocity ($\sim$ mm s^{-1}) even for large currents.
- **Semiconductors** have lower n $\Rightarrow$ higher v for the same current.
- At a junction where the wire narrows (A decreases), v must increase (since I, n, q are constant).
- Increasing temperature in a metal barely changes n but reduces v (increased resistance).

Potential Difference and Power

Potential Difference

The **potential difference** (p.d.) across a component is defined as the **energy transferred per unit charge** passing through it.

$$V = \frac{W}{Q}$$

- V: potential difference (V); W: energy transferred (J); Q: charge (C).
- Unit: volt (V) $\equiv$ J C^{-1}.
- 1 V: 1 joule of energy transferred per coulomb of charge.

Electrical Power

$$P = VI = I^2R = \frac{V^2}{R}$$

Derivation: Power is energy per unit time. $P = W/t = (QV)/t = IV$. Substituting $V = IR$: $P = I(IR) = I^2R$; or $I = V/R$: $P = (V/R)V = V^2/R$.

- All three forms are equivalent for a resistor at any instant.
- Use $P = VI$ when both V and I are known.
- Use $P = I^2R$ when current and resistance are known.
- Use $P = V^2/R$ when voltage and resistance are known.

Common Mistake

Energy is $W = Pt = VIt$, not $W = VI$. Always multiply power by time to get energy. Also, $P = V^2/R$ and $P = I^2R$ apply to a single resistor; for a circuit with multiple components, identify V and I for each component separately.

Resistance and Ohm's Law

Resistance

The **resistance** of a component is defined as the ratio of the potential difference across it to the current flowing through it:

$$R = \frac{V}{I}$$

- Unit: ohm (Ω) $\equiv$ V A^{-1}.
- Resistance is a property of the component — it describes how much it opposes current flow.

Ohm's Law

Ohm's law states that the current through a metallic conductor is **directly proportional** to the potential difference across it, **provided the temperature remains constant**:

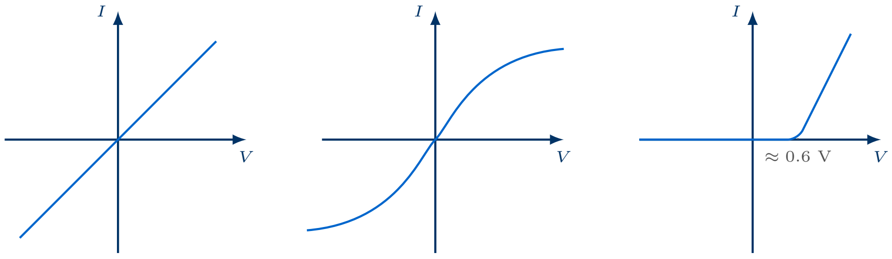
$$V \propto I \quad \Leftrightarrow \quad V = IR \quad (R \text{ constant})$$

A component that obeys Ohm's law is called **ohmic**. The I–V graph for an ohmic conductor is a straight line through the origin.

Ohm's Law is not a Definition of Resistance

$R = V/I$ *defines* resistance for any component. Ohm's law is a separate statement that R is *constant* (independent of V and I) for a metallic conductor at constant temperature. Not all components obey Ohm's law.

I–V Characteristics



Metallic conductor (constant temp.)
Filament lamp (gradient decreases)
Semiconductor diode (threshold ≈ 0.6 V)

Interpreting I–V Graphs

- **Metallic conductor (ohmic):** straight line through origin — gradient = $1/R$ (constant). Both forward and reverse biased sections are identical straight lines.
- **Filament lamp:** S-shaped curve — gradient decreases at high V and I, meaning resistance increases. Cause: increasing current ⇒ increasing temperature ⇒ increased resistance (more collisions between electrons and vibrating lattice ions).
- **Semiconductor diode:** conducts appreciably only above the **threshold voltage** (≈ 0.6 V for silicon) in the forward direction; in reverse bias, current is essentially zero (tiny reverse leakage).

Resistivity

Resistivity

The resistance of a uniform conductor depends on its material, length and cross-sectional area:

$$R = \frac{\rho L}{A}$$

- ρ: **resistivity** of the material (Ω m).
- L: length of conductor (m).
- A: cross-sectional area (m^2).
- Resistivity is a **material property** — it does not depend on the shape of the sample.
- $R \propto L$ (double the length ⇒ double the resistance).
- $R \propto 1/A$ (double the area ⇒ halve the resistance).

Resistivity

The **resistivity** ρ of a material is defined by:

$$\rho = \frac{RA}{L}$$

It is the resistance of a 1 m cube of the material between opposite faces. Unit: Ω m.

LDRs and Thermistors

Light-Dependent Resistor (LDR)

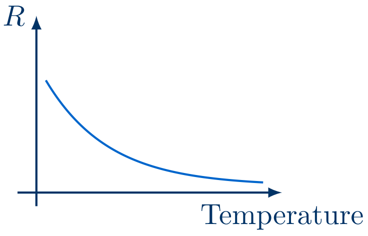
The **resistance of an LDR decreases** as **light intensity increases**.

- In darkness: resistance can be $\sim$ MΩ.
- In bright light: resistance falls to $\sim$ kΩ or lower.
- Mechanism: photons give electrons enough energy to break free and become charge carriers, increasing n and thus conductivity.

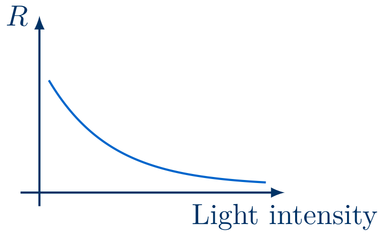
Thermistor (NTC)

The **resistance of an NTC thermistor decreases** as **temperature increases**.

- NTC = Negative Temperature Coefficient.
- Mechanism: rising temperature gives more electrons enough energy to act as charge carriers, increasing n and reducing resistivity.
- Opposite behaviour to metals (where resistance *increases* with temperature because v is reduced).



Thermistor (NTC)



LDR

Comparing Metals Thermistors and LDRs

Component	Effect on resistance	Reason
Metal wire	Resistance **increases** with T	More lattice vibration; v decreases
NTC Thermistor	Resistance **decreases** with T	More charge carriers; n increases
LDR	Resistance **decreases** with light	More charge carriers; n increases
Filament lamp	Resistance **increases** with I	Temperature rises (same as metal)

Formula Summary Sheet

Formula	Quantity	Units
$Q = It$	Charge	C
$Q = ne$	Quantisation of charge	C
$I = Anvq$	Current (drift velocity)	A
$V = W/Q$	Potential difference	V
$P = VI = I^2R = V^2/R$	Electrical power	W
$W = VIt$	Electrical energy	J
$R = V/I$	Resistance (definition)	Ω
$V = IR$	Ohm's law	V
$R = \rho L/A$	Resistivity	Ω

Key definitions to learn word-for-word:
P.d.: energy transferred per unit charge through a component.
Resistance: ratio of p.d. across a component to current through it ($R = V/I$).
Ohm's law: current is directly proportional to p.d. at constant temperature.
Resistivity: $\rho = RA/L$; a material property independent of sample dimensions.

Worked Examples

Example 1 — Drift Velocity

Question: A copper wire of cross-sectional area 1.5 mm^2 carries a current of 3.0 A. The number density of free electrons in copper is 8.5×10^{28} m^{-3}. Calculate the mean drift velocity of the electrons.

Solution

Rearrange $I = Anvq$ for v:

$$v = \frac{I}{Anq} = \frac{3.0}{(1.5 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})}$$

$$v = \frac{3.0}{1.5 \times 10^{-6} \times 1.36 \times 10^{10}} = \frac{3.0}{2.04 \times 10^{4}} = \mathbf{1.47 \times 10^{-4}}\ \text{m s}^{-1}$$

This is about 0.15 mm s^{-1} — extremely slow, yet the effect of the current is felt almost instantaneously because the electric field propagates at close to the speed of light.

Example 2 — Power and Energy

Question: A 60 Ω resistor is connected to a 12 V supply for 5.0 min. Calculate (a) the current, (b) the power dissipated, and (c) the total energy transferred.

Solution

(a) $I = V/R = 12/60 = \mathbf{0.20}$ A

(b) $P = V^2/R = 144/60 = \mathbf{2.4}$ W (or $P = I^2R = 0.04 \times 60 = 2.4$ W ✓)

(c) $W = Pt = 2.4 \times (5.0 \times 60) = 2.4 \times 300 = \mathbf{720}$ J

Example 3 — Resistivity

Question: A nichrome wire of length 0.80 m and diameter 0.50 mm has resistance 4.4 Ω. Calculate the resistivity of nichrome.

Solution

Cross-sectional area: $A = \pi r^2 = \pi(0.25 \times 10^{-3})^2 = 1.96 \times 10^{-7}$ m^2

$$\rho = \frac{RA}{L} = \frac{4.4 \times 1.96 \times 10^{-7}}{0.80} = \frac{8.63 \times 10^{-7}}{0.80} = \mathbf{1.08 \times 10^{-6}\ \Omega}\ \text{m}$$

Example 4 — I–V Characteristic

Question: The current through a filament lamp increases from 0.20 A to 0.60 A when the voltage increases from 3.0 V to 6.0 V. Show that the lamp does not obey Ohm's law.

Solution

At $V = 3.0$ V: $R_1 = V/I = 3.0/0.20 = 15$ Ω
At $V = 6.0$ V: $R_2 = V/I = 6.0/0.60 = 10$ Ω
$R_1 \neq R_2$ — resistance is not constant; it decreases as current decreases (i.e. as temperature decreases). This is characteristic of a filament lamp: at higher current the filament is

hotter and resistance is higher. The lamp **does not obey Ohm's law**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) electric current and (b) potential difference. State the SI unit of each.

[4 marks]

Q2. Explain what is meant by saying that charge is *quantised*. A current of 2.5 mA flows for 4.0 min. Calculate the charge transferred and the number of electrons passing a point.

[4 marks]

Q3. State Ohm's law and sketch the I–V characteristics of (a) a metallic conductor at constant temperature and (b) a filament lamp. Explain why the graphs have different shapes.

[5 marks]

Q4. A semiconductor diode is connected in a circuit. Sketch the I–V characteristic and mark the threshold voltage. Explain what happens in reverse bias.

[3 marks]

Section B — Longer Structured Questions

Q5. A conductor has cross-sectional area A, number density of charge carriers n, and carries a current I.

(a) Derive the expression $I = Anvq$, defining all symbols.

[3 marks]

(b) A copper wire ($n = 8.5 \times 10^{28}$ m^{-3}, diameter 1.2 mm) carries a current of 5.0 A. Calculate the mean drift velocity of the electrons.

[3 marks]

(c) The wire is replaced with one of the same material but half the diameter. The current remains 5.0 A. State and explain what happens to the drift velocity.

[2 marks]

Q6. A uniform resistance wire of length 1.20 m and cross-sectional area 3.5×10^{-7} m^2 is made of a material of resistivity 4.9×10^{-7} Ω m.

(a) Calculate the resistance of the wire.

[2 marks]

(b) A potential difference of 6.0 V is applied across the wire. Calculate the current through it and the power dissipated.

[3 marks]

(c) The wire is stretched so its length doubles but its volume remains constant. Show that the resistance increases by a factor of 4.

[3 marks]

Q7. A thermistor is connected in series with a 4.7 kΩ fixed resistor across a 9.0 V supply. At 20 °C the thermistor has resistance 8.2 kΩ.

(a) Calculate the current in the circuit and the voltage across the thermistor at 20 °C.

[3 marks]

(b) Describe and explain what happens to the current as the temperature increases.

[2 marks]

(c) Calculate the power dissipated in the fixed resistor at 20 °C.

[2 marks]

Mark Scheme and Answers

Q1(a). Electric current is the rate of flow of charge [1]; unit: ampere (A) [1].
Q1(b). Potential difference is the energy transferred per unit charge through a component [1]; unit: volt (V) [1].

Q2. Charge is quantised means it only exists in discrete multiples of $e = 1.60 \times 10^{-19}$ C [1]. $Q = It = 2.5 \times 10^{-3} \times 240 = \mathbf{0.60}$ C [1]. Number of electrons: $N = Q/e = 0.60/(1.60 \times 10^{-19}) = \mathbf{3.75 \times 10^{18}}$ [2].

Q3. Ohm's law: current is proportional to p.d. at constant temperature [1]. Metallic conductor: straight line through origin [1]. Filament lamp: S-shaped curve, decreasing gradient [1]. Metal: R constant so linear [1]; lamp: higher $I \Rightarrow$ higher $T \Rightarrow$ higher R, so gradient I/V decreases [1].

Q4. Correct exponential-ish forward curve with threshold ≈ 0.6 V marked [2]; in reverse bias current is (approximately) zero — the diode blocks current [1].

Q5(a). In time t, carriers move distance vt; volume $= Avt$; number $= nAvt$; charge $= nAvtq$; current $I = Q/t = nAvq$ [3].

Q5(b). $A = \pi(0.6 \times 10^{-3})^2 = 1.131 \times 10^{-6}$ m^2; $v = I/(Anq) = 5.0/(1.131 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.60 \times 10^{-19}) = \mathbf{3.26 \times 10^{-4}}$ m s^{-1} [3].

Q5(c). Diameter halves $\Rightarrow$ radius halves $\Rightarrow$ area reduces by factor 4 [1]; since $I = Anvq$ is constant and A is quartered, v must increase by factor 4 [1].

Q6(a). $R = \rho L/A = (4.9 \times 10^{-7} \times 1.20)/(3.5 \times 10^{-7}) = \mathbf{1.68\ \Omega}$ [2].

Q6(b). $I = V/R = 6.0/1.68 = \mathbf{3.57}$ A [1]; $P = VI = 6.0 \times 3.57 = \mathbf{21.4}$ W [2].

Q6(c). Volume constant: $A'L' = AL$; $L' = 2L$ so $A' = A/2$ [1]; new $R' = \rho L'/A' = \rho(2L)/(A/2) = 4\rho L/A = 4R$ [2].

Q7(a). $R_{\text{total}} = 8200 + 4700 = 12900\ \Omega$; $I = 9.0/12900 = \mathbf{6.98 \times 10^{-4}}$ A [2]; $V_T = IR_T = 6.98 \times 10^{-4} \times 8200 = \mathbf{5.72}$ V [1].

Q7(b). As temperature increases, thermistor resistance decreases [1]; total resistance decreases so current increases (from $V = IR$) [1].

Q7(c). $P = I^2R = (6.98 \times 10^{-4})^2 \times 4700 = 4.87 \times 10^{-7} \times 4700 = \mathbf{2.29 \times 10^{-3}}$ W $\approx$ 2.3 mW [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Understand that current is a flow of charge carriers	
☐ State that charge is quantised ($Q = ne$)	
☐ Use $Q = It$ to find charge, current or time	
☐ Use and derive $I = Anvq$, defining all terms	
☐ Define p.d. as energy transferred per unit charge; use $V = W/Q$	
☐ Use all three power formulae: $P = VI$, $P = I^2R$, $P = V^2/R$	
☐ Define resistance as $R = V/I$	
☐ State Ohm's law and identify ohmic/non-ohmic behaviour	
☐ Sketch and interpret I–V graphs for metal, filament lamp and diode	
☐ Explain why filament lamp resistance increases with current	
☐ Use $R = \rho L/A$ and define resistivity	
☐ Explain how resistance of an LDR depends on light intensity	
☐ Explain how resistance of an NTC thermistor depends on temperature	
☐ Distinguish the behaviour of metals from thermistors as temperature changes	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Everything in electricity connects through two equations: $Q = It$ and $V = W/Q$. All the power formulae, resistance definitions and drift velocity expression follow from those two ideas. Once you see that, the whole topic becomes a matter of careful substitution.

Topic 10

D.C. Circuits

Revision Booklet

This booklet covers:

- Practical Circuits and E.M.F.
- Internal Resistance
- Kirchhoff's Laws
- Resistors in Series and Parallel
- Potential Dividers
- Thermistors and LDRs in Potential Dividers

Practical Circuits and E.M.F.

Electromotive Force (E.M.F.)

The **electromotive force** (e.m.f., symbol ε) of a source is defined as the **energy transferred per unit charge** in driving charge around a complete circuit.

$$\varepsilon = \frac{W}{Q}$$

- W: work done by the source (J); Q: charge (C).
- Unit: volt (V) $\equiv$ J C^{-1}.
- E.m.f. is a **property of the source**; it represents energy input per coulomb.

Potential Difference (P.D.)

The **potential difference** (p.d.) between two points is the **energy transferred per unit charge** by a component as charge passes between those points.

$$V = \frac{W}{Q}$$

- P.d. describes **energy released** (e.g. by a resistor); e.m.f. describes **energy supplied** (by a source).
- Both have unit: volt (V).

E.M.F. vs P.D. — Energy Perspective

- **E.m.f.**: energy is *given to* each coulomb of charge by the source (chemical $\rightarrow$ electrical).
- **P.d.**: energy is *taken from* each coulomb of charge by a component (electrical $\rightarrow$ heat, light, etc.).
- Around any complete circuit: total e.m.f. = total p.d. (conservation of energy — see Kirchhoff's Second Law).

Internal Resistance

Internal Resistance

A real source of e.m.f. has **internal resistance** r — resistance within the source itself (e.g. chemical paste in a battery). As current flows, some energy is dissipated internally.

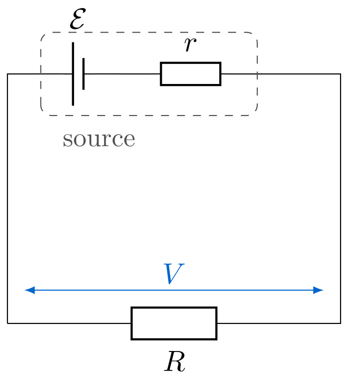
Terminal P.D. and Internal Resistance

$$\varepsilon = V + Ir \qquad \Longrightarrow \qquad V = \varepsilon - Ir$$

- $V = \varepsilon - Ir$: **terminal p.d.** (voltage across the external circuit).

- Ir: **lost volts** — p.d. across internal resistance.
- $V < \varepsilon$ whenever current flows.
- For an external resistance R: $I = \dfrac{\varepsilon}{R + r}$

Circuit diagram for a source with internal resistance

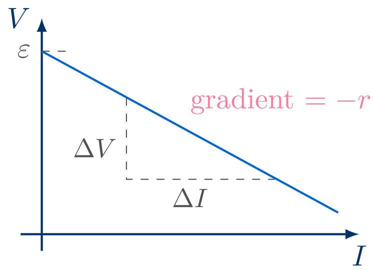


Common Mistake

The e.m.f. is *not* the same as the terminal p.d. except when $I = 0$ (open circuit). Under load, $V = \varepsilon - Ir < \varepsilon$. Always check whether the question gives e.m.f. or terminal voltage.

Graphical determination of ε and r

Rearranging: $V = \varepsilon - Ir$ has the form $y = c - mx$.



- **y-intercept**: $V = \varepsilon$ (e.m.f. when $I = 0$, i.e. open circuit).
- **Gradient**: $-r$ (magnitude of gradient gives internal resistance).
- **x-intercept**: $I = \varepsilon/r$ (short-circuit current; never reached safely).

Kirchhoff's Laws

Kirchhoff's First Law (KCL)

The sum of currents **entering** a junction equals the sum of currents **leaving** that junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

This is a consequence of **conservation of charge** — charge does not accumulate at a junction.

Kirchhoff's Second Law (KVL)

The sum of the e.m.f.s around any **closed loop** equals the sum of the potential differences (i.e. IR products) around that loop.

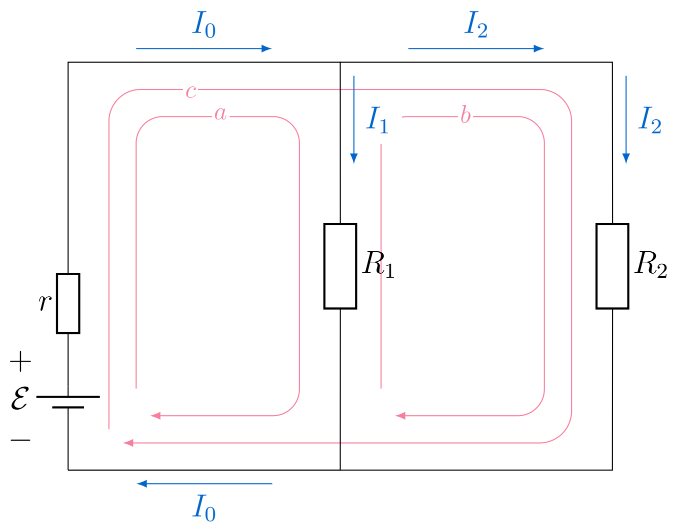
$$\sum \mathcal{E} = \sum IR$$

This is a consequence of **conservation of energy** — a charge returning to its starting point has the same potential energy.

Sign Convention for Kirchhoff's Second Law

- Choose a direction to traverse the loop.
- **E.m.f.**: positive if traversed from $-$ to $+$ terminal (source adds energy); negative if $+$ to $-$.
- IR: positive if traversed in the same direction as current; negative if opposite.
- Apply consistently — the choice of direction does not affect the final answer.

Circuit Diagram to show Kirchoff's Laws



Kirchoff's Closed Loops

There are three **closed loops** shown in the diagram above, **a**,**b** and **c**.
For **a**

$$\mathcal{E} = I_0 r + I_1 R_1$$

For **b**

$$0 = -I_1 R_1 + I_2 R_2$$

For **c**

$$\mathcal{E} = I_0 r + I_2 R_2$$

Also from Kirchoff's first law we know

$$I_0 = I_1 + I_2$$

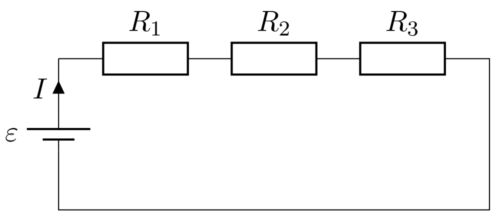
Resistors in Series and Parallel

Series Combination

For n resistors in series, the **same current** flows through each.

$$R_{\text{total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

Derivation (KVL): $V = V_1 + V_2 + \cdots$; since I is the same, $IR = IR_1 + IR_2 + \cdots$, hence $R = R_1 + R_2 + \cdots$



Series circuit — same I throughout

Parallel Combination

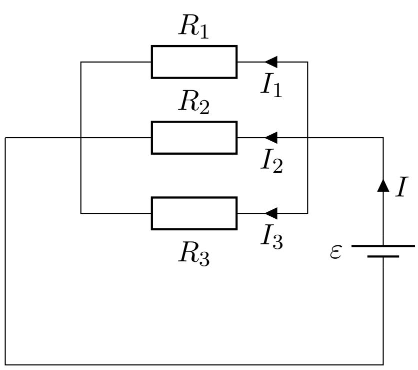
For n resistors in parallel, the **same p.d.** exists across each.

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Derivation (KCL): $I = I_1 + I_2 + \cdots$; since V is the same, $V/R = V/R_1 + V/R_2 + \cdots$, hence $1/R = 1/R_1 + 1/R_2 + \cdots$

Special case for two resistors in parallel:

$$R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



Parallel circuit — same V across each

Common Mistake

For two resistors in parallel, R_{total} is *always less* than the smaller of the two. If your answer is larger than either resistor, recheck your arithmetic — a common error is adding reciprocals incorrectly.

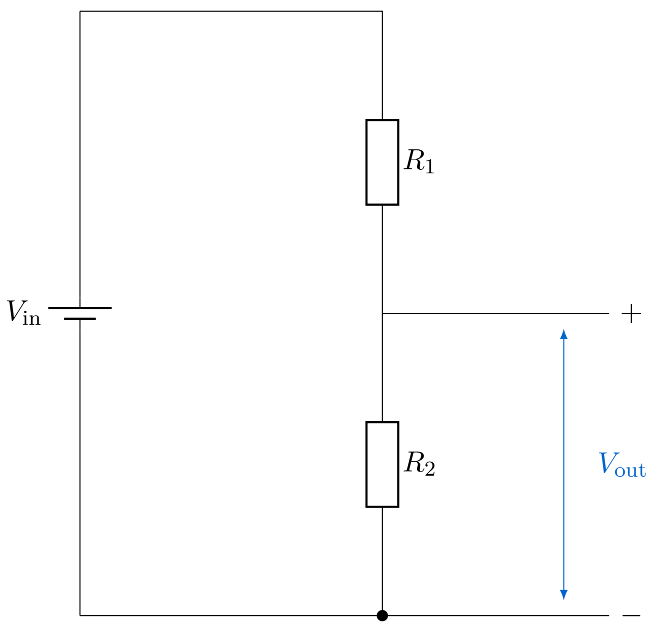
Potential Dividers

Potential Divider Principle

A **potential divider** uses two (or more) resistors in series across a supply to produce a fraction of the supply voltage as an output.

$$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1 + R_2}$$

- V_{out} is taken across R_2.
- The ratio $R_2/(R_1 + R_2)$ determines the fraction of V_{in} that appears across R_2.
- No current is drawn from the output (ideal case, or high-resistance load).



Potentiometer as a Comparison Tool

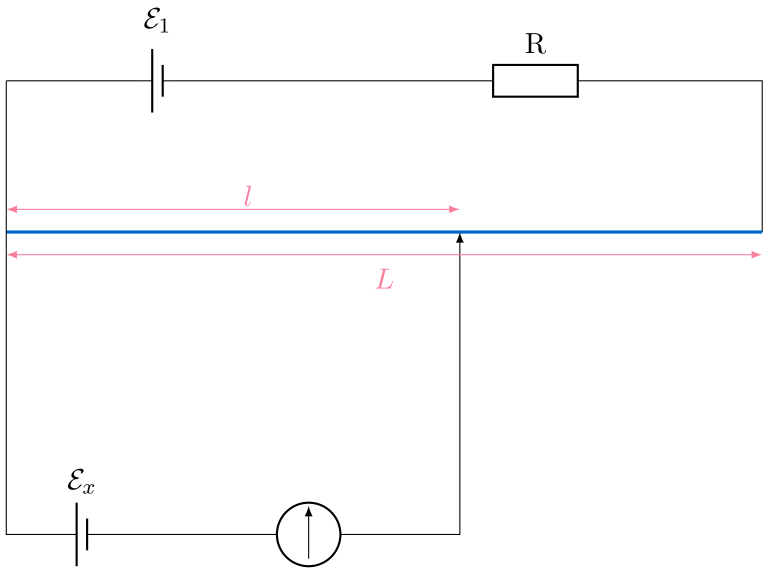
A **potentiometer** is a sliding-contact potential divider used to **compare** e.m.f.s or p.d.s precisely:

- A uniform resistance wire carries a **driver current** from a stable supply.
- The p.d. across a length l of the wire is proportional to l.
- At **balance** (null deflection on galvanometer), no current is drawn from the test source — so the measurement is unaffected by internal resistance.
- $\mathcal{E}_x = \dfrac{\mathcal{E}_1 l}{L}$ at their respective balance lengths.

Null Method

A **null method** is a measurement technique in which the quantity being measured is compared against a known standard and the detector (e.g. galvanometer) is adjusted to read **zero**. Because no current flows at balance, the result is independent of any resistance in the measuring instrument.

Circuit Diagram for a Null Method Measurement



Thermistors and LDRs in Potential Dividers

Thermistor (NTC)

A **thermistor** (negative temperature coefficient, NTC) is a resistor whose resistance **decreases** as temperature **increases**. It is used as a temperature sensor in potential divider circuits.

Light-Dependent Resistor (LDR)

An **LDR** has resistance that **decreases** as light intensity **increases**. It is used as a light sensor in potential divider circuits.

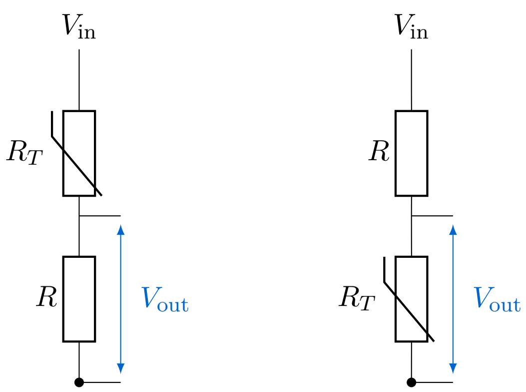
Using Sensors in Potential Dividers

For a potential divider with a thermistor (R_T) in series with a fixed resistor R, with output taken across R:

$$V_{\text{out}} = V_{\text{in}} \times \frac{R}{R_T + R}$$

- **Temperature rises** $\Rightarrow$ R_T decreases $\Rightarrow$ V_{out} **increases**.
- **Temperature falls** $\Rightarrow$ R_T increases $\Rightarrow$ V_{out} **decreases**.
- Swap the positions of R_T and R to **invert** the response (output high when cold, low when hot).

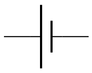
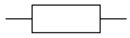
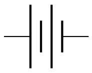
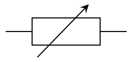
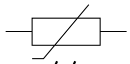
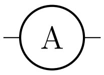
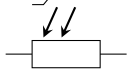
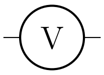
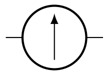
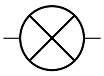
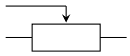
The same logic applies to an LDR replacing R_T, with light intensity replacing temperature.



V_{out} high when hot V_{out} high when cold

Circuit Symbols (Section 6 Reference)

The following symbols from CIE Section 6 are required for Topic 10 circuit diagrams.

Component	Symbol	Component	Symbol
Cell		Fixed resistor	
Battery of cells		Variable resistor	
Switch		Thermistor	
Ammeter		Light-dependent resistor	
Voltmeter		Galvanometer	
Lamp		Potentiometer	

Formula Summary Sheet

Formula	Quantity	Units
$\varepsilon = W/Q$	E.m.f. definition	V
$V = \varepsilon - Ir$	Terminal p.d.	V
$I = \varepsilon/(R + r)$	Current in circuit	A
$R_{\text{total}} = R_1 + R_2 + \cdots$	Series resistance	Ω
$1/R_{\text{total}} = 1/R_1 + 1/R_2 + \cdots$	Parallel resistance	Ω
$R_{\text{total}} = R_1R_2/(R_1 + R_2)$	Two resistors in parallel	Ω
$V_{\text{out}} = V_{\text{in}} \times R_2/(R_1 + R_2)$	Potential divider output	V
$\varepsilon_1/\varepsilon_2 = l_1/l_2$	Potentiometer balance	—

KCL: $\sum I_{\text{in}} = \sum I_{\text{out}}$ at any junction (conservation of charge).
KVL: $\sum \mathcal{E} = \sum IR$ around any closed loop (conservation of energy).
Note: Lost volts $= Ir$; terminal p.d. $< \mathcal{E}$ whenever current flows.

Worked Examples

Example 1 — Internal Resistance

Question: A battery of e.m.f. 12 V and internal resistance 0.5 Ω is connected to an external resistor of 3.5 Ω. Calculate (a) the current, (b) the terminal p.d., and (c) the power dissipated internally.

Solution

(a) $I = \dfrac{\varepsilon}{R+r} = \dfrac{12}{3.5+0.5} = \dfrac{12}{4.0} = \mathbf{3.0}\ \text{A}$

(b) $V = \varepsilon - Ir = 12 - (3.0)(0.5) = 12 - 1.5 = \mathbf{10.5}\ \text{V}$

(c) $P_{\text{internal}} = I^2 r = (3.0)^2 \times 0.5 = 9 \times 0.5 = \mathbf{4.5}\ \text{W}$

Example 2 — Kirchhoff's Laws

Question: In the circuit below, $\varepsilon_1 = 10$ V, $\varepsilon_2 = 4$ V, $R_1 = 3$ Ω, $R_2 = 2$ Ω, $R_3 = 5$ Ω. Find the current flowing in the circuit (treat as a single loop, both sources driving in the same direction).

Solution

Applying KVL around the loop (taking the direction of current as positive):

$$\sum \varepsilon = \sum IR$$

$$10 - 4 = I(3 + 2 + 5)$$

$$6 = 10I$$

$$I = \mathbf{0.60}\ \text{A}$$

Note: if ε_2 had opposed ε_1, we would write $10 - 4$ on the left side (one e.m.f. is negative in our sign convention).

Example 3 — Resistors in Parallel

Question: Two resistors of 6 Ω and 12 Ω are connected in parallel across a 6 V supply of negligible internal resistance. Find (a) the combined resistance, (b) the total current, and (c) the current through each resistor.

Solution

(a) $R_{\text{total}} = \dfrac{6 \times 12}{6+12} = \dfrac{72}{18} = \mathbf{4.0\ \Omega}$

(b) $I_{\text{total}} = V/R_{\text{total}} = 6/4.0 = \mathbf{1.5}\ \text{A}$

(c) $I_1 = 6/6 = \mathbf{1.0}$ A; $I_2 = 6/12 = \mathbf{0.50}$ A

Check: $1.0 + 0.5 = 1.5$ A ✓(KCL satisfied)

Example 4 — Potential Divider with Thermistor

Question: A potential divider consists of a thermistor R_T in series with a 1.5 kΩ fixed resistor, connected across a 5.0 V supply. The output is taken across the fixed resistor. At 20 °C the thermistor has resistance 3.0 kΩ. Calculate (a) V_{out} at 20 °C, and (b) whether V_{out} increases or decreases as temperature rises.

Solution

(a) $$V_{\text{out}} = 5.0 \times \frac{1500}{3000 + 1500} = 5.0 \times \frac{1500}{4500} = 5.0 \times \frac{1}{3} = \mathbf{1.67}\ \text{V}$$

(b) As temperature **rises**, R_T **decreases**. The fraction $R/(R_T + R)$ **increases**, so V_{out} **increases**.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electromotive force (e.m.f.) and distinguish it from potential difference in terms of energy.

[3 marks]

Q2. A cell has e.m.f. 9.0 V and internal resistance 0.8 Ω. When connected to a resistor, a current of 3.0 A flows. Calculate (a) the terminal p.d. and (b) the power lost to internal resistance.

[3 marks]

Q3. State Kirchhoff's first and second laws and give the physical principle underlying each.

[4 marks]

Q4. Three resistors of 2 Ω, 4 Ω and 6 Ω are connected in parallel. Calculate the combined resistance.

[2 marks]

Section B — Longer Structured Questions

Q5. A battery of e.m.f. ε and internal resistance r is connected to a variable external resistor R.

(a) Show that the terminal p.d. $V = \varepsilon - Ir$ and hence that $V = \varepsilon - \dfrac{\varepsilon r}{R + r}$.

[2 marks]

(b) A graph of V against I is plotted as R is varied. Describe the graph and explain how ε and r can be determined from it.

[3 marks]

(c) Data from the graph gives $\varepsilon = 6.0$ V and $r = 0.4\ \Omega$. Calculate the current and terminal p.d. when $R = 2.6\ \Omega$.

[3 marks]

Q6. The circuit shown consists of a supply of e.m.f. 12 V and negligible internal resistance connected to resistors $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$ and $R_3 = 12\ \Omega$, where R_2 and R_3 are in parallel with each other and R_1 is in series with this parallel combination.

(a) Calculate the combined resistance of R_2 and R_3 in parallel.

[2 marks]

(b) Calculate the total resistance of the circuit and the current drawn from the supply.

[2 marks]

(c) Calculate the p.d. across the parallel combination and hence the current through each of R_2 and R_3.

[3 marks]

(d) Verify your answers using Kirchhoff's first law.

[1 mark]

Q7. A potential divider consists of a 2.0 kΩ resistor and a light-dependent resistor (LDR) connected in series across a 10 V supply. The output voltage is taken across the LDR. In bright light, the LDR has resistance 500 Ω; in darkness, its resistance is 20 kΩ.

(a) Calculate V_{out} in bright light.

[2 marks]

(b) Calculate V_{out} in darkness.

[2 marks]

(c) Explain how the circuit could be modified so that V_{out} is high in bright light rather than in darkness.

[1 mark]

Mark Scheme and Answers

Q1. E.m.f. is the energy transferred *to* each coulomb of charge by the source [1]; p.d. is the energy transferred *from* each coulomb of charge by a component [1]; e.m.f. relates to energy input (e.g. chemical to electrical), p.d. relates to energy output (e.g. electrical to heat) [1].

Q2(a). $V = \varepsilon - Ir = 9.0 - (3.0)(0.8) = 9.0 - 2.4 = \mathbf{6.6}$ V [2].
Q2(b). $P = I^2 r = (3.0)^2 \times 0.8 = \mathbf{7.2}$ W [1].

Q3. KCL: sum of currents into a junction = sum of currents out [1]; consequence of conservation of charge [1]. KVL: sum of e.m.f.s around a closed loop = sum of IR products [1]; consequence of conservation of energy [1].

Q4. $\dfrac{1}{R} = \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} = \dfrac{6+3+2}{12} = \dfrac{11}{12}$; $\quad R = \mathbf{12/11} \approx \mathbf{1.1\ \Omega}$ [2].

Q5(a). By KVL: $\varepsilon = IR + Ir = I(R + r)$; rearranging gives $V = IR = \varepsilon - Ir$ [2].

Q5(b). Straight line [1]; y-intercept = ε (open-circuit voltage) [1]; magnitude of gradient = r [1].

Q5(c). $I = 6.0/(2.6{+}0.4) = 6.0/3.0 = \mathbf{2.0}$ A [1]; $V = 6.0{-}(2.0)(0.4) = 6.0{-}0.8 = \mathbf{5.2}$ V [2].

Q6(a). $R_{23} = (6 \times 12)/(6 + 12) = 72/18 = \mathbf{4.0\ \Omega}$ [2].
Q6(b). $R_{\text{total}} = 4 + 4 = \mathbf{8.0\ \Omega}$; $I = 12/8 = \mathbf{1.5}$ A [2].
Q6(c). $V_{23} = 1.5 \times 4 = \mathbf{6.0}$ V; $I_2 = 6/6 = \mathbf{1.0}$ A; $I_3 = 6/12 = \mathbf{0.50}$ A [3].
Q6(d). $I_2 + I_3 = 1.0 + 0.5 = 1.5\ \text{A} = I_{\text{total}}$ ✓[1].

Q7(a). $V_{\text{out}} = 10 \times 500/(2000 + 500) = 5000/2500 = \mathbf{2.0}$ V [2].
Q7(b). $V_{\text{out}} = 10 \times 20000/(2000 + 20000) = 200000/22000 = \mathbf{9.1}$ V [2].
Q7(c). Swap the positions of the LDR and the 2.0 kΩ resistor so that the output is taken across the fixed resistor instead [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Recall and use the circuit symbols from Section 6	
☐ Define e.m.f. as energy transferred per unit charge by a source	
☐ Distinguish between e.m.f. and p.d. in terms of energy	
☐ Use $V = \varepsilon - Ir$ and $I = \varepsilon/(R + r)$ for circuits with internal resistance	
☐ Determine ε and r from a V–I graph (intercept and gradient)	
☐ State and apply Kirchhoff's first law (conservation of charge)	
☐ State and apply Kirchhoff's second law (conservation of energy)	
☐ Derive and use $R_{\text{total}} = R_1 + R_2 + \cdots$ for series resistors	
☐ Derive and use $1/R_{\text{total}} = 1/R_1 + 1/R_2 + \cdots$ for parallel resistors	
☐ Use Kirchhoff's laws to solve multi-loop circuit problems	
☐ Understand the principle of a potential divider circuit	
☐ Use $V_{\text{out}} = V_{\text{in}} \times R_2/(R_1 + R_2)$ for a potential divider	
☐ Explain the potentiometer as a null method for comparing p.d.s	
☐ Explain how a thermistor or LDR is used in a potential divider	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Kirchhoff's laws are nothing more than conservation of charge and conservation of energy written in circuit language. Once you see that, every circuit problem becomes a matter of careful bookkeeping — choose your loops, track your signs, and the algebra does the rest.

CIE International A Level Physics

Topic 11

Particle Physics

Revision Booklet

This booklet covers:

- Atoms, Nuclei and Radiation
- Types of Radiation: α, β and γ
- Radioactive Decay Equations
- Fundamental Particles: Quarks
- Hadrons, Baryons and Mesons
- Leptons and Beta Decay

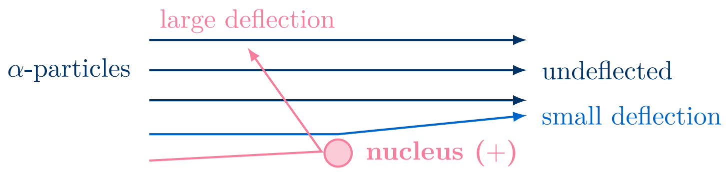
Atoms, Nuclei and Radiation

The Nuclear Atom

The **Geiger–Marsden** α-particle scattering experiment (directed α-particles at thin gold foil) showed:

- Most α-particles passed straight through $\Rightarrow$ the atom is mostly empty space.
- A small fraction were deflected through large angles $\Rightarrow$ a dense, positively charged nucleus exists.
- A very few bounced back ($> 90°$) $\Rightarrow$ the nucleus is very small compared to the atom.

Rutherford scattering diagram



Nuclear Notation

A **nuclide** is a specific nuclear species characterised by its proton and nucleon numbers:

$$^{A}_{Z}\mathrm{X}$$

- A = **nucleon number** (mass number): total number of protons + neutrons.
- Z = **proton number** (atomic number): number of protons.
- Number of neutrons $N = A - Z$.
- **Isotopes**: atoms of the same element (same Z) with different numbers of neutrons (different A).

Conservation Laws in Nuclear Processes

In every nuclear reaction or decay, the following are always conserved:

- **Nucleon number** A (total count of protons + neutrons).
- **Charge** (equivalently, proton number Z).

These two conservation laws are the key tool for balancing decay equations.

Unified Atomic Mass Unit

The **unified atomic mass unit** (u) is defined as exactly $\frac{1}{12}$ of the mass of a carbon-12 atom.

$$1\ \mathrm{u} = 1.661 \times 10^{-27}\ \mathrm{kg}$$

Approximate masses: proton ≈ 1 u; neutron ≈ 1 u; electron ≈ 0.00055 u.

Types of Radiation: α, β and γ

Radiation	Composition	Charge	Mass	Stopped by
α (alpha)	2 protons + 2 neutrons (helium-4 nucleus)	$+2e$	4 u	A few cm of air; thin paper
β^- (beta-minus)	Electron (e^-)	$-e$	≈ 0 ($\frac{1}{1836}$ u)	A few mm of aluminium
β^+ (beta-plus)	Positron (e^+)	$+e$	≈ 0 ($\frac{1}{1836}$ u)	Annihilates with electron
γ (gamma)	Electromagnetic wave / photon	0	0	Several cm of lead

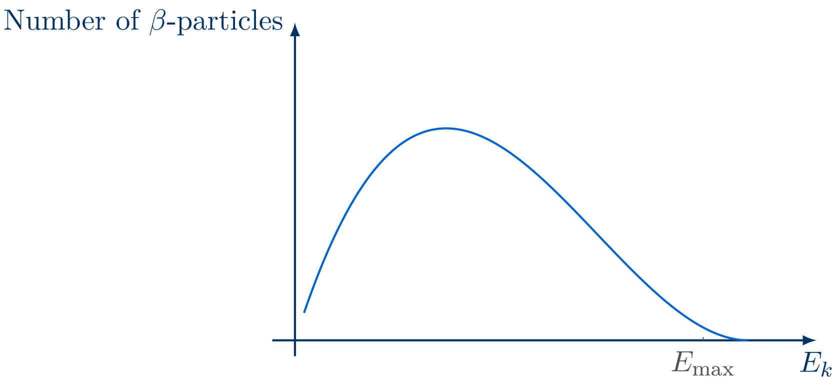
Antiparticles

Every particle has a corresponding **antiparticle** with:

- The **same mass** as the particle.
- The **opposite charge** (and opposite quantum numbers).
- The **positron** (β^+, e^+) is the antiparticle of the electron (e^-).
- When a particle meets its antiparticle they **annihilate**, producing energy (usually two γ-ray photons).

Energies of Emitted Particles

- **α-particles** are emitted with **discrete (fixed) energies**: each decay of a given nuclide always releases an α-particle of the same kinetic energy, giving a sharp line spectrum.
- **β-particles** have a **continuous range of energies** from zero up to a maximum: the total energy of the decay is shared between the β-particle and an (anti)neutrino, which can take any share — hence the continuous spectrum.
- **(Anti)neutrinos** are produced in β-decay to account for this energy sharing and to conserve momentum and angular momentum.



Continuous β energy spectrum

Common Mistake

Students often state that β-particles have a fixed energy like α-particles. They do not — the continuous spectrum is direct evidence for neutrino emission. The **maximum** β energy equals the total energy available from the decay (as if no neutrino were present).

Radioactive Decay Equations

Alpha decay

In α-decay, the nucleus emits a helium-4 nucleus ($^4_2\alpha$). Both A and Z decrease:

$$^A_Z\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\alpha$$

Example:

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^4_2\alpha$$

Check: A: $238 = 234 + 4$ ✓; Z: $92 = 90 + 2$ ✓

Beta-minus decay (β^-)

A neutron converts to a proton; an electron and an electron antineutrino are emitted. A is unchanged; Z increases by 1:

$$^A_Z\text{X} \longrightarrow {}_{Z+1}^{\ \ A}\text{Y} + {}^{\ \ 0}_{-1}\text{e} + \bar{\nu}_e$$

Example:

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + {}^{\ \ 0}_{-1}\text{e} + \bar{\nu}_e$$

Beta-plus decay (β^+)

A proton converts to a neutron; a positron and an electron neutrino are emitted. A is unchanged; Z decreases by 1:

$$^A_Z\text{X} \longrightarrow {}_{Z-1}^{\ \ A}\text{Y} + {}^{\ \ 0}_{+1}\text{e} + \nu_e$$

Gamma emission

γ-radiation accompanies α or β decay when the daughter nucleus is left in an excited state. Neither A nor Z changes:

$$^A_Z\text{X}^* \longrightarrow {}^A_Z\text{X} + \gamma$$

Decay Equation Summary

Decay	ΔA	ΔZ	Particles emitted
α	-4	-2	${}^4_2\alpha$
β^-	0	$+1$	${}^{0}_{-1}\text{e}$, $\bar{\nu}_e$
β^+	0	-1	${}^{0}_{+1}\text{e}$, ν_e
γ	0	0	γ photon

Fundamental Particles: Quarks

Quarks

A **quark** is a fundamental, point-like particle. There are **six flavours** of quark (and six corresponding antiquarks):

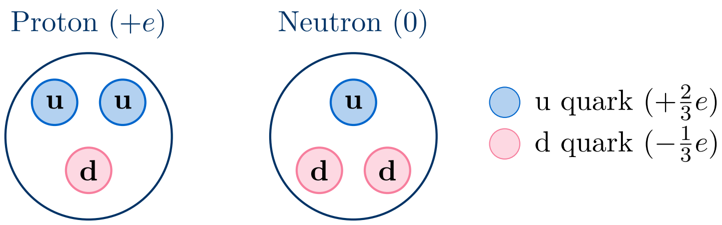
Flavour	Symbol	Charge
up	u	$+\frac{2}{3}e$
down	d	$-\frac{1}{3}e$
strange	s	$-\frac{1}{3}e$
charm	c	$+\frac{2}{3}e$
top	t	$+\frac{2}{3}e$
bottom	b	$-\frac{1}{3}e$

Antiquark	Symbol	Charge
anti-up	$\bar{\text{u}}$	$-\frac{2}{3}e$
anti-down	$\bar{\text{d}}$	$+\frac{1}{3}e$
anti-strange	$\bar{\text{s}}$	$+\frac{1}{3}e$
anti-charm	$\bar{\text{c}}$	$-\frac{2}{3}e$
anti-top	$\bar{\text{t}}$	$-\frac{2}{3}e$
anti-bottom	$\bar{\text{b}}$	$+\frac{1}{3}e$

Only up and down quarks are required for protons and neutrons; strange, charm, top and bottom are examined only for their charges.

Quark Composition of Proton and Neutron

Particle	Quarks	Charge calculation	Total charge
Proton	uud	$\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e$	$+e$
Neutron	udd	$\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e$	0



Hadrons, Baryons and Mesons

Hadrons

A **hadron** is any particle made of quarks. Hadrons are subdivided into two classes:

- **Baryons**: composed of **three quarks** (qqq). Examples: proton (uud), neutron (udd).
- **Mesons**: composed of **one quark and one antiquark** ($q\bar{q}$). Examples: pion ($\pi^+ = u\bar{d}$).

Protons and neutrons are **not** fundamental particles — they are baryons made of quarks.

Class	Quark content	Example	Charge
Baryon	qqq (three quarks)	Proton (uud)	$+e$
Baryon	qqq (three quarks)	Neutron (udd)	0
Meson	$q\bar{q}$ (quark + antiquark)	π^+ ($u\bar{d}$)	$+e$
Meson	$q\bar{q}$ (quark + antiquark)	π^- ($\bar{u}d$)	$-e$
Meson	$q\bar{q}$ (quark + antiquark)	π^0 ($u\bar{u}$ or $d\bar{d}$)	0

Leptons and Beta Decay

Leptons

Leptons are fundamental (point-like, not made of quarks) particles. The leptons required at A-level are:

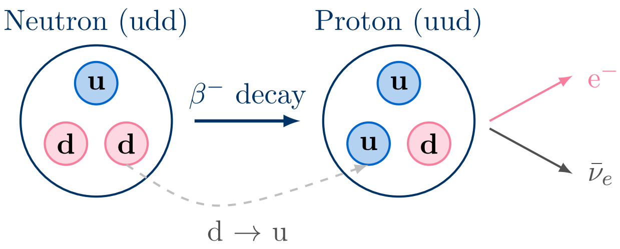
- **Electron** (e^-) — charge $-e$.
- **Positron** (e^+) — antiparticle of electron; charge $+e$.
- **Electron neutrino** (ν_e) — charge 0; produced in β^+ decay.
- **Electron antineutrino** ($\bar{\nu}_e$) — charge 0; produced in β^- decay.

Quark Changes in Beta Decay

Beta decay occurs because a quark changes flavour inside a nucleon:

Decay	Quark change	Nucleon change	Particles emitted
β^-	$d \to u$	Neutron → Proton	$e^- + \bar{\nu}_e$
β^+	$u \to d$	Proton → Neutron	$e^+ + \nu_e$

Quark-level picture of β^- decay



Neutrinos vs Antineutrinos

This is a common source of error in exams:

- β^- decay produces an **electron anti**neutrino ($\bar{\nu}_e$) — the antiparticle of the neutrino.
- β^+ decay produces an **electron neutrino** (ν_e).

A helpful check: the lepton number must be conserved. In β^-, a lepton of number +1 (the electron) is created alongside a lepton of number −1 (the antineutrino), giving net lepton number zero — matching the original nucleon.

Key Facts Summary

Particle	Symbol	Charge	Type
Proton	p	$+e$	Baryon (uud)
Neutron	n	0	Baryon (udd)
Electron	e$^-$	$-e$	Lepton
Positron	e$^+$	$+e$	Lepton (antiparticle of e$^-$)
Electron neutrino	ν_e	0	Lepton
Electron antineutrino	$\bar{\nu}_e$	0	Lepton
α-particle	${}^4_2\alpha$	$+2e$	Baryon (not fundamental)

Quark charges (to recall): u: $+\frac{2}{3}e$; d: $-\frac{1}{3}e$; s: $-\frac{1}{3}e$; c: $+\frac{2}{3}e$; t: $+\frac{2}{3}e$; b: $-\frac{1}{3}e$

Antiquark charge = opposite of quark charge.

Conservation laws in every nuclear/particle process: nucleon number A; charge Z; lepton number.

Worked Examples

Example 1 — Completing a Decay Equation

Question: Complete the following decay equation and identify the type of decay:

$$^{210}_{84}\text{Po} \longrightarrow {}^{A}_{Z}\text{X} + {}^{4}_{2}\alpha$$

Solution

Conserve nucleon number A: $210 = A + 4 \Rightarrow A = \mathbf{206}$
Conserve charge Z: $84 = Z + 2 \Rightarrow Z = \mathbf{82}$ (lead, Pb)

$$^{210}_{84}\text{Po} \longrightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\alpha$$

This is **alpha decay** — the nucleon number decreases by 4 and the proton number by 2.

Example 2 — Identifying Decay Type from Equation

Question: Identify the particle X and the type of decay in:

$$^{90}_{38}\text{Sr} \longrightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + X$$

Solution

A is conserved: $90 = 90 + 0 + 0$ ✓
Z: $38 = 39 + (-1) + 0$ ✓
The emitted particles are an electron (β^-) and X. This is β^- decay, so X must be an **electron antineutrino**: $X = \bar{\nu}_e$.

$$^{90}_{38}\text{Sr} \longrightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

Example 3 — Quark Composition and Charge

Question: (a) State the quark composition of an antiproton. (b) Calculate its charge.

Solution

(a) The proton has quark composition uud. The antiproton is its antiparticle, so every quark is replaced by its antiquark:

$$\text{Antiproton} = \bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$$

(b) Charge $= -\frac{2}{3}e + (-\frac{2}{3}e) + \frac{1}{3}e = -\frac{3}{3}e = -\mathbf{e}$
This confirms the antiproton has charge $-e$, equal in magnitude but opposite in sign to the proton.

Example 4 — Quark Changes in β^+ Decay

Question: A proton undergoes β^+ decay. Write a full decay equation and describe the quark-level change.

Solution

$$^{1}_{1}\text{p} \longrightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\text{e} + \nu_e$$

At the quark level, one **up quark converts to a down quark**:

$$\text{Proton (uud)} \longrightarrow \text{Neutron (udd)} + \text{e}^+ + \nu_e$$

Charge check: $+e \to 0 + e + 0$ ✓; Nucleon number: $1 = 1 + 0 + 0$ ✓

Practice Exam Questions

Section A — Short Answer Questions

Q1. Describe the key observations from the α-particle scattering experiment and explain what each tells us about atomic structure.

[4 marks]

Q2. Distinguish between nucleon number and proton number. Define the term *isotope*.

[3 marks]

Q3. State the composition, charge and approximate mass (in u) of α-, β^-- and γ-radiation.

[5 marks]

Q4. Explain why β-particles have a continuous energy spectrum, whereas α-particles from a given nuclide have a fixed energy.

[3 marks]

Section B — Longer Structured Questions

Q5. The nuclide $^{226}_{88}\text{Ra}$ undergoes α-decay.

(a) Write a balanced nuclear equation for this decay, identifying the daughter nuclide.

[2 marks]

(b) State the two conservation laws used to balance the equation.

[2 marks]

(c) Explain why the α-particles emitted in this decay all have the same kinetic energy.

[2 marks]

Q6. The nuclide $^{14}_{6}\text{C}$ undergoes β^- decay.

(a) Write a full balanced equation for this decay, including all particles emitted.

[2 marks]

(b) Describe the quark-level change that occurs during this decay.

[2 marks]

(c) State and explain whether the β^- particles emitted have a fixed or continuous range of energies.

[2 marks]

Q7. (a) State what is meant by a *hadron* and distinguish between a baryon and a meson.

[3 marks]

(b) The particle π^+ is a meson with quark composition $\mathrm{u\bar{d}}$. Show that this gives a charge of $+e$.

[2 marks]

(c) State the quark composition of an antiproton and calculate its charge.

[2 marks]

(d) Explain why electrons are *not* classified as hadrons.

[1 mark]

Mark Scheme and Answers

Q1. Most α-particles pass straight through $\Rightarrow$ atom is mostly empty space [1]; small fraction deflected through large angles $\Rightarrow$ concentrated positive charge (nucleus) exists [1]; very few back-scattered $\Rightarrow$ nucleus is very small / very dense [1]; all deflections from positive charge are consistent with electrostatic repulsion [1].

Q2. Nucleon number (A): total number of protons + neutrons in a nucleus [1]; **proton number** (Z): number of protons only [1]; **isotopes**: atoms of the same element (same Z) with different numbers of neutrons (different A) [1].

Q3. α: 2 protons + 2 neutrons; charge $+2e$; mass 4 u [2]. β^-: electron; charge $-e$; mass ≈ 0 ($\frac{1}{1836}$ u) [2]. γ: electromagnetic radiation/photon; charge 0; mass 0 [1].

Q4. In β-decay, an (anti)neutrino is also emitted [1]; the total energy is shared between the β-particle and the (anti)neutrino in varying proportions [1]; so the β-particle can have any energy from zero to a maximum — continuous spectrum [1]. For α: only two products (daughter + α); by conservation of energy and momentum, the α always takes the same fixed share [implied/bonus].

Q5(a). ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha$ [2] (1 mark if equation has correct concept but arithmetic error).

Q5(b). Conservation of nucleon number A [1]; conservation of charge (proton number Z) [1].

Q5(c). Only two products (daughter + α) [1]; by conservation of energy and momentum, the α always receives the same fixed kinetic energy (discrete energy levels) [1].

Q6(a). ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$ [2].

Q6(b). A down quark converts to an up quark (d $\rightarrow$ u) [1]; inside a neutron, converting it to a proton [1].

Q6(c). Continuous range [1]; because the energy is shared between the β^- particle and the antineutrino in varying proportions [1].

Q7(a). A hadron is a particle made of quarks [1]; a baryon consists of three quarks [1]; a meson consists of one quark and one antiquark [1].

Q7(b). Charge $= +\frac{2}{3}e + \frac{1}{3}e$ [1] $= +e$ ✓[1].

Q7(c). $\bar{u}\bar{u}\bar{d}$ [1]; charge $= -\frac{2}{3}e - \frac{2}{3}e + \frac{1}{3}e = -e$ [1].

Q7(d). Electrons are leptons, not composed of quarks / not hadrons by definition [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Describe the α-scattering experiment and interpret its results	
☐	Describe the nuclear model: protons, neutrons and orbital electrons	
☐	Distinguish nucleon number (A) from proton number (Z)	
☐	Define isotopes and use the notation ${}^{A}_{Z}X$	
☐	State that nucleon number and charge are conserved in nuclear processes	
☐	Describe composition, mass and charge of α, β^-, β^+ and γ	
☐	State that a positron is the antiparticle of an electron (same mass, opposite charge)	
☐	State that $\bar{\nu}_e$ is produced in β^- decay and ν_e in β^+ decay	
☐	Explain why β-particles have a continuous energy spectrum	
☐	Write balanced α- and β-decay equations	
☐	Use the unified atomic mass unit (u)	
☐	State the six quark flavours and recall their charges	
☐	State the quark composition of the proton (uud) and neutron (udd)	
☐	Distinguish hadrons (baryons: qqq; mesons: q$\bar{\text{q}}$) from leptons	
☐	Describe the quark change in β^- (d→u) and β^+ (u→d) decay	
☐	Recall that electrons and neutrinos are leptons	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Particle physics is really just careful bookkeeping. Every decay equation is balanced by two rules — conserve A, conserve Z — and every quark change in β-decay is just one quark swapping flavour. Get those two ideas solid and the whole topic clicks into place.

Topic 12

Motion in a Circle

Revision Booklet

This booklet covers:

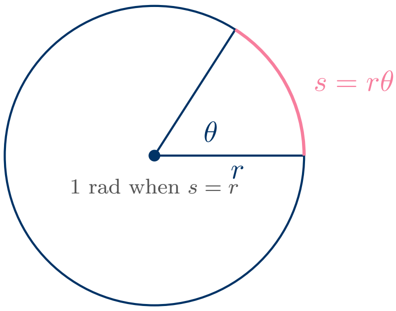
- Angular Displacement and the Radian
- Angular Speed and Period
- Centripetal Acceleration
- Centripetal Force
- Examples of Circular Motion

Angular Displacement and the Radian

The Radian

The **radian** (rad) is the SI unit of angle. One radian is the angle subtended at the centre of a circle by an arc whose length equals the radius.

$$\theta \text{ (rad)} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$



Key Angle Conversions

Full circle:	360ř $= 2\pi$ rad
Half circle:	180ř $= \pi$ rad
Quarter circle:	90ř $= \pi/2$ rad
To convert degrees to radians:	multiply by $\pi/180$
To convert radians to degrees:	multiply by $180/\pi$

Angular Speed and Linear Speed

Angular Speed

The **angular speed** ω of an object moving in a circle is the rate of change of angular displacement:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \text{units: rad s}^{-1}$$

For uniform circular motion (constant speed), ω is constant.

Angular Speed and Period

$$\omega = \frac{2\pi}{T} = 2\pi f$$

ω = angular speed (rad s^{-1})
T = period — time for one complete revolution (s)
f = frequency (Hz = s^{-1})

Linear Speed and Angular Speed

$$v = r\omega$$

v = linear (tangential) speed ($m\ s^{-1}$)
r = radius of the circular path (m)
ω = angular speed ($rad\ s^{-1}$)

Although ω is constant for uniform circular motion, the **velocity** is not — its direction changes continuously. The speed v is constant but the direction of motion is always **tangential** to the circle.

Speed vs Velocity in Circular Motion

In uniform circular motion:

- **Speed** is constant — the magnitude of velocity does not change.
- **Velocity** is not constant — its direction changes at every point.
- Because velocity changes, the object is **accelerating**, even though its speed is constant.

Centripetal Acceleration

Centripetal Acceleration

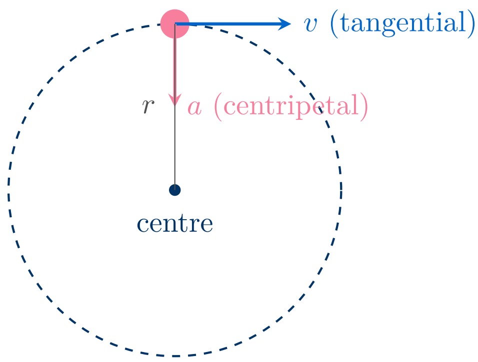
For an object moving in a circle at constant speed, the acceleration is directed **towards the centre** of the circle. This is called **centripetal acceleration**.

- It arises because the **direction** of velocity is continuously changing.
- A force of **constant magnitude** that is always **perpendicular to the velocity** produces centripetal acceleration.
- Because the force is perpendicular to motion, it does **no work** — kinetic energy (and hence speed) remains constant.

Centripetal Acceleration

$$a = r\omega^2 = \frac{v^2}{r}$$

a = centripetal acceleration, directed towards centre ($m\ s^{-2}$)
r = radius of circular path (m)
ω = angular speed ($rad\ s^{-1}$)
v = linear speed ($m\ s^{-1}$)



Centripetal Force

Centripetal Force

The **centripetal force** is the resultant force directed towards the centre of the circle that causes centripetal acceleration. It is not a new type of force — it is whatever force (or combination of forces) acts towards the centre in a given situation.

Centripetal Force

Applying Newton's second law ($F = ma$) with $a = r\omega^2 = v^2/r$:

$$F = mr\omega^2 = \frac{mv^2}{r}$$

F = centripetal force, directed towards centre (N)
m = mass of the object (kg)
r = radius of circular path (m)
ω = angular speed (rad s^{-1})
v = linear speed (m s^{-1})

What Provides the Centripetal Force?

The centripetal force is provided by different physical forces depending on the situation:

Situation	**Force providing centripetal force**
Planet/satellite orbiting a star	Gravitational attraction
Car turning on a flat road	Friction between tyres and road
Ball on a string, horizontal circle	Tension in the string
Electron orbiting nucleus (Bohr model)	Electrostatic (Coulomb) attraction
Object on the inside of a curved loop	Normal contact force

"Centrifugal Force" is Not Real

There is no outward "centrifugal force" acting on the object. In the reference frame of the ground (inertial frame), the only horizontal force on an object in circular motion is the **inward** centripetal force. The sensation of being "pushed outward" is the result of inertia — the body's tendency to continue in a straight line.

Examples of Circular Motion

Conical Pendulum

A mass on a string of length L makes angle θ with the vertical, moving in a horizontal circle of radius $r = L\sin\theta$.

Conical Pendulum

Resolving forces:

$$\text{Vertical:} \quad T\cos\theta = mg$$
$$\text{Horizontal (centripetal):} \quad T\sin\theta = \frac{mv^2}{r} = mr\omega^2$$

Dividing: $\tan\theta = \dfrac{r\omega^2}{g} = \dfrac{v^2}{rg}$

Car on a Banked Track

On a banked track (angle θ), the horizontal component of the normal force provides the centripetal force, reducing the need for friction.

Ideal Banking Angle

For a vehicle travelling at speed v on a track banked at angle θ, with no friction:

$$\tan\theta = \frac{v^2}{rg}$$

Vertical Circular Motion

For an object on the inside of a vertical loop of radius r at the **top** of the loop:

$$mg + N = \frac{mv^2}{r}$$

The minimum speed for the object to maintain contact: $N \geq 0 \Rightarrow v_{\text{min}} = \sqrt{gr}$.

Formula Summary Sheet

Formula	Quantity	Units
$\theta = s/r$	Angular displacement	rad
$\omega = \Delta\theta/\Delta t$	Angular speed	rad s^{-1}
$\omega = 2\pi/T = 2\pi f$	Angular speed from period	rad s^{-1}
$v = r\omega$	Linear speed	m s^{-1}
$a = r\omega^2 = v^2/r$	Centripetal acceleration	m s^{-2}
$F = mr\omega^2 = mv^2/r$	Centripetal force	N

Useful: 2π rad = 360ř; 1 revolution = 2π rad; $v = r\omega$ links linear and angular quantities.

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy

1. **Find** ω: use $\omega = 2\pi/T$ or $\omega = 2\pi f$ — convert rpm or revolutions per second to rad s^{-1} first.
2. **Find** v: use $v = r\omega$ if needed.
3. **Identify the centripetal force**: decide which physical force (tension, gravity, normal force, friction) provides it.
4. **Apply** $F = mr\omega^2$ **or** $F = mv^2/r$: equate to the expression for that force and solve.

Common Errors — Avoid These!

- Using **degrees** instead of radians in $v = r\omega$ or $a = r\omega^2$ — ω must always be in rad s^{-1}.
- Confusing **period** T with frequency f — remember $T = 1/f$.
- Forgetting to **identify the centripetal force** physically — in a free-body diagram, only real forces appear; there is no "centripetal force" arrow separate from, say, tension or gravity.
- In vertical circle problems, failing to account for **the component of gravity** that contributes to (or subtracts from) the centripetal force.
- Confusing r (radius) with **diameter** — always halve the diameter.

Worked Examples

Example 1 — Angular and Linear Speed

Question: A CD rotates at 500 rpm. Calculate (a) the angular speed, and (b) the linear speed of a point 6.0 cm from the centre.

Solution

Solution:
(a) Convert rpm to rad s^{-1}:

$$\omega = \frac{500 \times 2\pi}{60} = \mathbf{52.4} \text{ rad s}^{-1}$$

(b) $v = r\omega = 0.060 \times 52.4 = \mathbf{3.14}$ m s^{-1}

Example 2 — Centripetal Force on a Car

Question: A car of mass 1200 kg travels at 18 m s^{-1} around a flat circular bend of radius 45 m. Calculate the centripetal force required and state what provides it.

Solution

Solution:

$$F = \frac{mv^2}{r} = \frac{1200 \times 18^2}{45} = \frac{1200 \times 324}{45} = \mathbf{8640} \text{ N}$$

This force is provided by **friction** between the tyres and the road surface, acting towards the centre of the bend.

Example 3 — Satellite Orbit

Question: A satellite orbits Earth at a radius of 7.5×10^6 m. The gravitational field strength at this altitude is 7.1 N kg^{-1}. Calculate the orbital period.

Solution

Solution:
Gravitational force provides centripetal force:

$$mg = mr\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{7.1}{7.5 \times 10^6}} = 9.73 \times 10^{-4} \text{ rad s}^{-1}$$

Period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{9.73 \times 10^{-4}} = 6454 \text{ s} \approx \mathbf{1.79} \text{ hours}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the radian.

[1 mark]

Q2. A wheel of radius 0.35 m completes 120 revolutions per minute. Calculate (a) the angular speed and (b) the linear speed of a point on the rim.

[3 marks]

Q3. Explain why an object moving in a circle at constant speed is accelerating, and state the direction of this acceleration.

[2 marks]

Q4. A stone of mass 0.15 kg is attached to a string and swung in a horizontal circle of radius 0.80 m at 3.0 rev s^{-1}. Calculate the tension in the string. (Assume the string is horizontal.)

[3 marks]

Q5. Explain why the centripetal force does no work on an object moving in a circle at constant speed.

[2 marks]

Section B — Longer Structured Questions

Q6. A car of mass 950 kg travels over the top of a hill that has a circular cross-section of radius 120 m.

(a) Draw a free-body diagram for the car at the top of the hill, showing and labelling the forces acting.

[2 marks]

(b) Write an equation relating the forces at the top of the hill to the centripetal acceleration. Hence find the speed at which the car just loses contact with the road.

[4 marks]

(c) At a speed of 20 m s^{-1}, calculate the normal contact force on the car at the top of the hill.

[2 marks]

Q7. A conical pendulum consists of a mass of 0.25 kg on a string of length 0.60 m, rotating so that the string makes an angle of 30ř with the vertical.

(a) Calculate the radius of the circular path.

[1 mark]

(b) Calculate the tension in the string.

[2 marks]

(c) Calculate the angular speed of the mass.

[3 marks]

Mark Scheme and Answers

Q1. The radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle [1].

Q2. (a) $\omega = 120 \times 2\pi/60 = \mathbf{4\pi} = \mathbf{12.6}$ rad $\text{s}^{-\mathbf{1}}$ [2]. (b) $v = r\omega = 0.35 \times 12.6 = \mathbf{4.4}$ m $\text{s}^{-\mathbf{1}}$ [1].

Q3. The direction of the velocity changes continuously [1]; acceleration is the rate of change of velocity — a change in direction (even at constant speed) constitutes acceleration, directed towards the centre of the circle [1].

Q4. $\omega = 3.0 \times 2\pi = 18.85$ rad s^{-1} [1]; $T = F = mr\omega^2 = 0.15 \times 0.80 \times 18.85^2$ [1] $= \mathbf{42.6}$ N [1].

Q5. The centripetal force is always perpendicular to the velocity (direction of motion) [1]; work done $= F\cos 90$ř $\times\, d = 0$ — a perpendicular force does no work [1].

Q6(a). Diagram showing weight mg downward and normal force N upward, with $mg > N$ at speed [2].

Q6(b). At top of hill, net downward force provides centripetal force: $mg - N = mv^2/r$ [1]; at the point of losing contact, $N = 0$: $mg = mv^2/r$ [1]; $v = \sqrt{gr} = \sqrt{9.81 \times 120}$ [1] $= \mathbf{34.3}$ m $\text{s}^{-\mathbf{1}}$ [1].

Q6(c). $mg - N = mv^2/r$; $N = m(g - v^2/r) = 950(9.81 - 20^2/120) = 950(9.81 - 3.33)$ [1] $= 950 \times 6.48 = \mathbf{6160}$ N [1].

Q7(a). $r = L\sin\theta = 0.60\sin 30$ř $= \mathbf{0.30}$ m [1].

Q7(b). $T\cos\theta = mg$; $T = mg/\cos 30$ř $= (0.25 \times 9.81)/\cos 30$ř [1] $= \mathbf{2.83}$ N [1].

Q7(c). $T \sin\theta = mr\omega^2$ [1]; $\omega^2 = T\sin\theta/(mr) = 2.83 \times \sin 30$ř$/(0.25 \times 0.30)$ [1]; $\omega = \sqrt{2.83 \times 0.5/0.075} = \sqrt{18.9} = \mathbf{4.34}$ rad s^{-1} [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define the radian and convert between degrees and radians	
☐ Define angular speed ω and use $\omega = 2\pi/T = 2\pi f$	
☐ Use $v = r\omega$ to relate linear and angular speed	
☐ Explain why an object in uniform circular motion is accelerating	
☐ State that centripetal acceleration is directed towards the centre	
☐ Use $a = r\omega^2$ and $a = v^2/r$	
☐ Use $F = mr\omega^2$ and $F = mv^2/r$	
☐ Identify the physical force providing centripetal force in a given situation	
☐ Explain why centripetal force does no work	
☐ Solve problems involving objects at the top/bottom of vertical circles	
☐ Solve conical pendulum problems by resolving tension components	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Circular motion is the bridge between mechanics and fields — once you are confident with centripetal force and acceleration, gravitational and electric field problems fall into place naturally. Always ask: what real force is pointing towards the centre?

CIE International A Level Physics

Topic 13

Gravitational Fields

Revision Booklet

This booklet covers:

- Newton's Law of Gravitation
- Gravitational Field Strength
- Gravitational Potential
- Orbital Motion & Kepler's Third Law
- Escape Velocity

Core Concepts and Definitions

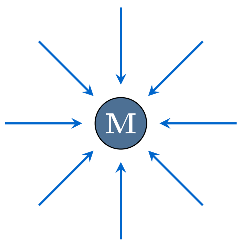
Gravitational Field

A **gravitational field** is a region of space in which a mass experiences a force due to the gravitational attraction of another mass.

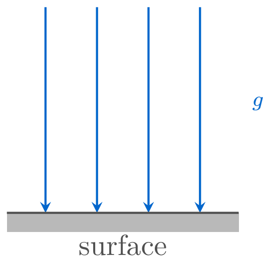
- Gravitational fields are always **attractive** — there is no gravitational repulsion.
- Any mass creates a gravitational field in the space around it.
- A **test mass** placed in the field will experience a force towards the source mass.

Radial vs Uniform Fields

- **Radial field** (e.g., around a planet): field lines point towards the centre; g decreases with distance.
- **Uniform field** (e.g., near Earth's surface over small distances): field lines are parallel and equally spaced; g is approximately constant at 9.81 N kg^{-1}.



Radial Field



Uniform Field

Newton's Law of Gravitation

Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

F = gravitational force between the two masses (N)
G = gravitational constant = 6.67×10^{-11} N m^{2}kg^{-2}
m_1, m_2 = the two point masses (or uniform spheres) (kg)
r = separation between centres of mass (m)

Key Points

- The force is always **attractive**.
- It applies strictly to **point masses**, but also works for uniform spheres if r is measured from the centre.
- r is the **centre-to-centre** distance — not surface to surface.

- The force obeys an **inverse-square law**: double the distance, quarter the force.

Gravitational Field Strength

Definition of Gravitational Field Strength g

The **gravitational field strength** at a point is the gravitational force exerted per unit mass on a small test mass placed at that point.

$$g = \frac{F}{m} \qquad \text{units: N kg}^{-1} \equiv \text{m s}^{-2}$$

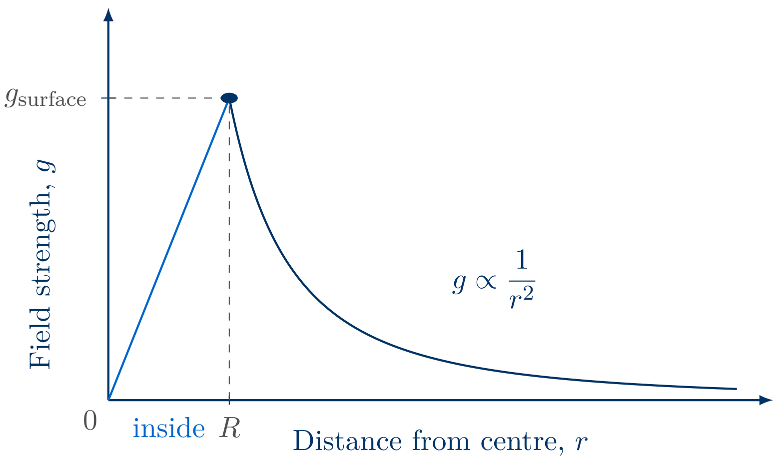
g is a **vector** quantity, directed towards the source mass.

Field Strength Formulae

$$g = \frac{GM}{r^2}$$

g = gravitational field strength at distance r (N kg^{-1})
G = gravitational constant (N m^2 kg^{-2})
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Variation of g with distance r



Common Mistake

Do not confuse g (field strength, N kg^{-1}) with G (the universal gravitational constant, N m^2 kg^{-2}). They are completely different quantities!

Gravitational Potential

Definition of Gravitational Potential ϕ

The **gravitational potential** at a point is the work done per unit mass to move a small test mass from infinity to that point.

$$\phi = \frac{W}{m} \qquad \text{units: J kg}^{-1}$$

Gravitational potential is always **negative** (zero at infinity; work is done *by* gravity as mass moves inwards, so the system loses potential energy).

Gravitational Potential Formulae

$$\phi = -\frac{GM}{r} \qquad E_p = m\phi = -\frac{GMm}{r}$$

ϕ = gravitational potential at distance r (J kg^{-1})
E_p = gravitational potential energy (J)
G = 6.67×10^{-11} N m^2 kg^{-2}
M = mass of body creating the field (kg)
r = distance from centre of M (m)

Relationship Between g and ϕ

Field Strength from Potential Gradient

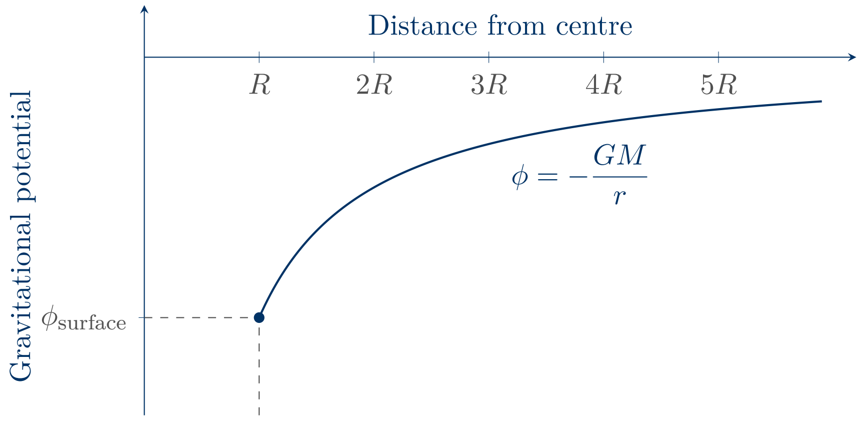
$$g = -\frac{\Delta\phi}{\Delta r}$$

Interpreting the Gradient

g is the **negative gradient** of the potential–distance graph. On a ϕ–r graph:

- A steeper gradient $\Rightarrow$ stronger field (larger g)
- The gradient becomes shallower as r increases $\Rightarrow$ g decreases
- The area under a g–r graph gives the change in potential $\Delta\phi$

Graph of ϕ against r



$\phi \to 0$ as $r \to \infty$

Equipotential Surfaces

An **equipotential surface** is a surface on which the gravitational potential is the same everywhere.

- No work is done moving a mass along an equipotential surface.
- Equipotentials are always **perpendicular** to field lines.
- For a point mass (or uniform sphere), equipotentials are concentric spheres.

Circular Orbits and Satellites

For an object of mass m in a circular orbit of radius r around a mass M, the gravitational force provides the centripetal force:

Orbital Mechanics

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$\therefore \quad v = \sqrt{\frac{GM}{r}} \qquad \text{(orbital speed)}$$

$$T^2 = \frac{4\pi^2}{GM}r^3 \qquad \text{(Kepler's Third Law)}$$

Kepler's Third Law

$$T^2 \propto r^3$$

The square of the orbital period is proportional to the cube of the orbital radius. The constant of proportionality is $\frac{4\pi^2}{GM}$.

Energy in Circular Orbits

Energy of a Satellite in Circular Orbit

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r} \qquad \text{(kinetic energy — always positive)}$$
$$E_p = -\frac{GMm}{r} \qquad \text{(potential energy — always negative)}$$
$$E_{total} = E_k + E_p = -\frac{GMm}{2r} \qquad \text{(total energy — always negative)}$$

As r increases: v decreases, E_k decreases, E_p increases, E_{total} increases (becomes less negative).

Geostationary Orbits

Geostationary Satellite

A **geostationary** satellite has:

- An orbital period of exactly **24 hours**
- An orbit in the **equatorial plane**
- Movement in the **same direction** as Earth's rotation (west to east)
- An orbital radius of approximately 4.2×10^7 m ($\approx$ 36 000 km altitude)

It appears **stationary** relative to a point on Earth's surface.
Uses: satellite TV, telecommunications, weather monitoring.

Escape Velocity

Definition of Escape Velocity

The **escape velocity** is the minimum speed at which an object must be projected from the surface of a body so that it can escape to infinity against the gravitational field, without any further energy input.

Escape Velocity

Setting total energy = 0 at infinity:

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Note: $v_{esc} = \sqrt{2}\, v_{orbital}$ at the same radius.

Formula Summary Sheet

Formula	Quantity	Units
$F = \frac{Gm_1m_2}{r^2}$	Gravitational force	N
$g = \frac{F}{m}$	Field strength (definition)	N kg^{-1}
$g = \frac{GM}{r^2}$	Field strength (point mass)	N kg^{-1}
$\phi = -\frac{GM}{r}$	Gravitational potential	J kg^{-1}
$E_p = -\frac{GMm}{r}$	Gravitational PE	J
$g = -\frac{\Delta\phi}{\Delta r}$	Field from potential gradient	N kg^{-1}
$v = \sqrt{\frac{GM}{r}}$	Orbital speed	m s^{-1}
$T^2 = \frac{4\pi^2r^3}{GM}$	Kepler's Third Law	s^2, m^3
$v_{esc} = \sqrt{\frac{2GM}{R}}$	Escape velocity	m s^{-1}

Constants: $G = 6.67 \times 10^{-11}$ N m^{2}kg^{-2}, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Calculation Questions

1. **Identify** what you are asked to find.
2. **List** the quantities given; convert units if needed (e.g. days → seconds, km → m).
3. **Select** the appropriate formula.
4. **Substitute** values carefully, showing all working.
5. **Check** units and significant figures in your answer.

Common Errors — Avoid These!

- Using **diameter** instead of radius in $g = GM/r^2$.
- Forgetting the **negative sign** in $\phi = -GM/r$.
- Not converting **hours/days to seconds** before applying Kepler's Law.
- Confusing g (field strength, vector) with ϕ (potential, scalar).
- Saying $\phi = 0$ at the surface — it is zero only **at infinity**.

- Thinking higher orbit ⇒ faster orbital speed — it is **slower**.

Worked Examples

Example 1 — Field Strength at Altitude

Question: Calculate the gravitational field strength at a height of 400 km above Earth's surface. ($M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m)

Solution

Solution:
$r = R_E + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6$ m

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.77 \times 10^6)^2}$$

$$g = \frac{3.98 \times 10^{14}}{4.58 \times 10^{13}} = \mathbf{8.69} \text{ N kg}^{-1}$$

Note: this is the field strength at the ISS orbit — astronauts are *not* weightless because gravity is zero!

Example 2 — Orbital Period Using Kepler's Third Law

Question: A satellite orbits Earth at a radius of 4.2×10^7 m. Calculate its orbital period.

Solution

Solution:

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 \times (4.2 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}$$

$$T^2 = \frac{4\pi^2 \times 7.41 \times 10^{22}}{3.98 \times 10^{14}} = 7.36 \times 10^9 \text{ s}^2$$

$$T = \sqrt{7.36 \times 10^9} = 8.58 \times 10^4 \text{ s} \approx \mathbf{23.8} \text{ hours}$$

This is a geostationary orbit ($T \approx 24$ h).

Example 3 — Change in Gravitational Potential Energy

Question: A spacecraft of mass 2500 kg is moved from the Earth's surface to an altitude of 800 km. Calculate the increase in gravitational potential energy.

Solution

Solution:
$r_1 = 6.37 \times 10^6$ m, $\quad r_2 = 6.37 \times 10^6 + 8.00 \times 10^5 = 7.17 \times 10^6$ m

$$\Delta E_p = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta E_p = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 2500 \times \left(\frac{1}{6.37 \times 10^6} - \frac{1}{7.17 \times 10^6}\right)$$

$$\Delta E_p = 9.95 \times 10^{17} \times 1.75 \times 10^{-8} = \mathbf{1.74 \times 10^{10}} \text{ J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State Newton's Law of Gravitation.

[2 marks]

Q2. Define gravitational field strength and state its units.

[2 marks]

Q3. Explain why gravitational potential is always a negative quantity.

[2 marks]

Q4. The gravitational field strength at Earth's surface is 9.81 N kg^{-1}. Calculate the field strength at a distance of $3R_E$ from Earth's centre, where R_E is Earth's radius.

[2 marks]

Q5. Two planets, X and Y, orbit the same star. Planet X has an orbital radius twice that of planet Y. Determine the ratio T_X/T_Y.

[3 marks]

Section B — Longer Structured Questions

Q6. A satellite of mass m orbits a planet of mass M in a circular orbit of radius r.

(a) Show that the orbital speed of the satellite is given by $v = \sqrt{GM/r}$.

[2 marks]

(b) Show that the period of the orbit satisfies $T^2 = \dfrac{4\pi^2 r^3}{GM}$.

[2 marks]

(c) The satellite is moved to a lower orbit. Explain what happens to its:

- speed
- kinetic energy
- total energy

[3 marks]

Q7. The Moon orbits the Earth with a period of 27.3 days at a mean orbital radius of 3.84×10^8 m.

(a) Use this data to calculate the mass of the Earth.

[3 marks]

(b) Calculate the gravitational potential at the Moon's orbital radius.

[2 marks]

(c) A 75 kg astronaut travels from Earth's surface to the Moon's orbital radius. Calculate the change in gravitational potential energy.

[3 marks]

Mark Scheme and Answers

Q1. Any two masses exert an attractive force on each other [1]; the force is proportional to the product of their masses and inversely proportional to the square of their separation [1].

Q2. Gravitational field strength is the gravitational force per unit mass acting on a (small test) mass placed at that point [1]; units: N kg^{-1} (or m s^{-2}) [1].

Q3. Gravitational potential is defined as zero at infinity [1]; work is done by gravity as mass moves from infinity inward, so the potential decreases below zero — it is always negative [1].

Q4. $g \propto 1/r^2$; at $3R_E$, $g = 9.81/3^2 = 9.81/9 = \mathbf{1.09}$ N kg^{-1} [2].

Q5. By Kepler's Third Law: $T^2 \propto r^3$, so $\left(\frac{T_X}{T_Y}\right)^2 = \left(\frac{2r_Y}{r_Y}\right)^3 = 8$ [2]; $\frac{T_X}{T_Y} = \sqrt{8} = \mathbf{2\sqrt{2} \approx 2.83}$ [1].

Q6(a). Gravitational force = centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$ [1]; cancel m, rearrange: $v = \sqrt{GM/r}$ [1].

Q6(b). Substitute $v = 2\pi r/T$ into result from (a) [1]; rearrange to get $T^2 = 4\pi^2 r^3/GM$ [1].

Q6(c). Speed **increases** [1]; KE **increases** [1]; total energy **decreases** (becomes more negative) [1].

Q7(a). $T = 27.3 \times 24 \times 3600 = 2.36 \times 10^6$ s; $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84 \times 10^8)^3}{6.67 \times 10^{-11}(2.36 \times 10^6)^2}$ [1] $= \mathbf{6.02 \times 10^{24}}$ kg [2].

Q7(b). $\phi = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24}}{3.84 \times 10^8} = \mathbf{-1.05 \times 10^6}$ J kg^{-1} [2].

Q7(c). $\Delta E_p = m\Delta\phi = m(\phi_{\text{Moon orbit}} - \phi_{\text{surface}})$ [1]; $\phi_{\text{surface}} = -GM/R_E = -6.25 \times 10^7$ J kg^{-1}; $\Delta\phi = -1.05 \times 10^6 - (-6.25 \times 10^7) = 6.14 \times 10^7$ J kg^{-1}; $\Delta E_p = 75 \times 6.14 \times 10^7 = \mathbf{4.6 \times 10^9}$ J [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	State and apply Newton's Law of Gravitation	
☐	Define gravitational field strength; use $g = F/m$ and $g = GM/r^2$	
☐	Sketch field line and equipotential diagrams	
☐	Define gravitational potential and explain why it is negative	
☐	Use $\phi = -GM/r$ and $E_p = -GMm/r$	
☐	Apply the relationship $g = -\Delta\phi/\Delta r$	
☐	Derive the expression for orbital speed	
☐	State and apply Kepler's Third Law ($T^2 \propto r^3$)	
☐	Analyse energy changes in satellite orbits	
☐	Describe the properties of a geostationary satellite	
☐	Derive and apply the formula for escape velocity	
☐	Interpret graphs of g vs r and ϕ vs r	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: understanding *why* formulas work is more powerful than memorising them. Practice drawing diagrams and deriving key results from first principles.

CIE International A Level Physics

Topic 14, 15 & 16

Thermal Physics

Revision Booklet

This booklet covers:

- Temperature and Thermal Equilibrium
- Temperature Scales
- Specific Heat Capacity and Latent Heat
- The Mole and Ideal Gases
- Kinetic Theory of Gases
- Internal Energy
- The First Law of Thermodynamics

Thermal Equilibrium

Thermal Energy Transfer

Thermal (heat) energy is transferred from a region of **higher temperature** to a region of **lower temperature**. Transfer continues until the two regions reach the same temperature.

Thermal Equilibrium

Two objects are in **thermal equilibrium** when there is no net transfer of thermal energy between them. This occurs when they are at the **same temperature**.

Zeroth Law of Thermodynamics: If object A is in thermal equilibrium with object C, and object B is also in thermal equilibrium with object C, then A and B are in thermal equilibrium with each other.

Temperature Scales

Thermometric Property

A **thermometric property** is a physical property that varies continuously and measurably with temperature. Examples include:

- Resistance of a metal wire (e.g. platinum resistance thermometer)
- e.m.f. of a thermocouple
- Volume of a gas at constant pressure
- Density of a liquid

Thermodynamic Temperature Scale

The **thermodynamic (Kelvin) temperature scale** does not depend on the property of any particular substance. It is an absolute scale with:

- **Absolute zero** (0 K) — the lowest possible temperature
- **Triple point of water** (273.16 K) — fixed reference point

Temperature Conversion

$$T/\mathrm{K} = \theta/{}^\circ\mathrm{C} + 273.15$$

T = thermodynamic temperature (K)
θ = Celsius temperature (°C)

Note: a temperature *difference* of 1 K equals a difference of 1 °C.

Absolute Zero

At absolute zero (0 K = −273.15 °C):

- Molecules have minimum possible kinetic energy
- It is impossible to reach in practice, only to approach asymptotically
- All ideal gases would have zero volume (zero pressure at constant volume)

Specific Heat Capacity and Latent Heat

Specific Heat Capacity

The **specific heat capacity** c of a substance is the energy required to raise the temperature of **unit mass** by **one kelvin** (or one degree Celsius), without change of state.

$$c = \frac{Q}{m\,\Delta T} \qquad \text{units: J kg}^{-1}\text{K}^{-1}$$

Specific Heat Capacity

$$Q = mc\Delta T$$

Q = thermal energy transferred (J)
m = mass (kg)
c = specific heat capacity ($\text{J kg}^{-1}\,\text{K}^{-1}$)
ΔT = temperature change (K or °C)

Specific Latent Heat

The **specific latent heat** L of a substance is the energy required to change the state of **unit mass** at constant temperature.

$$L = \frac{Q}{m} \qquad \text{units: J kg}^{-1}$$

- **Specific latent heat of fusion** L_f: solid → liquid (melting)
- **Specific latent heat of vaporisation** L_v: liquid → gas (boiling)

Note: $L_v > L_f$ for any given substance, since greater work is done separating molecules completely during vaporisation.

Specific Latent Heat

$$Q = mL$$

Q = thermal energy transferred (J)
m = mass (kg)
L = specific latent heat (J kg^{-1})

Common Mistake

During a change of state, temperature remains **constant** even though energy is being supplied. The energy goes into breaking intermolecular bonds, not increasing kinetic energy.

The Mole and Ideal Gases

Amount of Substance

The **mole** (mol) is the SI base unit for amount of substance. One mole of any substance contains exactly $N_A = 6.02 \times 10^{23}$ particles (the Avogadro constant).

Equation of State for an Ideal Gas

$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

p = pressure (Pa)
V = volume (m^3)
n = number of moles (mol)
R = molar gas constant = 8.31 $\text{J mol}^{-1}\text{K}^{-1}$
T = thermodynamic temperature (K)
N = number of molecules
k = Boltzmann constant = 1.38×10^{-23} J K^{-1}

The Boltzmann constant: $k = R/N_A$

Ideal Gas

An **ideal gas** is one that obeys $pV \propto T$ (where T is thermodynamic temperature) under all conditions. Real gases approximate ideal behaviour at low pressures and high temperatures.

Gas Laws — special cases of $pV = nRT$

- **Boyle's Law** (constant T): pV = constant, so $p_1V_1 = p_2V_2$
- **Charles' Law** (constant p): V/T = constant, so $V_1/T_1 = V_2/T_2$
- **Pressure Law** (constant V): p/T = constant, so $p_1/T_1 = p_2/T_2$

Common Mistake

Always use **thermodynamic temperature in kelvin** in gas law calculations — never degrees Celsius.

Kinetic Theory of Gases

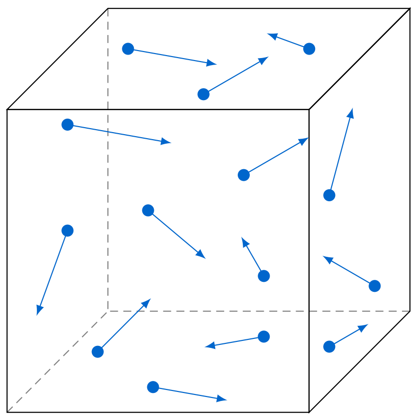
Assumptions of the Kinetic Model

Basic Assumptions of Kinetic Theory

1. The gas contains a **large number** of molecules moving in **random directions** with a range of speeds.
2. The molecules occupy **negligible volume** compared to the volume of the gas.
3. All collisions are **perfectly elastic** (no kinetic energy lost).
4. **Intermolecular forces are negligible** except during collisions.
5. The duration of collisions is **negligible** compared to the time between collisions.
6. Molecules obey **Newton's laws of motion**.

Pressure from Kinetic Theory

The pressure exerted by a gas arises from molecules colliding with the container walls. Each collision exerts a force; the average of many such collisions gives a steady pressure.



Kinetic Theory Equation

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

p = pressure (Pa)
V = volume (m^3)
N = total number of molecules
m = mass of one molecule (kg)
$\langle c^2\rangle$ = mean square speed ($m^2\,s^{-2}$)

The **root-mean-square speed**: $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$

Note: $c_{\text{r.m.s.}}$ is not the same as the mean speed $\bar{c}$. Because we square first then average, faster molecules contribute more. In general $c_{\text{r.m.s.}} > \bar{c}$.

Derivation of $pV = \frac{1}{3}Nm\langle c^2\rangle$

Consider one molecule of mass m moving with speed c_x in the x-direction inside a box of side L.

Step 1 — change in momentum at one wall: The molecule hits the right wall and bounces back elastically:

$$\Delta p = mc_x - (-mc_x) = 2mc_x$$

Step 2 — time between collisions with the same wall: The molecule must travel $2L$ before returning:

$$\Delta t = \frac{2L}{c_x}$$

Step 3 — force from one molecule:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mc_x}{2L/c_x} = \frac{mc_x^2}{L}$$

Step 4 — pressure from N molecules: Summing over all N molecules and dividing by area L^2:

$$p = \frac{Nm\langle c_x^2\rangle}{L^3} = \frac{Nm\langle c_x^2\rangle}{V}$$

Step 5 — extend to 3 dimensions: By symmetry of random motion: $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle = \frac{1}{3}\langle c^2\rangle$

$$\boxed{pV = \tfrac{1}{3}Nm\langle c^2\rangle}$$

Molecular Kinetic Energy

Derivation of $\langle E_k\rangle = \frac{3}{2}kT$

Start with the two equations for an ideal gas:

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle \qquad \text{and} \qquad pV = NkT$$

Equating the right-hand sides:

$$\tfrac{1}{3}Nm\langle c^2\rangle = NkT$$

Divide both sides by N and multiply by $\frac{3}{2}$:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

Since $\langle E_k\rangle = \frac{1}{2}m\langle c^2\rangle$:

$$\boxed{\langle E_k\rangle = \tfrac{3}{2}kT}$$

Average Translational Kinetic Energy

$$\langle E_k\rangle = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

The average kinetic energy of a molecule is **directly proportional to thermodynamic temperature**.

Key Deductions

- At absolute zero, molecules have minimum kinetic energy
- Molecules of different gases at the same temperature have the **same average KE**
- Heavier molecules move more **slowly** on average at the same temperature: $c_{\text{r.m.s.}} = \sqrt{3kT/m}$
- $c_{\text{r.m.s.}} \propto \sqrt{T}$ — doubling thermodynamic temperature increases r.m.s. speed by factor $\sqrt{2}$

Internal Energy

Internal Energy

The **internal energy** of a system is the sum of the **random kinetic energies** and **potential energies** of all the molecules in the system.

$$U = E_{k,\text{total}} + E_{p,\text{total}}$$

- Internal energy is a **function of state**.
- For an **ideal gas**: $E_p = 0$, so $U = E_{k,\text{total}}$ only.
- A rise in temperature $\Rightarrow$ increase in average KE $\Rightarrow$ increase in U.
- During a change of state: temperature (and KE) constant; E_p increases as bonds break; U increases.

The First Law of Thermodynamics

Work Done by/on a Gas

When a gas changes volume at **constant pressure**:

$$W = p\Delta V$$

W = work done **by** the gas (J)
p = pressure (Pa)
ΔV = change in volume (m^3)

First Law of Thermodynamics

$$\Delta U = q + W$$

ΔU = increase in internal energy of the system (J)
q = energy supplied **to** the system by heating (J)
W = work done **on** the system (J)

Sign Convention

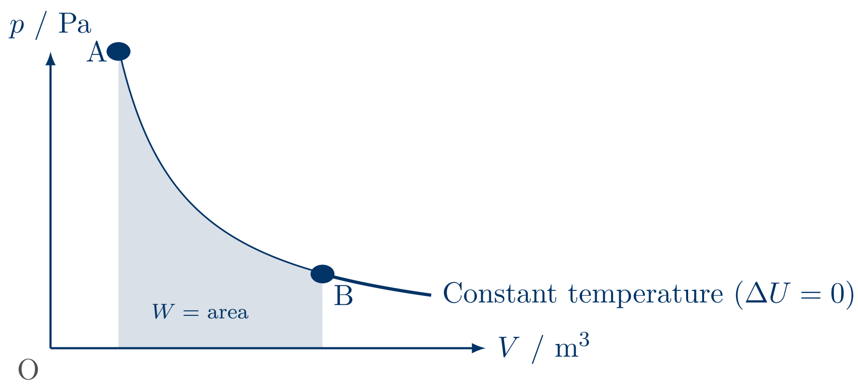
CIE uses $\Delta U = q + W$ where W is work done **on** the system. Some textbooks use $\Delta U = q - W$ where W is work done **by** the system. Always check which convention is being used.

Special Cases of the First Law

Process	Condition	Result
Constant temperature	$\Delta T = 0 \Rightarrow \Delta U = 0$	$q = -W$
No heat transfer	$q = 0$	$\Delta U = W$
Constant volume	$\Delta V = 0 \Rightarrow W = 0$	$\Delta U = q$
Constant pressure	p constant	$\Delta U = q - p\Delta V$

p–V Diagrams

The **area under a p–V curve** equals the work done by the gas during that process.



Formula Summary

Formula	Quantity	Units
$T/\text{K} = \theta/°\text{C} + 273.15$	Temperature conversion	K
$Q = mc\Delta T$	Sensible heat	J
$Q = mL$	Latent heat	J
$pV = nRT$	Ideal gas (moles)	Pa, m^3, K
$pV = NkT$	Ideal gas (molecules)	Pa, m^3, K
$k = R/N_A$	Boltzmann constant	J K^{-1}
$pV = \frac{1}{3}Nm\langle c^2\rangle$	Kinetic theory	Pa
$\langle E_k\rangle = \frac{3}{2}kT$	Mean molecular KE	J
$W = p\Delta V$	Work done by gas	J
$\Delta U = q + W$	First law	J

Constants: $R = 8.31$ J mol^{-1}K^{-1} $k = 1.38 \times 10^{-23}$ J K^{-1} $N_A = 6.02 \times 10^{23}$ mol^{-1}

Worked Examples

Example 1 — Specific Heat Capacity

Question: A 2.0 kg block of aluminium ($c = 900$ J kg^{-1}K^{-1}) is heated from 20 °C to 80 °C. Calculate the energy supplied.

Solution

$$Q = mc\Delta T = 2.0 \times 900 \times (80 - 20) = \mathbf{1.08 \times 10^5} \text{ J}$$

Example 2 — Ideal Gas Law

Question: A fixed mass of ideal gas has pressure 1.2×10^5 Pa and volume 3.0×10^{-3} m^3 at 27 °C. It is heated at constant pressure until its volume doubles. Find the final temperature.

Solution

$T_1 = 27 + 273.15 = 300.15$ K. At constant pressure, $V \propto T$:

$$T_2 = T_1 \times \frac{V_2}{V_1} = 300.15 \times 2 = \mathbf{600} \text{ K } (\mathbf{327} \text{ °C})$$

Example 3 — r.m.s. Speed

Question: Calculate the r.m.s. speed of nitrogen molecules ($M = 0.028\ \text{kg}\,\text{mol}^{-1}$) at 300 K.

Solution

Mass of one molecule: $m = 0.028/6.02 \times 10^{23} = 4.65 \times 10^{-26}$ kg

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.65 \times 10^{-26}}} = \sqrt{2.67 \times 10^5} = \mathbf{517}\ \text{m}\,\text{s}^{-1}$$

Example 4 — First Law

Question: A gas is compressed. 650 J of work is done on the gas and 200 J of heat flows out. Find the change in internal energy.

Solution

$W = +650$ J (work done *on* gas), $\quad q = -200$ J (heat leaves)

$$\Delta U = q + W = -200 + 650 = +\mathbf{450}\ \text{J}$$

Internal energy **increases** by 450 J.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define *specific latent heat of vaporisation* and explain why it is greater than the specific latent heat of fusion for the same substance.

[3 marks]

Q2. State the basic assumptions of the kinetic theory of gases.

[4 marks]

Q3. Explain what is meant by *internal energy* and state how it differs for an ideal gas compared to a real gas.

[3 marks]

Q4. A gas at pressure 2.0×10^5 Pa and temperature 300 K occupies a volume of 0.40 m^3. Calculate the number of molecules present.

[2 marks]

Q5. Show that the mean kinetic energy of a gas molecule is $\frac{3}{2}kT$ by combining the kinetic theory equation with the ideal gas equation.

[3 marks]

Section B — Longer Structured Questions

Q6. A sample of water of mass 0.50 kg is heated from 20 °C to 100 °C and then completely vaporised.
($c_{\text{water}} = 4200\ \text{J kg}^{-1}\text{K}^{-1}$, $L_v = 2.26 \times 10^6\ \text{J kg}^{-1}$)

(a) Calculate the energy needed to heat the water from 20 °C to 100 °C.

[2 marks]

(b) Calculate the energy needed to vaporise the water at 100 °C.

[2 marks]

(c) Explain in terms of molecular behaviour why energy is needed to vaporise water even though the temperature does not change.

[2 marks]

Q7. A fixed mass of ideal gas undergoes the cycle A → B → C → A on a p–V diagram where A → B is constant temperature expansion, B → C is constant volume pressure decrease, and C → A is constant pressure compression.

(a) State the change in internal energy during A → B and justify your answer.
[2 marks]

(b) Using the first law, find q for A → B given that the work done by the gas is 440 J.
[2 marks]

(c) Describe the energy changes during the constant volume process B → C.
[2 marks]

Mark Scheme and Answers

Q1. Specific latent heat of vaporisation is the energy per unit mass required to change a substance from liquid to gas at constant temperature [1]; greater than L_f because molecules must be completely separated, requiring work against intermolecular forces over a much greater distance than in melting [2].

Q2. Large number of molecules moving randomly in all directions with a range of speeds [1]; occupying negligible volume compared to container [1]; collisions perfectly elastic [1]; intermolecular forces negligible except during collisions; collisions of negligible duration; obey Newton's laws [1].

Q3. Internal energy is the sum of random kinetic and potential energies of all molecules [1]; for an ideal gas, $E_p = 0$ so internal energy is kinetic energy only [1]; for a real gas, molecules have potential energy due to intermolecular forces [1].

Q4. $N = pV/kT = (2.0 \times 10^5 \times 0.40)/(1.38 \times 10^{-23} \times 300) = \mathbf{1.93 \times 10^{25}}$ molecules [2].

Q5. $pV = \frac{1}{3}Nm\langle c^2\rangle$ [1]; equate with $pV = NkT$: $\frac{1}{3}m\langle c^2\rangle = kT$ [1]; so $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ [1].

Q6(a). $Q = 0.50 \times 4200 \times 80 = \mathbf{1.68 \times 10^5}$ J [2].

Q6(b). $Q = 0.50 \times 2.26 \times 10^6 = \mathbf{1.13 \times 10^6}$ J [2].

Q6(c). KE of molecules unchanged (temperature constant) [1]; energy breaks intermolecular bonds as molecules separate completely, increasing potential energy [1].

Q7(a). $\Delta U = 0$ [1]; temperature is constant and for an ideal gas internal energy depends only on temperature [1].

Q7(b). $\Delta U = 0$, work done on gas $= -440$ J; $q = +440$ J — heat flows into gas [2].

Q7(c). No work done ($\Delta V = 0$) [1]; temperature and KE decrease; heat flows out; internal energy decreases [1].

Revision Checklist

	Learning Objective	Confidence (1–3)
☐	Explain thermal equilibrium and direction of heat flow	
☐	Convert between Celsius and Kelvin	
☐	State examples of thermometric properties	
☐	Define and use specific heat capacity: $Q = mc\Delta T$	
☐	Define and use specific latent heat: $Q = mL$	
☐	State the assumptions of kinetic theory	
☐	Use $pV = nRT$ and $pV = NkT$	
☐	Derive and use $pV = \frac{1}{3}Nm\langle c^2\rangle$	
☐	Show that mean KE of a molecule is $\frac{3}{2}kT$	
☐	Define internal energy and relate to temperature	
☐	Apply the first law $\Delta U = q + W$	
☐	Interpret and use p–V diagrams	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Remember: understanding *why* formulas work is more powerful than memorising them.
Practice drawing p–V diagrams and deriving key results from first principles.

CIE International A Level Physics

Topic 17

Oscillations

Revision Booklet

This booklet covers:

- Simple Harmonic Motion
- Displacement, Velocity & Acceleration
- Energy in SHM
- Damped Oscillations
- Forced Oscillations & Resonance

Core Concepts and Definitions

Oscillatory Motion — Key Terms

- **Displacement** x: the distance from the equilibrium position, with direction (m).
- **Amplitude** x_0: the maximum displacement from equilibrium (m).
- **Period** T: the time for one complete oscillation (s).
- **Frequency** f: the number of complete oscillations per second (Hz); $f = 1/T$.
- **Angular frequency** ω: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}).
- **Phase difference** ϕ: the fraction of a cycle by which one oscillation leads or lags another, expressed in radians.

Simple Harmonic Motion (SHM)

An oscillation is **simple harmonic** if the acceleration of the object is:

- **proportional** to its displacement from a fixed equilibrium point, and
- always directed **towards** that equilibrium point (opposite to displacement).

$$a \propto -x$$

Equations of Simple Harmonic Motion

Defining Equation of SHM

$$a = -\omega^2 x$$

a = acceleration (m s^{-2})
ω = angular frequency (rad s^{-1})
x = displacement from equilibrium (m)

The negative sign confirms acceleration is always directed **opposite** to displacement.

Displacement; Velocity and Acceleration

$$x = x_0 \sin \omega t \qquad \text{(starting from equilibrium)}$$

$$v = v_0 \cos \omega t \qquad \text{where } v_0 = \omega x_0$$

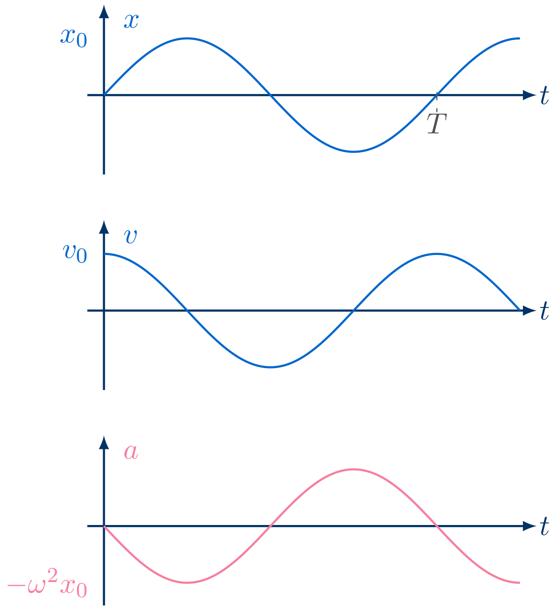
$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

x_0 = amplitude (m)
v_0 = maximum speed = ωx_0 (m s^{-1})
t = time (s)

Maxima and Minima

- Acceleration is **maximum** at maximum displacement ($x = \pm x_0$): $|a_{\text{max}}| = \omega^2 x_0$
- Speed is **maximum** at the equilibrium position ($x = 0$): $v_{\text{max}} = \omega x_0$
- Speed is **zero** at the turning points ($x = \pm x_0$).

Graphs of x, v and a against time



Phase Relationships

- v leads x by $\frac{\pi}{2}$ rad (quarter period ahead).
- a leads v by $\frac{\pi}{2}$ rad, so a is **antiphase** (π rad ahead) with x.

Common Mistake

The equation $x = x_0 \sin \omega t$ assumes the object starts at the **equilibrium position** at $t = 0$. If it starts at maximum displacement, use $x = x_0 \cos \omega t$ instead. Always check the initial conditions.

Energy in Simple Harmonic Motion

Total Energy of a SHM System

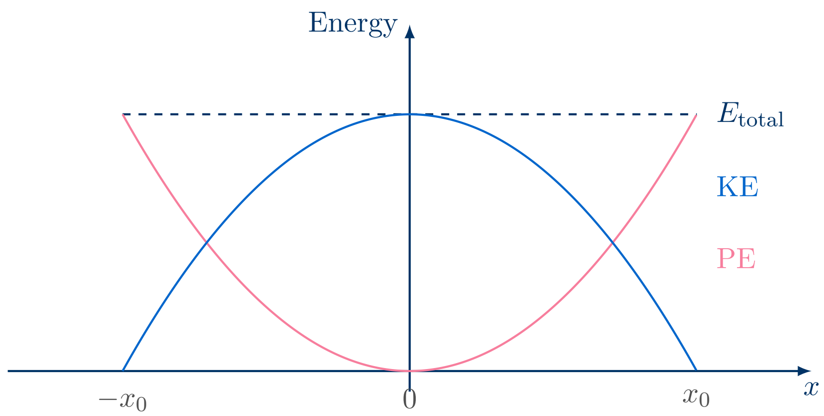
$$E = \frac{1}{2}m\omega^2 x_0^2$$

E = total mechanical energy (J)
m = mass of the oscillating object (kg)
ω = angular frequency (rad s^{-1})
x_0 = amplitude (m)

Energy Interchange During SHM

- At the **equilibrium position** ($x = 0$): KE is maximum, PE is zero.
- At the **turning points** ($x = \pm x_0$): KE is zero, PE is maximum.
- The **total energy remains constant** throughout (assuming no damping).
- $\text{KE} = \frac{1}{2}m\omega^2(x_0^2 - x^2)$ $\quad$ $\text{PE} = \frac{1}{2}m\omega^2 x^2$

Energy–displacement graph



Damped and Forced Oscillations

Damping

Damping

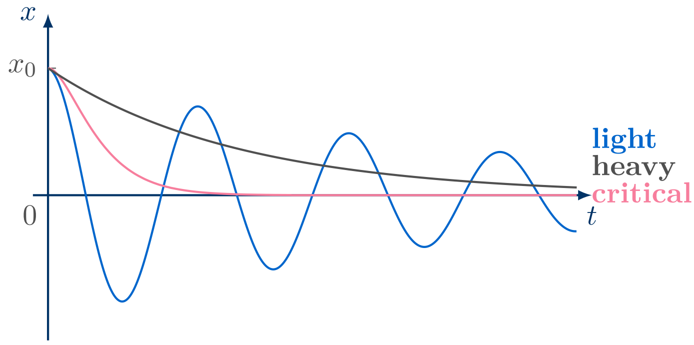
Damping occurs when a resistive force (e.g., air resistance, viscosity) acts on an oscillating system. Energy is removed from the system, causing the amplitude to **decrease over time**. The frequency is approximately unchanged for light damping.

Types of Damping

- **Light damping**: amplitude decreases gradually over many oscillations; the system oscillates for a long time before coming to rest.

- **Critical damping**: the system returns to equilibrium in the **shortest possible time** without oscillating. Used in car suspension and door closers.
- **Heavy (overdamping)**: the system returns to equilibrium **slowly** without oscillating; slower return than critical damping.

Displacement–time graphs for the three types of damping



Forced Oscillations and Resonance

Forced Oscillations and Natural Frequency

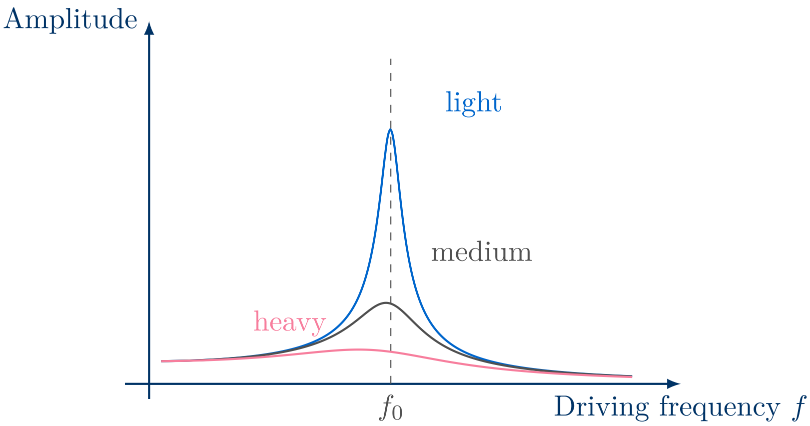
- Every oscillating system has a **natural frequency** f_0 at which it oscillates freely when displaced and released.
- A **forced oscillation** occurs when a periodic **driving force** is applied to the system at a **driving frequency** f.
- The system oscillates at the **driving frequency**, not necessarily at its natural frequency.

Resonance

Resonance occurs when the driving frequency equals the natural frequency of the system ($f = f_0$). At resonance:

- The amplitude of oscillation is a **maximum**.
- Energy transfer from the driver to the system is most efficient.
- The degree of damping determines how sharp the resonance peak is and the maximum amplitude reached.

Amplitude–frequency graph (resonance curves)



Effect of Damping on Resonance

- More damping ⇒ lower and broader resonance peak.
- More damping ⇒ peak shifts **below** f_0 (towards lower frequencies).
- Less damping ⇒ sharper, taller peak with maximum amplitude closer to f_0.
- With no damping the peak would be infinite at exactly f_0.

Resonance in Real Life

Resonance can be **useful** (e.g. MRI scanners, musical instruments, microwave ovens) or **destructive** (e.g. bridges vibrating in wind, buildings in earthquakes). Engineers use damping to control unwanted resonance.

Formula Summary Sheet

Formula	Quantity	Units
$a = -\omega^2 x$	SHM defining equation	m s^{-2}
$\omega = 2\pi f = \dfrac{2\pi}{T}$	Angular frequency	rad s^{-1}
$x = x_0 \sin \omega t$	Displacement (from equilibrium at $t = 0$)	m
$v = v_0 \cos \omega t$	Velocity	m s^{-1}
$v = \pm\,\omega\sqrt{x_0^2 - x^2}$	Speed at displacement x	m s^{-1}
$v_0 = \omega x_0$	Maximum speed	m s^{-1}
$a_{\max} = \omega^2 x_0$	Maximum acceleration	m s^{-2}
$E = \frac{1}{2} m\omega^2 x_0^2$	Total energy of SHM system	J
$\mathrm{KE} = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$	Kinetic energy at displacement x	J
$\mathrm{PE} = \frac{1}{2} m\omega^2 x^2$	Potential energy at displacement x	J

Key relationships: $T = 1/f$, $\omega = 2\pi f$, $v_{\max} = \omega x_0$, $a_{\max} = \omega^2 x_0$
Phase: v leads x by $\pi/2$; a is antiphase with x (leads by π)

Worked Examples

Example 1 — Finding Angular Frequency and Max Speed

Question: A mass oscillates with SHM, amplitude 4.0 cm and period 0.80 s. Calculate (a) the angular frequency and (b) the maximum speed.

Solution

(a) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.80} = \mathbf{7.85}$ rad s^{-1}

(b) $v_{\max} = \omega x_0 = 7.85 \times 0.040 = \mathbf{0.31}$ m s^{-1}

Example 2 — Speed at a Given Displacement

Question: The same mass ($\omega = 7.85$ rad s^{-1}, $x_0 = 4.0$ cm) has displacement $x = 2.5$ cm. Find its speed.

Solution

$$v = \omega\sqrt{x_0^2 - x^2} = 7.85 \times \sqrt{(0.040)^2 - (0.025)^2}$$

$$v = 7.85 \times \sqrt{1.60 \times 10^{-3} - 6.25 \times 10^{-4}} = 7.85 \times \sqrt{9.75 \times 10^{-4}}$$

$$v = 7.85 \times 0.0312 = \mathbf{0.245} \text{ m s}^{-1}$$

Example 3 — Total Energy of SHM System

Question: A 0.20 kg mass oscillates with $\omega = 7.85$ rad s^{-1} and amplitude 4.0 cm. Calculate the total energy.

Solution

$$E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.20 \times (7.85)^2 \times (0.040)^2$$

$$E = 0.10 \times 61.6 \times 1.60 \times 10^{-3} = \mathbf{9.9 \times 10^{-3}} \text{ J}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State the two conditions that define simple harmonic motion.

[2 marks]

Q2. Define the terms *amplitude*, *period* and *angular frequency* for an oscillating system.

[3 marks]

Q3. An object undergoes SHM with frequency 2.5 Hz and amplitude 6.0 cm.

(a) Calculate the angular frequency. *[1 mark]*

(b) Calculate the maximum acceleration. *[2 marks]*

(c) Calculate the speed when $x = 4.0$ cm. *[2 marks]*

Q4. Sketch displacement–time graphs for *light*, *critical* and *heavy* damping on the same axes, starting from the same initial displacement. Label each curve.

[3 marks]

Q5. Explain what is meant by *resonance* and state the condition under which it occurs.

[2 marks]

Section B — Longer Structured Questions

Q6. A 0.15 kg mass is attached to a spring and oscillates vertically with SHM. Its displacement is given by $x = 0.050 \sin(12t)$, where x is in metres and t in seconds.

(a) Write down the amplitude and angular frequency of the motion.

[2 marks]

(b) Show that the maximum speed is 0.60 m s^{-1}.

[2 marks]

(c) Calculate the total energy of the oscillation.

[2 marks]

(d) Sketch graphs on the same axes showing how the kinetic energy and potential energy vary with displacement x. Label your axes clearly.

[3 marks]

Q7. A child on a swing is pushed periodically by a parent.

(a) Explain why the amplitude of the swing increases when the pushing frequency equals the natural frequency of the swing.

[2 marks]

(b) The parent now pushes at a frequency higher than the natural frequency. Describe and explain what happens to the amplitude.

[2 marks]

(c) Air resistance acts on the swing. Describe how this affects the resonance curve compared to an undamped system.

[2 marks]

Mark Scheme and Answers

Q1. Acceleration is proportional to displacement from a fixed (equilibrium) point [1]; acceleration is always directed towards that point / opposite to displacement [1].

Q2. Amplitude: maximum displacement from equilibrium [1]. Period: time for one complete oscillation [1]. Angular frequency: $\omega = 2\pi f = 2\pi/T$ (rad s^{-1}) [1].

Q3(a). $\omega = 2\pi f = 2\pi \times 2.5 = \mathbf{15.7}$ **rad s$^{\mathbf{-1}}$** [1].

Q3(b). $a_{\text{max}} = \omega^2 x_0 = (15.7)^2 \times 0.060$ [1] $= \mathbf{14.8}$ **m s$^{\mathbf{-2}}$** [1].

Q3(c). $v = \omega\sqrt{x_0^2 - x^2} = 15.7 \times \sqrt{(0.060)^2 - (0.040)^2}$ [1] $= 15.7 \times 0.0447 = \mathbf{0.70}$ **m s$^{\mathbf{-1}}$** [1].

Q4. All three start from same x_0 [1]; light damping: decaying sinusoid, multiple oscillations; critical: returns to zero smoothly in shortest time without oscillating; heavy: slower return to zero, no oscillation — all three correctly labelled [2].

Q5. Resonance: when the driving frequency equals the natural frequency of the system [1]; the amplitude of oscillation reaches a maximum [1].

Q6(a). Amplitude $x_0 = 0.050$ m [1]; angular frequency $\omega = 12$ rad s^{-1} [1].

Q6(b). $v_{\text{max}} = \omega x_0 = 12 \times 0.050 = 0.60$ m s^{-1} [2] (must show substitution).

Q6(c). $E = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} \times 0.15 \times 144 \times 2.5 \times 10^{-3}$ [1] $= \mathbf{2.7 \times 10^{-2}}$ J [1].

Q6(d). PE: upward parabola, zero at $x = 0$, maximum E at $x = \pm x_0$ [1]; KE: inverted parabola, maximum E at $x = 0$, zero at $x = \pm x_0$ [1]; axes correctly labelled with E, $-x_0$, 0, x_0 [1].

Q7(a). Energy is transferred most efficiently from driver to system when $f_{\text{drive}} = f_0$ [1]; each push is in phase with the motion so energy is added each cycle, increasing amplitude [1].

Q7(b). Amplitude decreases [1]; driving frequency is not matched to natural frequency so energy transfer is less efficient / pushes are out of phase with motion [1].

Q7(c). The resonance peak is lower (smaller maximum amplitude) [1]; the peak is broader and shifts slightly to a frequency below f_0 [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define displacement, amplitude, period, frequency and angular frequency	
☐	State and apply the two conditions for SHM	
☐	Use $a = -\omega^2 x$ to identify and analyse SHM	
☐	Use $x = x_0 \sin \omega t$ and $v = v_0 \cos \omega t$	
☐	Use $v = \pm\omega\sqrt{x_0^2 - x^2}$ to find speed at any displacement	
☐	Sketch and interpret graphs of x, v and a against t	
☐	Describe the phase relationships between x, v and a	
☐	Describe the interchange between KE and PE during SHM	
☐	Use $E = \frac{1}{2}m\omega^2 x_0^2$ for total energy	
☐	Sketch and interpret KE and PE against displacement graphs	
☐	Explain and distinguish light, critical and heavy damping	
☐	Sketch displacement–time graphs for the three types of damping	
☐	Explain resonance and the condition for it to occur	
☐	Describe the effect of damping on the resonance curve	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Remember: in SHM, acceleration and displacement are always linked by $a = -\omega^2 x$. If you can start from this equation and derive everything else, you truly understand the topic.

CIE International A Level Physics

Topic 18

Electric Fields

Revision Booklet

This booklet covers:

- Electric Fields and Field Lines
- Uniform Electric Fields
- Coulomb's Law
- Electric Field of a Point Charge
- Electric Potential

Core Concepts and Definitions

Electric Field

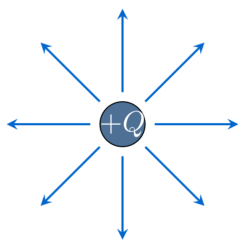
An **electric field** is a region of space in which a charged object experiences a force.

- Electric field is defined as the **force per unit positive charge** acting on a small stationary test charge placed at that point.
- It is a **vector** quantity, directed along the force on a positive test charge.
- Units: $\text{N C}^{-1} \equiv \text{V m}^{-1}$.

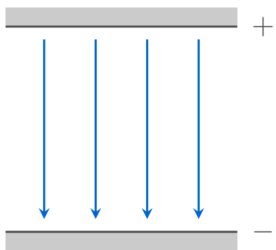
$$E = \frac{F}{q}$$

Radial vs Uniform Fields

- **Radial field** (around a point charge or sphere): field lines point radially inward (negative charge) or outward (positive charge); E decreases with distance.
- **Uniform field** (between parallel plates): field lines are parallel and equally spaced; E is constant throughout.



Radial Field (+)



Uniform Field

Uniform Electric Fields

Field Strength Between Parallel Plates

$$E = \frac{\Delta V}{\Delta d}$$

E = electric field strength (V m^{-1})
ΔV = potential difference between the plates (V)
Δd = separation of the plates (m)

Motion of Charged Particles in a Uniform Field

- A charge q in a uniform field E experiences a constant force $F = qE$.
- The motion is analogous to projectile motion in a gravitational field:
  - Along the field: **uniform acceleration** $a = qE/m$.

  - Perpendicular to the field: **constant velocity** (if no other forces).
- The path of the charge is **parabolic**.

Common Mistake

$E = \Delta V/\Delta d$ applies **only** to uniform fields (parallel plates). Do not apply it to the field around a point charge — use $E = Q/(4\pi\varepsilon_0 r^2)$ instead.

Coulomb's Law

Coulomb's Law

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

F = electrostatic force between the charges (N)
Q_1, Q_2 = the two point charges (C)
r = separation between the charges (m)
ε_0 = permittivity of free space = 8.85×10^{-12} F m^{-1}

Key Points

- Like charges **repel**; unlike charges **attract**.
- The force obeys an **inverse-square law**: double the distance, quarter the force.
- Applies strictly to **point charges**, and to uniform spheres (treat as point charge at centre).
- Compare with gravity: same inverse-square form but gravity is **always attractive**.

Comparison: Coulomb's Law vs Newton's Law of Gravitation

Coulomb's Law	Newton's Law
$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	$F = \frac{G m_1 m_2}{r^2}$
Can be attractive or repulsive	Always attractive
Acts between charges	Acts between masses
Much stronger force	Much weaker force

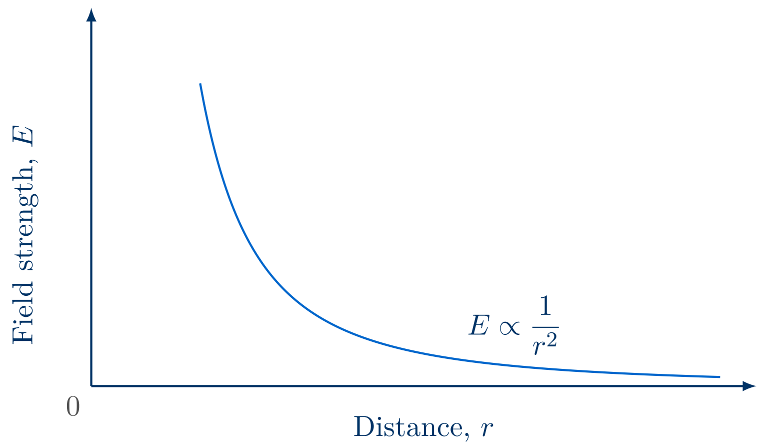
Electric Field of a Point Charge

Electric Field Strength due to a Point Charge

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

E = electric field strength at distance r (N C^{-1})
Q = point charge creating the field (C)
r = distance from the centre of Q (m)
ε_0 = 8.85×10^{-12} F m^{-1}

Variation of E with distance r



Comparing E and g fields

- Both follow an inverse-square law with distance.
- $E = Q/(4\pi\varepsilon_0 r^2)$ parallels $g = GM/r^2$.
- $F = qE$ parallels $F = mg$.
- Unlike gravitational fields, electric fields can point inward or outward depending on the sign of Q.

Electric Potential

Definition of Electric Potential V

The **electric potential** at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point.

$$V = \frac{W}{q} \qquad \text{units: J C}^{-1} \equiv \text{V}$$

- $V = 0$ at infinity (by convention).
- Around a positive charge: $V > 0$ (work must be done *against* repulsion).

- Around a negative charge: $V < 0$ (work is done *by* the field as the test charge moves in).

Electric Potential due to a Point Charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

V = electric potential at distance r (V)
Q = point charge (C)
r = distance from the charge (m)

Relationship Between E and V

Field Strength from Potential Gradient

$$E = -\frac{\Delta V}{\Delta r}$$

The electric field strength equals the **negative gradient** of the potential–distance graph. The area under an E–r graph gives the change in potential ΔV.

Electric Potential Energy

Potential Energy of Two Point Charges

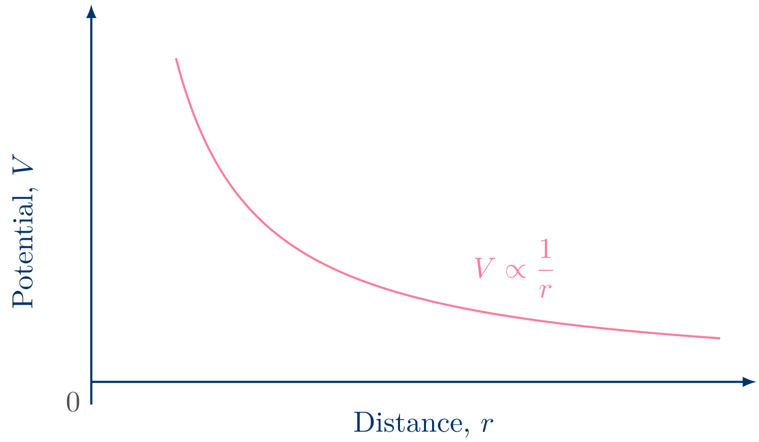
$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}$$

E_P = electric potential energy (J)
Q, q = the two point charges (C)
r = separation (m)
Note: $E_P = qV$, where V is the potential due to charge Q at the location of q.

Signs of Potential Energy

- Like charges ($Qq > 0$): $E_P > 0$ — energy must be supplied to bring them together.
- Unlike charges ($Qq < 0$): $E_P < 0$ — energy is released as they come together.
- $E_P = 0$ at infinite separation.

Graphs of V and E against r for a positive point charge



Common Mistake

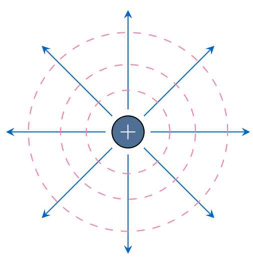
Note the difference in distance dependence: $E \propto 1/r^2$ but $V \propto 1/r$. Students often mix these up. Remember: potential falls off more *slowly* than field strength with distance.

Equipotential Surfaces

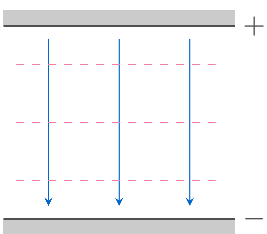
Equipotentials

An **equipotential surface** is a surface on which the electric potential is the same at every point.

- No work is done moving a charge *along* an equipotential.
- Equipotentials are always **perpendicular** to field lines.
- Around a point charge: equipotentials are concentric spheres.
- Between parallel plates: equipotentials are parallel planes equally spaced (for uniform field).



Point charge



Parallel plates

Formula Summary Sheet

Formula	Quantity	Units
$E = \dfrac{F}{q}$	Electric field strength (definition)	N C^{-1}
$F = qE$	Force on a charge in a field	N
$E = \dfrac{\Delta V}{\Delta d}$	Field between parallel plates (uniform)	V m^{-1}
$F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$	Coulomb's Law	N
$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$	Field due to a point charge	N C^{-1}
$V = \dfrac{Q}{4\pi\varepsilon_0 r}$	Potential due to a point charge	V
$E = -\dfrac{\Delta V}{\Delta r}$	Field from potential gradient	V m^{-1}
$E_P = \dfrac{Qq}{4\pi\varepsilon_0 r}$	Electric potential energy	J
$E_P = qV$	Potential energy of charge in field	J

Constants: $\varepsilon_0 = 8.85\times10^{-12}\ \text{F m}^{-1}$, $\dfrac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}\ \text{N m}^2\text{C}^{-2}$, $e = 1.60\times10^{-19}\ \text{C}$

Note: $E \propto 1/r^2$ (inverse-square) but $V \propto 1/r$ (inverse).

Worked Examples

Example 1 — Force Between Two Charges

Question: Calculate the electrostatic force between two charges of $+3.0\ \mu\text{C}$ and $-5.0\ \mu\text{C}$ separated by 0.12 m.

Solution

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{3.0\times10^{-6}\times5.0\times10^{-6}}{4\pi\times8.85\times10^{-12}\times(0.12)^2}$$

$$F = \frac{1.5\times10^{-11}}{1.61\times10^{-12}} = \mathbf{9.3}\ \text{N}$$

The force is **attractive** (unlike charges).

Example 2 — Field Strength and Potential at a Distance

Question: A point charge $Q = +4.0\ \mu\text{C}$. Calculate (a) the electric field strength and (b) the electric potential at a distance of 0.30 m.

Solution

(a) $E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.30)^2}$

$E = \dfrac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} \times (0.09)^{-1}$

$$E = \frac{4.0 \times 10^{-6}}{1.005 \times 10^{-10}} = \mathbf{4.0 \times 10^{5}} \text{ N C}^{-1}$$

(b) $V = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{4.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.30} = \mathbf{1.2 \times 10^{5}} \text{ V}$

Example 3 — Uniform Field Between Plates

Question: Two parallel plates are separated by 4.0 mm with a potential difference of 240 V. Calculate (a) the field strength and (b) the force on an electron between the plates.

Solution

(a) $E = \dfrac{\Delta V}{\Delta d} = \dfrac{240}{4.0 \times 10^{-3}} = \mathbf{6.0 \times 10^{4}} \text{ V m}^{-1}$

(b) $F = qE = 1.6 \times 10^{-19} \times 6.0 \times 10^{4} = \mathbf{9.6 \times 10^{-15}} \text{ N}$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define electric field strength and state its SI units.

[2 marks]

Q2. State Coulomb's Law and identify one similarity and one difference compared with Newton's Law of Gravitation.

[3 marks]

Q3. Define electric potential at a point. Explain why the potential around an isolated positive charge is positive, but around an isolated negative charge is negative.

[3 marks]

Q4. Two parallel plates are 6.0 mm apart. The electric field strength between them is 5.0×10^4 V m^{-1}. Calculate the potential difference between the plates.

[2 marks]

Q5. A point charge produces an electric field of 3.6×10^5 N C^{-1} at a distance of 0.10 m. Calculate the magnitude of the charge.

[3 marks]

Section B — Longer Structured Questions

Q6. Two point charges $Q_1 = +6.0\ \mu$C and $Q_2 = +6.0\ \mu$C are placed 0.20 m apart.

(a) Calculate the force between the two charges.

[2 marks]

(b) Calculate the electric field strength at the midpoint between the two charges. Explain your reasoning.

[3 marks]

(c) Calculate the electric potential at the midpoint between the two charges.

[3 marks]

Q7. An electron (mass 9.11×10^{-31} kg, charge -1.6×10^{-19} C) enters horizontally between two parallel plates. The plates are 30 mm apart and have a potential difference of 150 V. The electron enters midway between the plates with horizontal speed 4.0×10^{7} m s^{-1}.

(a) Calculate the electric field strength between the plates.

[1 mark]

(b) Calculate the vertical acceleration of the electron.

[2 marks]

(c) The plates are 60 mm long. Calculate the vertical deflection of the electron as it exits the plates.

[3 marks]

(d) State and explain whether the electron hits one of the plates before exiting.

[2 marks]

Mark Scheme and Answers

Q1. Electric field strength is the force per unit positive charge acting on a small stationary test charge placed at that point [1]; units: N C^{-1} or V m^{-1} [1].

Q2. The force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation [1]. Similarity: both follow an inverse-square law [1]. Difference: gravity is always attractive; electrostatic force can be attractive or repulsive [1].

Q3. Electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point [1]. Around a positive charge: the test charge is repelled, so work must be done *on* it to bring it in from infinity — the potential is positive [1]. Around a negative charge: the test charge is attracted, so work is done *by* the field — the potential is negative [1].

Q4. $\Delta V = E \times \Delta d = 5.0 \times 10^4 \times 6.0 \times 10^{-3} = \mathbf{300}$ V [2].

Q5. $E = Q/(4\pi\varepsilon_0 r^2)$ [1]; $Q = E \times 4\pi\varepsilon_0 r^2 = 3.6 \times 10^5 \times 4\pi \times 8.85 \times 10^{-12} \times (0.10)^2$ [1] $= \mathbf{4.0 \times 10^{-7}}$ C [1].

Q6(a). $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \dfrac{(6.0 \times 10^{-6})^2}{4\pi \times 8.85 \times 10^{-12} \times (0.20)^2}$ [1] $= \mathbf{8.1}$ N [1].

Q6(b). By symmetry, the two equal charges produce equal and opposite field contributions at the midpoint [1]; the fields cancel, so $E = \mathbf{0}$ at the midpoint [2].

Q6(c). $V_1 = V_2 = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{6.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 0.10}$ [1] $= 5.4 \times 10^5$ V; total $V = V_1 + V_2 = \mathbf{1.08 \times 10^6}$ V [2].

Q7(a). $E = \Delta V/\Delta d = 150/(30 \times 10^{-3}) = \mathbf{5.0 \times 10^3}$ V $\mathbf{m^{-1}}$ [1].

Q7(b). $F = qE = 1.6 \times 10^{-19} \times 5.0 \times 10^3 = 8.0 \times 10^{-16}$ N [1]; $a = F/m = 8.0 \times 10^{-16}/(9.11 \times 10^{-31}) = \mathbf{8.8 \times 10^{14}}$ m $\mathbf{s^{-2}}$ [1].

Q7(c). Time in field: $t = L/v = 0.060/(4.0 \times 10^7) = 1.5 \times 10^{-9}$ s [1]; vertical deflection: $y = \frac{1}{2}at^2 = \frac{1}{2} \times 8.8 \times 10^{14} \times (1.5 \times 10^{-9})^2$ [1] $= \mathbf{9.9 \times 10^{-4}}$ m $\approx \mathbf{1.0}$ mm [1].

Q7(d). The electron enters midway so has 15 mm to the nearest plate [1]; deflection of ≈ 1 mm $\ll 15$ mm, so the electron **does not** hit a plate [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define electric field strength as force per unit positive charge; use $E = F/q$	
☐	Represent electric fields using field lines for point charges and parallel plates	
☐	Use $E = \Delta V/\Delta d$ for a uniform field between parallel plates	
☐	Describe the parabolic motion of a charge in a uniform electric field	
☐	State and apply Coulomb's Law $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$	
☐	Use $E = Q/(4\pi\varepsilon_0 r^2)$ for the field due to a point charge	
☐	Define electric potential; explain why sign depends on sign of source charge	
☐	Use $V = Q/(4\pi\varepsilon_0 r)$ for potential due to a point charge	
☐	Apply $E = -\Delta V/\Delta r$ to relate field strength and potential gradient	
☐	Use $E_P = Qq/(4\pi\varepsilon_0 r)$ for potential energy of two charges	
☐	Sketch and interpret equipotential diagrams for point charges and parallel plates	
☐	Compare electric and gravitational fields (similarities and differences)	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

The key to electric fields is seeing the deep parallel with gravitational fields: same inverse-square laws, same potential formalism — but with charge replacing mass, and the crucial difference that charge can be positive or negative.

CIE International A Level Physics

Topic 19

Capacitance

Revision Booklet

This booklet covers:

- Capacitors and Capacitance
- Capacitors in Series and Parallel
- Energy Stored in a Capacitor
- Capacitor Discharge
- The Time Constant

Core Concepts and Definitions

Capacitance

The **capacitance** of a conductor is the charge stored per unit potential difference across it.

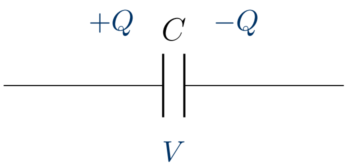
$$C = \frac{Q}{V} \qquad \text{units: F (farads)} \ \equiv \text{C V}^{-1}$$

- 1 F is a very large unit; practical capacitors are typically μF, nF or pF.
- Capacitance depends on the **geometry** of the conductor and the medium between the plates, not on Q or V individually.
- For an **isolated spherical conductor** of radius R: $C = 4\pi\varepsilon_0 R$.

The Parallel Plate Capacitor

Two conducting plates of area A separated by distance d:

- Charging the capacitor stores charge $+Q$ on one plate and $-Q$ on the other.
- The electric field between the plates is uniform: $E = V/d$.
- Capacitance increases with larger plate area A and smaller separation d.



Capacitors in Series and Parallel

Capacitors in Parallel

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots$$

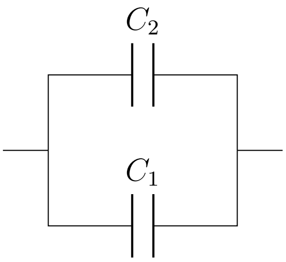
Each capacitor has the **same voltage**; charges add: $Q_{\text{total}} = Q_1 + Q_2 + \cdots$

Capacitors in Series

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Each capacitor has the **same charge**; voltages add: $V_{\text{total}} = V_1 + V_2 + \cdots$

Circuit diagrams



Parallel: $C_T = C_1 + C_2$

Series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

Derivation of Series Formula

For capacitors in series, each carries the same charge Q. The total voltage is:

$$V_{\text{total}} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Since $C_{\text{total}} = Q/V_{\text{total}}$, dividing through by Q gives $\dfrac{1}{C_{\text{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Common Mistake

Capacitors in series combine like **resistors in parallel** (reciprocal rule), and capacitors in parallel combine like **resistors in series** (direct sum). This is the opposite to resistors — don't mix them up.

Energy Stored in a Capacitor

Energy Stored

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

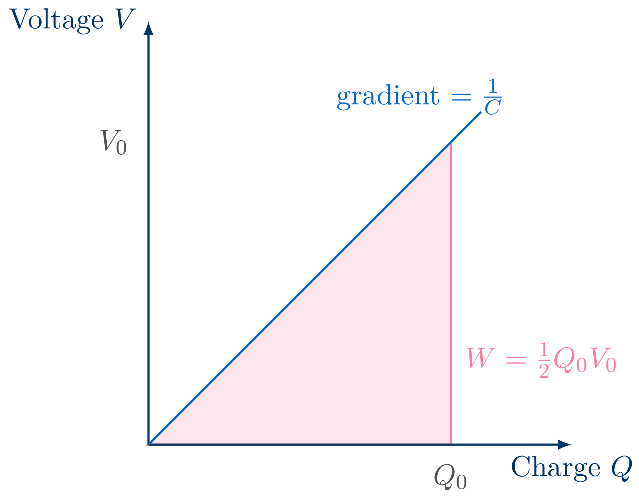
W = energy stored (J)
Q = charge stored (C)
V = potential difference across capacitor (V)
C = capacitance (F)

Energy from the V–Q Graph

The energy stored equals the **area under the V–Q graph** (a straight line through the origin with gradient $1/C$):

$$W = \text{area of triangle} = \tfrac{1}{2} \times Q \times V = \tfrac{1}{2}QV$$

V–Q graph for a capacitor



Discharging a Capacitor

Capacitor Discharge Through a Resistor

When a charged capacitor discharges through a resistor R, the charge, voltage and current all decay **exponentially** with time:

$$Q = Q_0\, e^{-t/RC} \qquad V = V_0\, e^{-t/RC} \qquad I = I_0\, e^{-t/RC}$$

where $I_0 = V_0/R = Q_0/RC$ is the initial current.

Time Constant

$$\tau = RC$$

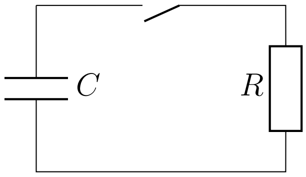
τ = time constant (s)
R = resistance of discharge path (Ω)
C = capacitance (F)

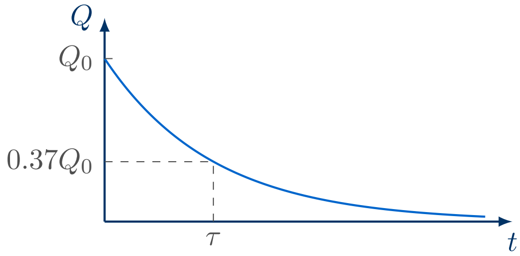
After one time constant ($t = \tau$), the charge/voltage/current has fallen to $e^{-1} \approx$ **37**% of its initial value.

Key Values During Discharge

- $t = \tau$: $Q = 0.37\, Q_0$ (37%)
- $t = 2\tau$: $Q = 0.135\, Q_0$ (13.5%)
- $t = 5\tau$: $Q \approx 0.007\, Q_0$ — capacitor considered fully discharged.
- A larger τ means slower discharge; a smaller τ means faster discharge.

Discharge circuit and Q–t graph





Analysing Discharge Graphs

- A **linear** $\ln Q$ vs t graph confirms exponential decay; gradient $= -1/RC$.
- The time constant τ can be read directly as the time for Q to fall to $0.37\,Q_0$.
- Doubling R or C doubles τ and halves the rate of discharge.

Linearising the discharge: $\ln Q$ against t

Taking logarithms of $Q = Q_0\,e^{-t/RC}$:

$$\ln Q = \ln Q_0 - \frac{1}{RC}\,t$$

This is of the form $y = mx + c$, so a graph of $\ln Q$ against t gives:

- gradient $= -\dfrac{1}{RC} = -\dfrac{1}{\tau}$
- y-intercept $= \ln Q_0$

Formula Summary Sheet

Formula	Quantity	Units
$C = Q/V$	Capacitance (definition)	F
$C_T = C_1 + C_2 + \cdots$	Capacitors in parallel	F
$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$	Capacitors in series	F
$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$	Energy stored	J
$\tau = RC$	Time constant	s
$Q = Q_0\, e^{-t/RC}$	Charge during discharge	C
$V = V_0\, e^{-t/RC}$	Voltage during discharge	V
$I = I_0\, e^{-t/RC}$	Current during discharge	A
$\ln Q = \ln Q_0 - \frac{t}{RC}$	Linearised discharge	—

Constants and values: $e^{-1} \approx 0.368$; after 1τ: 37% remains; after 5τ: < 1% remains

Units check: $[\tau] = [\Omega][\text{F}] = [\text{V A}^{-1}][\text{C V}^{-1}] = [\text{C A}^{-1}] = \text{s}$

Worked Examples

Example 1 — Capacitors in Series and Parallel

Question: Three capacitors of 2.0 μF, 3.0 μF and 6.0 μF are connected (a) in parallel and (b) in series. Find the total capacitance in each case.

Solution

(a) Parallel:

$$C_T = 2.0 + 3.0 + 6.0 = \mathbf{11.0}\ \mu\text{F}$$

(b) Series:

$$\frac{1}{C_T} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3+2+1}{6.0} = \frac{6}{6.0} = 1.0\ \mu\text{F}^{-1}$$

$$C_T = \mathbf{1.0}\ \mu\text{F}$$

Example 2 — Energy Stored

Question: A 470 μF capacitor is charged to 12 V. Calculate (a) the charge stored and (b) the energy stored.

Solution

(a) $Q = CV = 470 \times 10^{-6} \times 12 = \mathbf{5.64 \times 10^{-3}}$ C
(b) $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 470 \times 10^{-6} \times 12^2 = \mathbf{3.38 \times 10^{-2}}$ J

Example 3 — Capacitor Discharge

Question: A 220 μF capacitor charged to 9.0 V discharges through a 47 kΩ resistor. Calculate (a) the time constant and (b) the voltage after 5.0 s.

Solution

(a) $\tau = RC = 47 \times 10^3 \times 220 \times 10^{-6} = \mathbf{10.3}$ s
(b) $V = V_0\, e^{-t/RC} = 9.0 \times e^{-5.0/10.3} = 9.0 \times e^{-0.485} = 9.0 \times 0.616 = \mathbf{5.5}$ V

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define capacitance and state its SI unit.

[2 marks]

Q2. A capacitor stores 360 μC of charge when connected to a 12 V supply. Calculate its capacitance.

[2 marks]

Q3. Two capacitors of 4.0 μF and 12 μF are connected in series across a 6.0 V supply.

(a) Calculate the total capacitance. *[2 marks]*

(b) Calculate the total charge stored. *[1 mark]*

(c) Calculate the voltage across the 4.0 μF capacitor. *[2 marks]*

Q4. Explain, with reference to the V–Q graph, why the energy stored in a capacitor is $W = \frac{1}{2}QV$ and not $W = QV$.

[2 marks]

Q5. Define the time constant for a capacitor–resistor discharge circuit and state what fraction of the initial charge remains after two time constants.

[3 marks]

Section B — Longer Structured Questions

Q6. A 100 μF capacitor is charged to 20 V and then discharged through a 25 kΩ resistor.

(a) Calculate the initial charge stored on the capacitor.

[1 mark]

(b) Calculate the initial discharge current.

[2 marks]

(c) Calculate the time constant for the discharge.

[1 mark]

(d) Calculate the charge remaining after 4.0 s.

[2 marks]

(e) The student plots a graph of $\ln(Q/\text{C})$ against t/s. State the gradient and y-intercept of this graph.

[2 marks]

Q7. A 50 μF and a 200 μF capacitor are connected in series and charged from a 15 V supply.

(a) Calculate the combined capacitance.

[2 marks]

(b) Calculate the energy stored in the combination.

[2 marks]

(c) The two capacitors are now reconnected in parallel across the same supply. Calculate the new total energy stored and explain why it differs from part (b).

[3 marks]

Mark Scheme and Answers

Q1. Capacitance is the charge stored per unit potential difference [1]; unit: farad (F) or C V^{-1} [1].

Q2. $C = Q/V = 360 \times 10^{-6}/12 = \mathbf{30}\ \mu$F [2].

Q3(a). $\dfrac{1}{C_T} = \dfrac{1}{4.0} + \dfrac{1}{12} = \dfrac{3+1}{12} = \dfrac{4}{12}$ [1]; $C_T = \mathbf{3.0}\ \mu$F [1].

Q3(b). $Q = C_T V = 3.0 \times 10^{-6} \times 6.0 = \mathbf{1.8 \times 10^{-5}}$ C [1].

Q3(c). Same charge on each capacitor in series; $V_1 = Q/C_1 = 1.8 \times 10^{-5}/(4.0 \times 10^{-6}) =$ **4.5** V [2].

Q4. The V–Q graph is a straight line through the origin [1]; the energy is the area under this graph (a triangle), which is $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}QV$ — not QV because the voltage builds up gradually from 0 to V as charge is stored [1].

Q5. The time constant is the product RC [1]; it is the time taken for the charge (or voltage or current) to fall to $1/e \approx 37\%$ of its initial value [1]; after 2τ: $e^{-2} \approx \mathbf{13.5}\%$ remains [1].

Q6(a). $Q_0 = CV_0 = 100 \times 10^{-6} \times 20 = \mathbf{2.0 \times 10^{-3}}$ C [1].

Q6(b). $I_0 = V_0/R = 20/(25 \times 10^3)$ [1] $= \mathbf{8.0 \times 10^{-4}}$ A [1].

Q6(c). $\tau = RC = 25 \times 10^3 \times 100 \times 10^{-6} = \mathbf{2.5}$ s [1].

Q6(d). $Q = Q_0\, e^{-t/RC} = 2.0 \times 10^{-3} \times e^{-4.0/2.5}$ [1] $= 2.0 \times 10^{-3} \times e^{-1.6} = 2.0 \times 10^{-3} \times 0.202 = \mathbf{4.0 \times 10^{-4}}$ C [1].

Q6(e). Gradient $= -1/RC = -1/2.5 = \mathbf{-0.40}$ $\mathbf{s^{-1}}$ [1]; y-intercept $= \ln Q_0 = \ln(2.0 \times 10^{-3}) = \mathbf{-6.2}$ [1].

Q7(a). $\dfrac{1}{C_T} = \dfrac{1}{50} + \dfrac{1}{200} = \dfrac{4+1}{200} = \dfrac{5}{200}$ [1]; $C_T = \mathbf{40}$ μF [1].

Q7(b). $W = \frac{1}{2}C_T V^2 = \frac{1}{2} \times 40 \times 10^{-6} \times 15^2$ [1] $= \mathbf{4.5 \times 10^{-3}}$ J [1].

Q7(c). $C_T = 50 + 200 = 250$ μF; $W = \frac{1}{2} \times 250 \times 10^{-6} \times 15^2$ [1] $= \mathbf{2.8 \times 10^{-2}}$ J [1]; more energy is stored in parallel because the total capacitance is greater — more charge is drawn from the supply [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define capacitance; use $C = Q/V$	
☐	Calculate combined capacitance for series and parallel combinations	
☐	Derive the series and parallel formulae from $C = Q/V$	
☐	Use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$ for energy stored	
☐	Explain why energy stored is the area under a V–Q graph	
☐	Describe the exponential decay of Q, V and I during discharge	
☐	Use $x = x_0\, e^{-t/RC}$ for discharge of charge, voltage or current	
☐	Define the time constant $\tau = RC$ and state its physical significance	
☐	Determine τ from a discharge graph (directly or via $\ln Q$ vs t)	
☐	Sketch discharge curves for Q, V and I against time	
☐	Linearise the discharge equation and interpret gradient and intercept	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

The exponential decay of a capacitor is one of the most important mathematical forms in physics. Once you can recognise it, linearise it, and extract τ from a graph, you have a powerful tool that reappears throughout the course.

CIE International A Level Physics

Topic 20

Magnetic Fields

Revision Booklet

This booklet covers:

- Magnetic Fields and Field Lines
- Force on a Current-Carrying Conductor
- Force on a Moving Charge
- Hall Effect and Velocity Selector
- Magnetic Fields due to Currents
- Electromagnetic Induction

Magnetic Fields

Magnetic Field

A **magnetic field** is a region of space in which a moving charge, or a current-carrying conductor, experiences a force.

- Magnetic fields are produced by **moving charges** (electric currents) or by **permanent magnets**.
- The field is represented by **field lines** (flux lines); the direction is the direction of the force on a north pole.
- Field lines never cross; closer lines indicate a stronger field.
- Crosses ($\times$) represent field into the page; dots ($\bullet$) represent field out of the page.

Force on a Current-Carrying Conductor

Force on a Conductor

$$F = BIL\sin\theta$$

F = force on the conductor (N)
B = magnetic flux density (T)
I = current in the conductor (A)
L = length of conductor in the field (m)
θ = angle between the conductor and the field direction

Force is **maximum** when $\theta = 90°$ (conductor perpendicular to field): $F = BIL$.
Force is **zero** when $\theta = 0°$ (conductor parallel to field).

Magnetic Flux Density B

Magnetic flux density is defined as the force per unit current per unit length acting on a wire placed **at right angles** to the field.

$$B = \frac{F}{IL} \qquad \text{units: T (tesla)} \equiv \text{N A}^{-1}\text{m}^{-1}$$

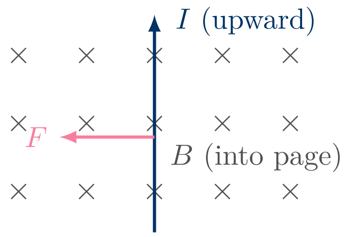
B is a vector quantity. 1 T is a strong field; Earth's field is $\approx 50\ \mu$T.

Fleming's Left-Hand Rule

For a **conventional current** in a magnetic field:

- **Thumb**: direction of force (motion) F
- **First finger**: direction of magnetic field B
- **Second finger**: direction of conventional current I

Remember: the rule gives the force on *positive* charges / conventional current. For electrons, reverse the direction.



Force on a Moving Charge

Force on a Moving Charge

$$F = BQv\sin\theta$$

F = magnetic force (N)
B = magnetic flux density (T)
Q = charge (C)
v = speed of the charge (m s^{-1})
θ = angle between velocity and field

For $\theta = 90°$: $F = BQv$, directed perpendicular to both v and B.

Circular Motion in a Magnetic Field

When a charged particle moves **perpendicular** to a uniform magnetic field, the magnetic force is always perpendicular to the velocity, so no work is done and the **speed is constant**. The particle moves in a **circle**:

$$BQv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{BQ}$$

A larger momentum or smaller B gives a larger radius.

The Hall Effect

Hall Voltage

When a current-carrying conductor is placed in a magnetic field perpendicular to the current, charge carriers experience a sideways force. They accumulate on one face until the electric force balances the magnetic force, producing the **Hall voltage**:

$$V_H = \frac{BI}{ntq}$$

V_H = Hall voltage (V)
B = magnetic flux density (T)
I = current through the conductor (A)
n = number density of charge carriers (m^{-3})
t = thickness of the conductor in the direction of B (m)
q = charge on each carrier (C)

A **Hall probe** uses this effect to measure B: since $V_H \propto B$ at constant I.

Velocity Selector

Velocity Selector

A velocity selector uses **crossed** electric and magnetic fields so that only particles with a specific speed pass through undeflected:

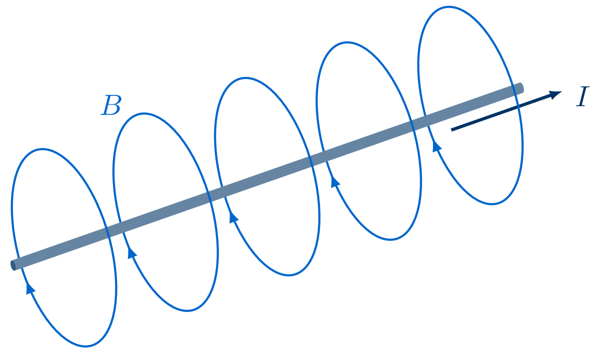
$$qE = BQv \quad \Rightarrow \quad v = \frac{E}{B}$$

- Electric force qE and magnetic force BQv act in **opposite** directions.
- Only particles where these forces balance travel in a straight line.
- Faster particles are deflected one way; slower particles the other.

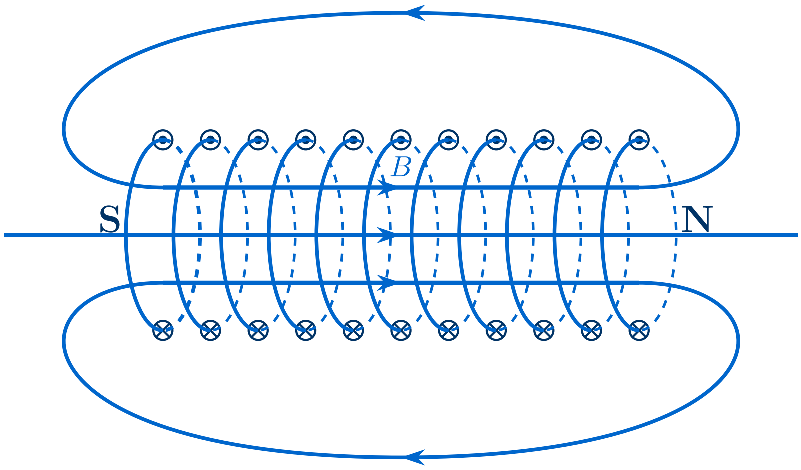
Magnetic Fields due to Currents

Field Patterns

- **Long straight wire**: concentric circles centred on the wire. Direction given by the **right-hand grip rule** — thumb points in direction of current, fingers curl in direction of field.
- **Flat circular coil**: field lines pass through the centre of the coil. Field at the centre is approximately uniform over a small region.
- **Long solenoid**: nearly uniform field inside, similar to a bar magnet outside. A **ferrous (iron) core** greatly increases the field strength.



Magnetic field around a straight wire



Magnetic field around a Solenoid

Forces Between Parallel Conductors

- **Same direction currents**: conductors **attract** each other.
- **Opposite direction currents**: conductors **repel** each other.
- Each conductor sits in the magnetic field produced by the other; the force is given by $F = BIL$.

Electromagnetic Induction

Magnetic Flux

Magnetic flux Φ is the product of the magnetic flux density and the cross-sectional area perpendicular to the field:

$$\Phi = BA\cos\theta \qquad \text{units: Wb (weber)} \equiv \text{T m}^2$$

When B is perpendicular to the area: $\Phi = BA$.

Flux linkage $N\Phi$ is the flux through a single turn multiplied by the number of turns N of the coil: units Wb-turns.

Faraday's Law and Lenz's Law

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

Faraday's Law: the induced e.m.f. is proportional to the rate of change of flux linkage.

Lenz's Law: the induced e.m.f. (and hence current) acts in a direction that **opposes** the change in flux that caused it (the minus sign above).

$\mathcal{E}$ = induced e.m.f. (V)
N = number of turns
$\Delta\Phi/\Delta t$ = rate of change of flux (Wb $s^{-1} \equiv$ V)

Factors Affecting the Induced E.M.F.

- **Rate of change** of flux: faster change ⇒ larger e.m.f.
- **Number of turns** N: more turns ⇒ larger e.m.f.
- **Strength of field** B: stronger field ⇒ larger flux change.
- **Area** of coil: larger area ⇒ more flux.

Lenz's law is a consequence of conservation of energy — the induced current creates a force that opposes the motion causing induction.

Common Mistake

An e.m.f. is induced only when the flux is **changing**. A conductor stationary in a steady field has zero induced e.m.f., even if the field is strong.

Formula Summary Sheet

Formula	Quantity	Units
$F = BIL\sin\theta$	Force on current-carrying conductor	N
$F = BQv\sin\theta$	Force on moving charge	N
$r = mv/(BQ)$	Radius of circular orbit in B field	m
$V_H = BI/(ntq)$	Hall voltage	V
$v = E/B$	Velocity selector condition	m s^{-1}
$\Phi = BA\cos\theta$	Magnetic flux	Wb
$\mathcal{E} = -\Delta(N\Phi)/\Delta t$	Faraday's / Lenz's law	V

Units: 1 T = 1 N A^{-1}m^{-1}; 1 Wb = 1 T m^2 = 1 V s
Right-hand grip rule: thumb ∥ current, fingers curl in direction of B field.

Worked Examples

Example 1 — Force on a Conductor

Question: A wire of length 0.15 m carries a current of 3.0 A at 60° to a uniform field of 0.25 T. Calculate the force on the wire.

Solution

$$F = BIL\sin\theta = 0.25 \times 3.0 \times 0.15 \times \sin 60^\circ$$
$$F = 0.25 \times 3.0 \times 0.15 \times 0.866 = \mathbf{9.7 \times 10^{-2}}\ \text{N}$$

Example 2 — Circular Motion of a Charged Particle

Question: A proton (mass 1.67×10^{-27} kg, charge 1.6×10^{-19} C) moves at 2.0×10^6 m s^{-1} perpendicular to a field of 0.15 T. Calculate the radius of its circular path.

Solution

$$r = \frac{mv}{BQ} = \frac{1.67 \times 10^{-27} \times 2.0 \times 10^6}{0.15 \times 1.6 \times 10^{-19}} = \frac{3.34 \times 10^{-21}}{2.40 \times 10^{-20}} = \mathbf{0.14}\ \text{m}$$

Example 3 — Induced E.M.F.

Question: A coil of 200 turns and area 50 cm^2 is in a field of 0.30 T perpendicular to the plane of the coil. The field drops to zero in 0.040 s. Calculate the induced e.m.f.

Solution

$$\Delta\Phi = B \times A = 0.30 \times 50 \times 10^{-4} = 1.5 \times 10^{-3}\ \text{Wb}$$
$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t} = 200 \times \frac{1.5 \times 10^{-3}}{0.040} = \mathbf{7.5}\ \text{V}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define magnetic flux density and state its SI unit.

[2 marks]

Q2. A wire of length 8.0 cm is placed perpendicular to a field of flux density 0.40 T and carries a current of 2.5 A. Calculate the force on the wire.

[2 marks]

Q3. State Faraday's Law and Lenz's Law of electromagnetic induction.

[3 marks]

Q4. Explain why the speed of a charged particle moving perpendicular to a uniform magnetic field remains constant.

[2 marks]

Q5. A Hall probe gives a voltage of 3.2 mV when a current of 50 mA passes through a slice of thickness 2.0 mm in a field B. Given $n = 8.5 \times 10^{28}$ m^{-3} and $q = 1.6 \times 10^{-19}$ C, calculate B.

[3 marks]

Section B — Longer Structured Questions

Q6. An electron (mass 9.11×10^{-31} kg, charge 1.6×10^{-19} C) enters a uniform magnetic field of 2.0×10^{-3} T perpendicular to the field with speed 5.0×10^{6} m s^{-1}.

(a) Calculate the radius of the circular path followed by the electron.

[2 marks]

(b) The field strength is doubled. State and explain the effect on the radius.

[2 marks]

(c) An electric field is now applied perpendicular to B so that the electron travels in a straight line. Calculate the electric field strength required.

[2 marks]

Q7. A rectangular coil of 80 turns and dimensions 4.0 cm × 6.0 cm is placed with its plane perpendicular to a uniform field of 0.50 T.

(a) Calculate the flux linkage through the coil.

[2 marks]

(b) The coil is rotated so that its plane becomes parallel to the field in 0.030 s. Calculate the mean induced e.m.f.

[2 marks]

(c) State and explain the direction of the induced current using Lenz's law.

[2 marks]

Mark Scheme and Answers

Q1. Magnetic flux density is the force per unit current per unit length on a wire placed at right angles to the field [1]; unit: tesla (T) or N A^{-1} m^{-1} [1].

Q2. $F = BIL \sin 90° = 0.40 \times 2.5 \times 0.080 = \mathbf{0.080}$ N [2].

Q3. Faraday's Law: the induced e.m.f. is proportional to the rate of change of flux linkage [1]; $\mathcal{E} = -\Delta(N\Phi)/\Delta t$ [1]. Lenz's Law: the induced e.m.f. acts in a direction to oppose the change in flux that caused it [1].

Q4. The magnetic force is always perpendicular to the velocity [1]; a perpendicular force does no work, so kinetic energy and hence speed remain unchanged [1].

Q5. $B = V_H ntq/I = (3.2 \times 10^{-3} \times 8.5 \times 10^{28} \times 2.0 \times 10^{-3} \times 1.6 \times 10^{-19})/(50 \times 10^{-3})$ [2] $= \mathbf{0.174}$ T [1].

Q6(a). $r = mv/(BQ) = (9.11 \times 10^{-31} \times 5.0 \times 10^{6})/(2.0 \times 10^{-3} \times 1.6 \times 10^{-19})$ [1] $= \mathbf{1.4 \times 10^{-2}}$ m [1].

Q6(b). Radius halves [1]; $r = mv/BQ$, so $r \propto 1/B$; doubling B halves r [1].

Q6(c). For straight-line motion: $qE = BQv$; $E = Bv = 2.0 \times 10^{-3} \times 5.0 \times 10^{6} = \mathbf{1.0 \times 10^{4}}$ V m^{-1} [2].

Q7(a). $\Phi = BA = 0.50 \times (0.040 \times 0.060) = 1.2 \times 10^{-3}$ Wb [1]; flux linkage $= N\Phi = 80 \times 1.2 \times 10^{-3} = \mathbf{9.6 \times 10^{-2}}$ Wb-turns [1].

Q7(b). $\Delta(N\Phi) = 9.6 \times 10^{-2}$ Wb-turns (falls to zero) [1]; $\mathcal{E} = 9.6 \times 10^{-2}/0.030 = \mathbf{3.2}$ V [1].

Q7(c). By Lenz's law the induced current opposes the decrease in flux [1]; the current flows in the direction that would create a field to oppose the rotation / maintain the flux through the coil [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	**Confidence (1–3)**
☐	Define magnetic flux density; use $F = BIL\sin\theta$	
☐	Apply Fleming's left-hand rule to find force directions	
☐	Use $F = BQv\sin\theta$ for force on a moving charge	
☐	Derive and use $r = mv/(BQ)$ for circular orbit radius	
☐	Explain and use the Hall effect; use $V_H = BI/(ntq)$	
☐	Explain the velocity selector condition $v = E/B$	
☐	Sketch field patterns for straight wire, circular coil and solenoid	
☐	Apply the right-hand grip rule for field direction around a wire	
☐	Explain forces between parallel current-carrying conductors	
☐	Define magnetic flux $\Phi = BA\cos\theta$ and flux linkage $N\Phi$	
☐	State and apply Faraday's and Lenz's laws	
☐	Identify factors that affect the magnitude of induced e.m.f.	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Faraday's law — one equation — explains the generator, the transformer, and the electric motor. Master flux linkage and you understand how almost all electrical power is generated.

Topic 21

Alternating Currents

Revision Booklet

This booklet covers:

- Characteristics of Alternating Currents
- R.M.S. Values and Power
- Half-Wave Rectification
- Full-Wave (Bridge) Rectification
- Smoothing with a Capacitor

Characteristics of Alternating Currents

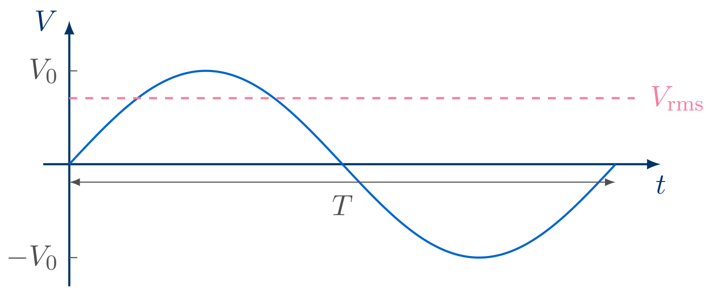
Alternating Current

An **alternating current** (a.c.) reverses direction periodically. For a sinusoidal a.c.:

$$I = I_0 \sin \omega t \qquad V = V_0 \sin \omega t$$

- I_0, V_0: **peak** (amplitude) values.
- $\omega = 2\pi f = 2\pi/T$: angular frequency (rad s^{-1}).
- T: period (s); f: frequency (Hz).
- UK mains supply: $f = 50$ Hz, $T = 20$ ms, $V_{\text{rms}} = 230$ V.

Sinusoidal a.c. voltage waveform



R.M.S. Values and Power

Root-Mean-Square (R.M.S.) Value

The **r.m.s. value** of an alternating current is defined as the value of steady direct current that would dissipate the **same power** in a purely resistive load.

R.M.S. Formulae

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

These apply to **sinusoidal** waveforms only. For other waveforms the factor $1/\sqrt{2}$ changes.

Mean Power in a Resistive Load

$$P_{\text{mean}} = \tfrac{1}{2}P_{\text{max}} = \tfrac{1}{2}I_0^2 R = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

The mean power is **half** the peak power for a sinusoidal waveform.

Why Use R.M.S.?

- The mean value of a sinusoidal current is **zero** — useless for power calculations.
- R.M.S. values allow direct use of d.c. power formulae ($P = IV$, $P = I^2R$, $P = V^2/R$).
- Meters and ratings (e.g. 230 V mains) always quote r.m.s. values.

Common Mistake

Do not use peak values in power calculations. Always convert to r.m.s. first: $V_{\text{rms}} = V_0/\sqrt{2}$. The peak mains voltage is $230\sqrt{2} \approx 325$ V, not 230 V.

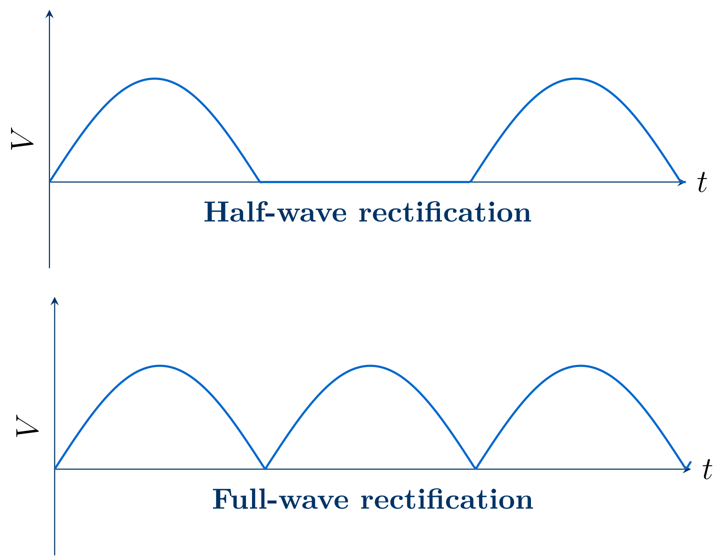
Rectification

Rectification

Rectification converts alternating current into direct current flowing in one direction only.

- **Half-wave rectification**: a single diode passes only the positive (or negative) half-cycles; output is a series of pulses with gaps.
- **Full-wave rectification**: a bridge rectifier (four diodes) inverts the negative half-cycles, producing a continuous pulsating d.c. output with no gaps.

Comparison of rectified outputs

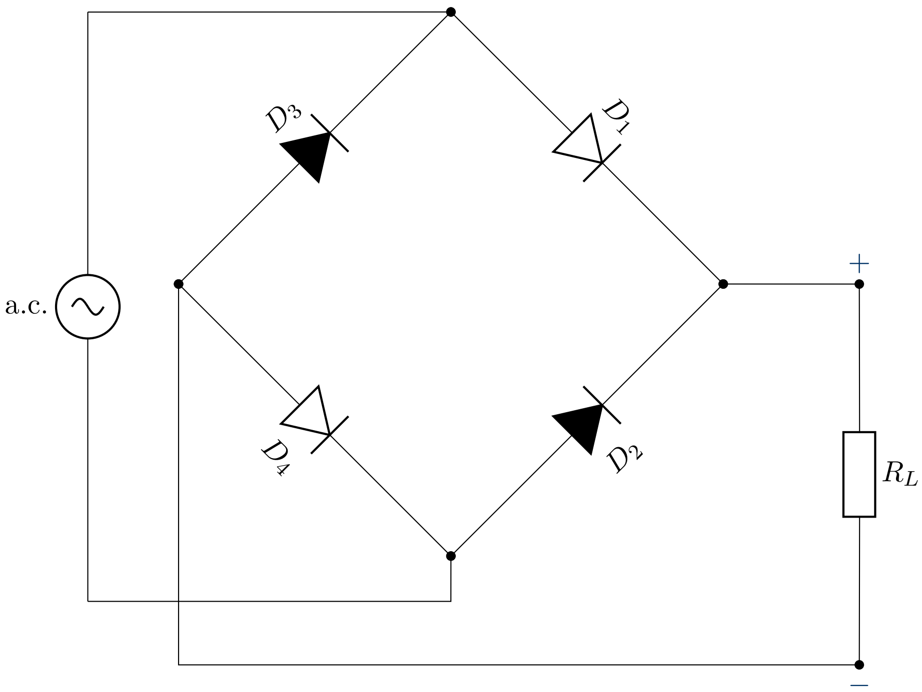


Bridge Rectifier

Bridge Rectifier

A bridge rectifier uses **four diodes** arranged in a diamond so that:

- During the **positive half-cycle**: current flows through diodes D1 and D4, through the load in the positive direction.
- During the **negative half-cycle**: current flows through diodes D2 and D3, but still through the load in the **same** direction.
- Both half-cycles contribute to the output — no wasted half-cycles.



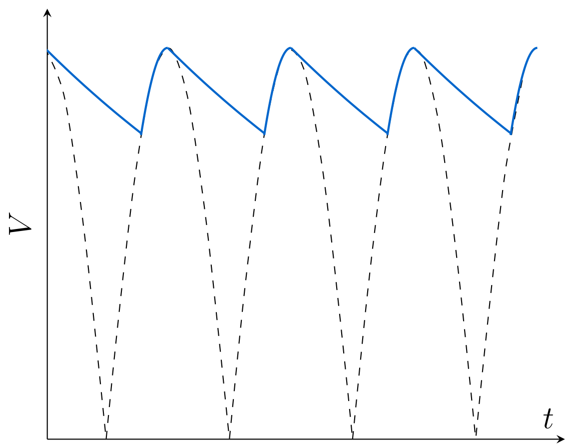
Smoothing with a Capacitor

Smoothing

A capacitor connected in **parallel with the load** reduces the ripple on a rectified output:

- The capacitor **charges up** rapidly to the peak voltage during each pulse.
- It **discharges slowly** through the load R_L between peaks, maintaining a more constant voltage.
- The output has a small residual **ripple voltage** rather than falling to zero between pulses.
- **Larger** C or **larger** R_L: longer time constant $\tau = CR_L$, less ripple.
- **Smaller** C or **smaller** R_L: faster discharge, larger ripple.

Effect of smoothing capacitor on full-wave rectified output



Common Mistake

In exam questions, always link ripple size explicitly to the time constant $\tau = CR_L$ — just saying “bigger capacitor” without explanation will not earn full marks.

Formula Summary Sheet

Formula	Quantity	Units
$I = I_0 \sin \omega t$	Sinusoidal alternating current	A
$V = V_0 \sin \omega t$	Sinusoidal alternating voltage	V
$\omega = 2\pi f = 2\pi/T$	Angular frequency	rad s^{-1}
$I_{rms} = I_0/\sqrt{2}$	R.M.S. current	A
$V_{rms} = V_0/\sqrt{2}$	R.M.S. voltage	V
$P_{mean} = \frac{1}{2} I_0^2 R$	Mean power (peak values)	W
$P_{mean} = I_{rms}^2 R = V_{rms}^2/R$	Mean power (r.m.s. values)	W

UK mains: $V_{rms} = 230$ V; $f = 50$ Hz; $V_0 = 230\sqrt{2} \approx 325$ V
Note: $1/\sqrt{2} \approx 0.707$; $P_{mean} = \frac{1}{2} P_{max}$ for sinusoidal waveform only.

Worked Examples

Example 1 — R.M.S. and Peak Values

Question: The mains supply has $V_{rms} = 230$ V at 50 Hz. Find (a) the peak voltage, (b) the angular frequency and (c) the mean power in a 1.2 kΩ resistor.

Solution

(a) $V_0 = V_{rms}\sqrt{2} = 230\sqrt{2} = \mathbf{325}$ V
(b) $\omega = 2\pi f = 2\pi \times 50 = \mathbf{314}$ rad s^{-1}
(c) $P = V_{rms}^2/R = 230^2/(1.2 \times 10^3) = 52900/1200 = \mathbf{44}$ W

Example 2 — Peak Power and Mean Power

Question: An a.c. supply has peak voltage $V_0 = 12$ V and is connected to a 60 Ω resistor. Calculate (a) the peak power and (b) the mean power dissipated.

Solution

(a) $P_{max} = V_0^2/R = 144/60 = \mathbf{2.4}$ W
(b) $P_{mean} = \frac{1}{2} P_{max} = \mathbf{1.2}$ W
Or equivalently: $V_{rms} = 12/\sqrt{2}$; $P = V_{rms}^2/R = 72/60 = 1.2$ W

Example 3 — Smoothing

Question: A full-wave rectifier feeds a 470 μF smoothing capacitor in parallel with a 2.2 kΩ load. Calculate the time constant and comment on the degree of smoothing for a 50 Hz supply.

Solution

$\tau = CR = 470 \times 10^{-6} \times 2200 = \mathbf{1.03}$ s
The period of the full-wave rectified signal is $T/2 = 1/(2 \times 50) = 10$ ms.
Since $\tau = 1.03$ s $\gg 10$ ms, the capacitor discharges very little between peaks — the output is well smoothed with very small ripple.

Practice Exam Questions

Section A — Short Answer Questions

Q1. Explain what is meant by the r.m.s. value of an alternating current and state why it is more useful than the peak value.

[3 marks]

Q2. An a.c. supply has peak voltage 340 V. Calculate (a) the r.m.s. voltage and (b) the mean power delivered to a 680 Ω resistor.

[3 marks]

Q3. Distinguish between half-wave and full-wave rectification. State the number of diodes required for each.

[3 marks]

Q4. Explain how a capacitor connected in parallel with a load resistor smooths a rectified output. State the effect of increasing the capacitance.

[3 marks]

Section B — Longer Structured Questions

Q5. The alternating voltage from a supply is given by $V = 170\sin(100\pi t)$, where V is in volts and t is in seconds.

(a) State the peak voltage and the frequency of the supply.

[2 marks]

(b) Calculate the r.m.s. voltage.

[1 mark]

(c) The supply is connected to a 500 Ω resistor. Calculate the mean power dissipated.

[2 marks]

(d) The supply is now passed through a bridge rectifier and a smoothing capacitor of 1000 μF is connected in parallel with the 500 Ω load. Calculate the time constant and comment on the effectiveness of smoothing.

[3 marks]

Q6. The graph below represents the output of a full-wave rectifier before smoothing. The peak voltage is 12 V and the supply frequency is 50 Hz.

(a) State the frequency of the rectified output.

[1 mark]

(b) On the same axes, sketch the output after connecting a large smoothing capacitor in parallel with the load. Indicate the approximate ripple voltage.

[2 marks]

(c) Explain what happens to the smoothing if the load resistance is reduced.

[2 marks]

Mark Scheme and Answers

Q1. The r.m.s. value is the equivalent steady d.c. that dissipates the same power in a resistive load [1]; it is more useful because power formulae ($P = I^2R$, $P = V^2/R$) can be applied directly [1]; the mean of a sinusoidal current is zero, so it gives no information about power [1].

Q2(a). $V_{\text{rms}} = V_0/\sqrt{2} = 340/\sqrt{2} =$ **240** V [1]. **Q2(b).** $P = V_{\text{rms}}^2/R = 240^2/680 = 57600/680 =$ **85** W [2].

Q3. Half-wave: uses **1 diode**; only one half of each cycle is passed; output has gaps of zero voltage [1]. Full-wave: uses **4 diodes** (bridge rectifier); both half-cycles are used; output is always positive with no gaps [2].

Q4. The capacitor charges to the peak voltage during each pulse [1]; between pulses it discharges slowly through the load, maintaining a more constant output voltage [1]; increasing C increases the time constant $\tau = CR$, so the capacitor discharges less between pulses and the ripple is smaller [1].

Q5(a). Peak voltage $V_0 =$ **170** V [1]; $\omega = 100\pi$, so $f = \omega/2\pi = 100\pi/2\pi =$ **50** Hz [1].

Q5(b). $V_{\text{rms}} = 170/\sqrt{2} =$ **120** V [1].

Q5(c). $P = V_{\text{rms}}^2/R = (120)^2/500$ [1] $= 14400/500 =$ **28.8** W [1].

Q5(d). $\tau = CR = 1000 \times 10^{-6} \times 500 =$ **0.50** s [1]; period of full-wave output $= 1/(2 \times 50) = 10$ ms [1]; $\tau \gg T/2$ so capacitor barely discharges between peaks — very effective smoothing with tiny ripple [1].

Q6(a). The full-wave rectified output has frequency $2 \times 50 =$ **100** Hz [1].

Q6(b). Sketch: smoothed output just below 12 V with small sawtooth ripple; ripple voltage is the small oscillation between the capacitor charge and discharge levels [2].

Q6(c). Reducing load resistance increases the discharge current [1]; the capacitor discharges faster between peaks ($\tau = CR$ decreases), so the ripple voltage increases and smoothing is less effective [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ Define period, frequency and peak value for an a.c. waveform	
☐ Use $x = x_0 \sin \omega t$ for sinusoidal current or voltage	
☐ Define r.m.s. value and explain why it is useful	
☐ Use $I_{\text{rms}} = I_0/\sqrt{2}$ and $V_{\text{rms}} = V_0/\sqrt{2}$	
☐ Calculate mean power using r.m.s. values: $P = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$	
☐ State that mean power is half peak power for sinusoidal a.c.	
☐ Distinguish half-wave and full-wave rectification graphically	
☐ Explain the action of a single diode for half-wave rectification	
☐ Explain the action of a bridge rectifier (four diodes)	
☐ Explain smoothing by a capacitor in terms of charge/discharge	
☐ Analyse the effect of C and R_L on the time constant and ripple	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

R.M.S. values are one of physics's most elegant ideas: a way to make a continuously varying quantity equivalent to a steady one. Once you see that $P_{\text{mean}} = \frac{1}{2}P_{\text{max}}$ comes directly from $\langle \sin^2 \rangle = \frac{1}{2}$, the whole topic falls into place.

CIE International A Level Physics

Topic 22

Quantum Physics

Revision Booklet

This booklet covers:

- The Photoelectric Effect
- Photon Energy and the Planck Relation
- Einstein's Photoelectric Equation
- Wave–Particle Duality & de Broglie Wavelength
- Energy Levels and Line Spectra

Core Concepts and Definitions

The Quantum Model of Light

Classical wave theory cannot explain certain phenomena involving light and matter. Quantum theory proposes that electromagnetic radiation is emitted and absorbed in discrete packets of energy called **photons**.

- A **photon** is a quantum of electromagnetic energy.
- The energy of a photon depends only on its **frequency**, not its intensity.
- At a fixed frequency, intensity is proportional to the **number of photons** per unit time. Increasing intensity does **not** increase the energy of individual photons.

Photon Energy — The Planck Relation

$$E = hf = \frac{hc}{\lambda}$$

E = energy of one photon (J)
h = Planck's constant = 6.63×10^{-34} J s
f = frequency of the radiation (Hz)
c = speed of light = 3.00×10^{8} m s^{-1}
λ = wavelength (m)

The Electronvolt

The **electronvolt** (eV) is a convenient unit of energy at the atomic scale.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

To convert eV → J: multiply by 1.60×10^{-19}.
To convert J → eV: divide by 1.60×10^{-19}.

The Photoelectric Effect

The Photoelectric Effect

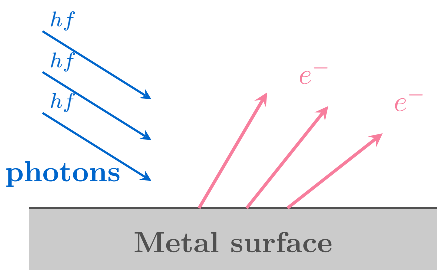
The **photoelectric effect** is the emission of electrons from a metal surface when electromagnetic radiation of sufficiently high frequency is incident on it.

- Electrons emitted are called **photoelectrons**.
- Emission occurs **instantaneously** if the frequency is above a threshold — there is no time delay.
- Below the **threshold frequency** f_0, no electrons are emitted regardless of intensity.
- Above f_0, increasing the intensity increases the **number** of photoelectrons per second, not their energy.

Why Classical Wave Theory Fails

- Wave theory predicts that any frequency of sufficient intensity should eventually eject electrons — **not observed**.
- Wave theory predicts a time delay before emission as energy builds up — **not observed** (emission is instantaneous).
- Wave theory predicts that intensity should increase the electron's kinetic energy — **not observed**.

These failures led Einstein to propose the photon model.



Einstein's Photoelectric Equation

$$hf = \phi + \tfrac{1}{2}mv_{\max}^2$$

hf = energy of the incident photon (J)
ϕ = **work function** — minimum energy to remove an electron from the surface (J)
$\frac{1}{2}mv_{\max}^2$ = maximum kinetic energy of emitted photoelectron (J)

The **work function** $\phi = hf_0$, where f_0 is the threshold frequency.
Maximum KE can also be written as eV_s, where V_s is the **stopping potential**.

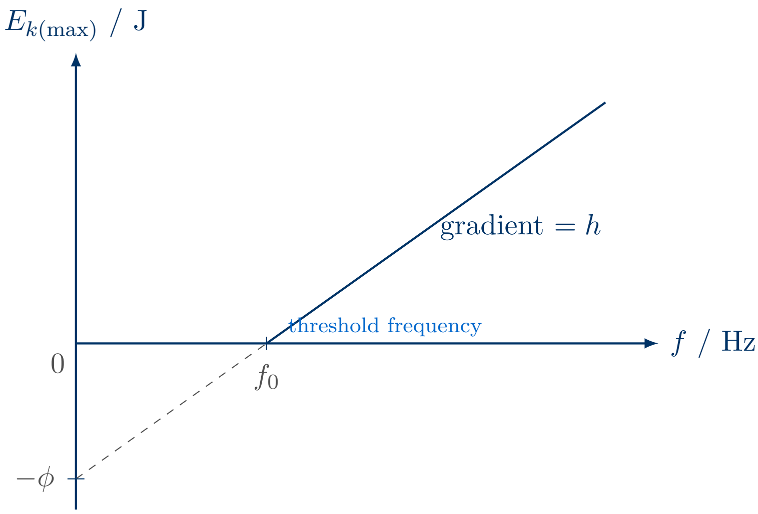
Stopping Potential V_s

The **stopping potential** is the minimum opposing potential difference required to prevent all photoelectrons from reaching the collecting electrode.

$$eV_s = \tfrac{1}{2}mv_{\max}^2 = hf - \phi$$

Measuring V_s at different frequencies allows h and ϕ to be determined experimentally.

Graph of Maximum KE against Frequency



Reading the Graph

- The **gradient** of the $E_{k(max)}$ vs f graph equals Planck's constant h.
- The **x-intercept** gives the threshold frequency f_0.
- The **y-intercept** (extrapolated) gives $-\phi$ (the negative of the work function).
- Different metals give **parallel lines** — same gradient (h), different intercepts (ϕ).

Wave–Particle Duality

Wave–Particle Duality

Wave–particle duality is the property of matter and radiation by which they exhibit both wave-like and particle-like behaviour depending on the experimental context.

- Light behaves as a **wave**: diffraction, interference, polarisation.
- Light behaves as **particles (photons)**: photoelectric effect.
- Electrons (and other particles) also show **wave behaviour**: electron diffraction.

de Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

λ = de Broglie wavelength (m)
h = Planck's constant (6.63×10^{-34} J s)
p = momentum of the particle (kg m s^{-1})
m = mass of the particle (kg)
v = speed of the particle (m s^{-1})

Electron Diffraction

When electrons are accelerated through a potential difference V and directed at a thin crystal or graphite film, a **diffraction pattern** of rings is observed — confirming their wave nature.

- The kinetic energy gained: $\frac{1}{2}mv^2 = eV$, giving $v = \sqrt{2eV/m}$.
- Therefore: $\lambda = \dfrac{h}{\sqrt{2meV}}$
- Increasing V increases p, decreasing $\lambda \Rightarrow$ rings become **smaller**.
- The wavelength must be comparable to the atomic spacing for diffraction to occur ($\sim 10^{-10}$ m).

Common Mistake — Confusing Photon and Particle Momentum

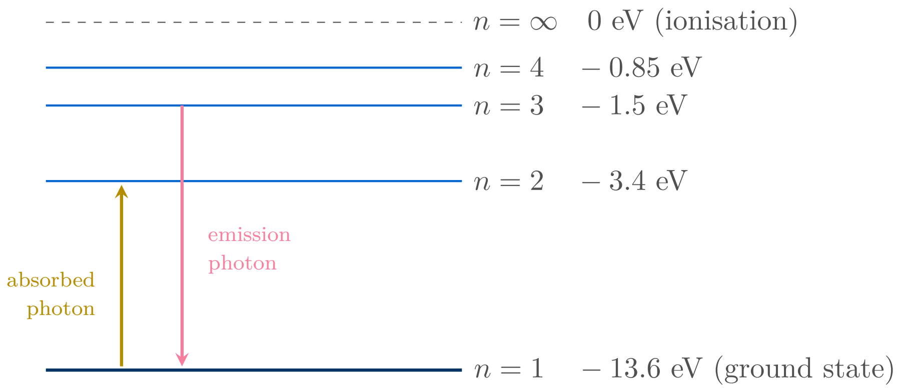
For a **photon** (massless): $p = E/c = hf/c = h/\lambda$.
For a **particle with mass**: $p = mv = h/\lambda$ (de Broglie).
Both use $\lambda = h/p$, but the relationship between energy and momentum differs. Do not use $E = hf$ for material particles.

Energy Levels and Line Spectra

Atomic Energy Levels

Electrons in an atom can only occupy discrete **energy levels**. These are quantised — only specific energies are allowed.

- The **ground state** is the lowest energy level (most stable; most negative value).
- **Excited states** have higher (less negative) energy.
- The **ionisation energy** is the energy required to remove an electron from the ground state to infinity ($E = 0$).
- Energy levels are usually expressed in electronvolts (eV) and are **negative** (bound states).



Hydrogen Energy Level Diagram (schematic)

Photon Energy from a Transition

When an electron moves between energy levels E_1 (lower) and E_2 (higher):

$$hf = E_2 - E_1$$

- **Emission**: electron falls to lower level ⇒ photon **emitted** with $hf = E_2 - E_1$.
- **Absorption**: electron promoted to higher level ⇒ photon **absorbed** with $hf = E_2 - E_1$.

Emission and Absorption Spectra

Line Spectra

- **Emission spectrum**: a series of **bright coloured lines** on a dark background. Each line corresponds to a specific photon frequency emitted during a downward electron transition. Produced by excited gases.
- **Absorption spectrum**: a **continuous spectrum** crossed by dark lines at the same frequencies as the emission lines. Cool gas in front of a broad-spectrum source absorbs specific photon energies.
- The **line positions are unique** to each element — used for identification (spectroscopy).

Why Line Spectra Prove Quantisation

Because electrons can only occupy discrete energy levels, only photons of specific energies (frequencies) can be absorbed or emitted. This produces a **line** spectrum rather than a continuous one. A continuous spectrum would imply electrons can have any energy — which they cannot.

Formula Summary Sheet

Formula	Quantity	Units
$E = hf$	Photon energy	J
$E = hc/\lambda$	Photon energy from wavelength	J
$hf = \phi + \frac{1}{2}mv_{\text{max}}^2$	Einstein's photoelectric equation	J
$\phi = hf_0$	Work function	J
$eV_s = \frac{1}{2}mv_{\text{max}}^2$	Stopping potential	J, V
$\lambda = h/p = h/mv$	de Broglie wavelength	m
$\lambda = h/\sqrt{2meV}$	de Broglie λ from accelerating pd	m
$hf = E_2 - E_1$	Photon from energy transition	J

Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Calculation Questions

1. **Identify** the relevant formula — is it a photon, a particle, or a transition?
2. **Convert units**: eV to J ($\times 1.60 \times 10^{-19}$); nm to m ($\times 10^{-9}$).
3. **Substitute** values carefully, showing all working.
4. **Quote** the final answer in appropriate units with correct significant figures.

Common Errors — Avoid These!

- Using the **wrong energy unit** — forgetting to convert eV to J before substituting.
- Confusing **threshold frequency** f_0 with threshold wavelength: a higher f_0 corresponds to a **shorter** threshold wavelength λ_0.
- Applying **intensity** changes to change photon energy — intensity only changes the **number** of photons.
- Using $E = hf$ for the de Broglie relation — this is for photons only; for matter use $\lambda = h/mv$.
- Forgetting that energy levels are **negative** — the transition energy is $|E_2 - E_1|$,

not just $E_2 - E_1$ as magnitudes.

- Confusing **emission** (bright lines) and **absorption** (dark lines) spectra.

Worked Examples

Example 1 — Photoelectric Effect Calculation

Question: Light of wavelength 250 nm is incident on a metal surface with a work function of 4.5 eV. Calculate the maximum kinetic energy of emitted photoelectrons and the stopping potential.

Solution

Solution:
Convert work function: $\phi = 4.5 \times 1.60 \times 10^{-19} = 7.20 \times 10^{-19}$ J
Photon energy:

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{250 \times 10^{-9}} = 7.96 \times 10^{-19} \text{ J}$$

Maximum KE:

$$E_{k(\text{max})} = hf - \phi = 7.96 \times 10^{-19} - 7.20 \times 10^{-19} = \mathbf{7.6 \times 10^{-20}} \text{ J}$$

Stopping potential:

$$V_s = \frac{E_{k(\text{max})}}{e} = \frac{7.6 \times 10^{-20}}{1.60 \times 10^{-19}} = \mathbf{0.48} \text{ V}$$

Example 2 — de Broglie Wavelength

Question: An electron is accelerated from rest through a potential difference of 3.0 kV. Calculate its de Broglie wavelength.

Solution

Solution:
Energy gained: $E_k = eV = 1.60 \times 10^{-19} \times 3000 = 4.80 \times 10^{-16}$ J
Speed: $v = \sqrt{\frac{2E_k}{m_e}} = \sqrt{\frac{2 \times 4.80 \times 10^{-16}}{9.11 \times 10^{-31}}} = 3.25 \times 10^{7}$ m s^{-1}
de Broglie wavelength:

$$\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.25 \times 10^{7}} = \mathbf{2.24 \times 10^{-11}} \text{ m}$$

This is comparable to atomic spacings ($\sim 10^{-10}$ m), confirming why electron diffraction is observed.

Example 3 — Energy Level Transition

Question: An electron in a hydrogen-like atom falls from an energy level of −1.5 eV to −3.4 eV. Find the frequency and wavelength of the emitted photon.

Solution

Solution:
Energy of emitted photon:

$$hf = E_2 - E_1 = -1.5 - (-3.4) = 1.9 \text{ eV} = 1.9 \times 1.60 \times 10^{-19} = 3.04 \times 10^{-19} \text{ J}$$

Frequency:

$$f = \frac{3.04 \times 10^{-19}}{6.63 \times 10^{-34}} = \mathbf{4.59 \times 10^{14}} \text{ Hz} \quad \text{(visible light — red/orange)}$$

Wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{4.59 \times 10^{14}} = \mathbf{654} \text{ nm}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. State what is meant by the *threshold frequency* in the photoelectric effect.

[2 marks]

Q2. Explain why the photoelectric effect cannot be explained by the wave model of light. Give two reasons.

[2 marks]

Q3. Light of frequency 8.0×10^{14} Hz is incident on a metal surface with work function 2.5 eV. Show whether photoelectric emission will occur and, if so, calculate the maximum kinetic energy of the emitted electrons.

[4 marks]

Q4. Calculate the de Broglie wavelength of a proton ($m_p = 1.67 \times 10^{-27}$ kg) moving at 2.0×10^6 m s^{-1}.

[2 marks]

Q5. An electron and a proton are each accelerated from rest through the same potential difference V. Show that the de Broglie wavelength of the electron is greater than that of the proton. ($m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg)

[3 marks]

Section B — Longer Structured Questions

Q6. A student investigates the photoelectric effect using a metal surface. The graph of maximum kinetic energy against frequency is a straight line. The threshold frequency of the metal is 5.5×10^{14} Hz.

(a) Calculate the work function of the metal in joules.

[2 marks]

(b) Light of frequency 9.0×10^{14} Hz is incident on the surface. Calculate the stopping potential.

[3 marks]

(c) The intensity of the incident light is doubled whilst keeping its frequency the same. State and explain what happens to (i) the rate of electron emission, and (ii) the maximum kinetic energy of emitted electrons.

[4 marks]

Q7. The energy levels of atomic hydrogen include $E_1 = -13.6$ eV (ground state), $E_2 = -3.4$ eV, and $E_3 = -1.5$ eV.

(a) Calculate the frequency of the photon emitted when an electron transitions from $n = 3$ to $n = 1$.

[3 marks]

(b) A photon of energy 12.1 eV is incident on a hydrogen atom in the ground state. Explain whether this photon will be absorbed.

[2 marks]

(c) Describe how the line spectrum of hydrogen provides evidence for the existence of discrete energy levels in atoms.

[3 marks]

Mark Scheme and Answers

Q1. The threshold frequency is the minimum frequency of electromagnetic radiation [1] that is capable of causing photoelectric emission from the surface [1].

Q2. Any two from: wave theory predicts a time delay before emission, but emission is instantaneous [1]; wave theory predicts any frequency of sufficient intensity should cause emission, but no emission occurs below f_0 regardless of intensity [1]; wave theory predicts intensity should increase electron KE, but maximum KE depends only on frequency [1].

Q3. Photon energy: $E = hf = 6.63 \times 10^{-34} \times 8.0 \times 10^{14} = 5.30 \times 10^{-19}$ J $= 3.32$ eV [1]. Since 3.32 eV $>$ 2.5 eV, emission **will** occur [1]. Maximum KE $= 3.32 - 2.5 = 0.82$ eV $= 1.31 \times 10^{-19}$ J [2].

Q4. $\lambda = h/(m_p v) = 6.63 \times 10^{-34}/(1.67 \times 10^{-27} \times 2.0 \times 10^6) = \mathbf{1.98 \times 10^{-13}}$ m [2].

Q5. Both particles gain the same kinetic energy eV, so $\frac{1}{2}mv^2 = eV \Rightarrow p = mv = \sqrt{2meV}$ [1]; therefore $\lambda = h/\sqrt{2meV}$ [1]; since $m_e \ll m_p$, the denominator is smaller for the electron, giving a **larger** λ for the electron [1].

Q6(a). $\phi = hf_0 = 6.63 \times 10^{-34} \times 5.5 \times 10^{14} = \mathbf{3.65 \times 10^{-19}}$ J [2].

Q6(b). $E_{k(\max)} = hf - \phi = 6.63 \times 10^{-34} \times 9.0 \times 10^{14} - 3.65 \times 10^{-19} = 5.97 \times 10^{-19} - 3.65 \times 10^{-19} = 2.32 \times 10^{-19}$ J [2]; $V_s = E_{k(\max)}/e = 2.32 \times 10^{-19}/1.60 \times 10^{-19} = \mathbf{1.45}$ V [1].

Q6(c). (i) Rate of electron emission **doubles** [1]: greater intensity $\Rightarrow$ more photons per second $\Rightarrow$ more photoelectron emissions per second [1]. (ii) Maximum KE is **unchanged** [1]: each photon still has the same energy (f unchanged); intensity affects number of photons, not energy per photon [1].

Q7(a). $\Delta E = -1.5 - (-13.6) = 12.1$ eV $= 12.1 \times 1.60 \times 10^{-19} = 1.936 \times 10^{-18}$ J [1]; $f = \Delta E/h = 1.936 \times 10^{-18}/6.63 \times 10^{-34} = \mathbf{2.92 \times 10^{15}}$ Hz (ultraviolet) [2].

Q7(b). The energy difference from $n = 1$ to $n = 3$ is $-1.5 - (-13.6) = 12.1$ eV [1]; the photon energy exactly matches this transition, so it **will** be absorbed and the electron promoted to $n = 3$ [1].

Q7(c). Each line in the emission spectrum corresponds to a specific photon frequency [1]; each frequency corresponds to a specific photon energy, given by $hf = E_2 - E_1$ [1]; the existence of discrete lines (rather than a continuum) shows that electrons can only transition between fixed energy values — i.e. energy levels are quantised [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	State that electromagnetic radiation is quantised into photons	
☐	Use $E = hf$ and $E = hc/\lambda$ to calculate photon energy	
☐	Convert between joules and electronvolts	
☐	Describe the photoelectric effect and state its key observations	
☐	Explain why wave theory fails to explain the photoelectric effect	
☐	Apply Einstein's equation $hf = \phi + \frac{1}{2}mv^2_{\text{max}}$	
☐	Define work function and threshold frequency; relate via $\phi = hf_0$	
☐	Explain the significance of stopping potential and use $eV_s = E_{k(\text{max})}$	
☐	State and apply the de Broglie relation $\lambda = h/p$	
☐	Describe electron diffraction as evidence for wave–particle duality	
☐	Explain what is meant by atomic energy levels and ground/excited states	
☐	Use $hf = E_2 - E_1$ for emission and absorption transitions	
☐	Interpret emission and absorption line spectra	
☐	Explain how line spectra provide evidence for energy quantisation	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Quantum physics is genuinely strange — but the maths is manageable. Focus on understanding *why* classical physics fails, and let the photon model guide the rest. The equations follow naturally from the physics.

CIE International A Level Physics

Topic 23

Nuclear Physics

Revision Booklet

This booklet covers:

- Mass–Energy Equivalence: $E = mc^2$
- Mass Defect and Binding Energy
- Binding Energy per Nucleon and Nuclear Stability
- Nuclear Fission and Fusion
- Radioactive Decay: Activity and Decay Constant
- Half-Life and Exponential Decay

Mass–Energy Equivalence

Einstein's Mass–Energy Relation

Einstein's special theory of relativity establishes that mass and energy are equivalent. A mass m at rest has an intrinsic energy given by:

$$E = mc^2$$

- $c = 3.00 \times 10^8$ m s^{-1} (speed of light in free space)
- A small mass corresponds to an enormous amount of energy.
- In nuclear reactions, small changes in mass Δm release measurable amounts of energy.

Atomic Mass Unit

The **unified atomic mass unit** (u) is defined as one-twelfth of the mass of a carbon-12 atom.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

The energy equivalent of 1 u:

$$E = mc^2 = 1.661 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$$

So 1 u $\equiv 931.5$ MeV$/c^2$.

Nuclear Notation and Equations

A nuclide is written ${}^A_Z$X, where A is the **nucleon number** (mass number) and Z is the **proton number** (atomic number).

Nuclear equations must conserve:

- **Nucleon number** A (top numbers balance)
- **Proton number** Z (bottom numbers balance)
- **Mass–energy** (energy is released or absorbed)
- **Charge** and **momentum**

Example: ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$

Mass Defect and Binding Energy

Mass Defect

The **mass defect** Δm of a nucleus is the difference between the total mass of the separate constituent nucleons and the actual mass of the nucleus.

$$\Delta m = Z\,m_p + (A - Z)\,m_n - m_{\text{nucleus}}$$

m_p = mass of a proton = 1.6726×10^{-27} kg
m_n = mass of a neutron = 1.6749×10^{-27} kg
m_{nucleus} = actual measured mass of the nucleus

The mass defect is always **positive**: the nucleus is always less massive than its parts.

Binding Energy

The **binding energy** of a nucleus is the energy required to completely separate a nucleus into its constituent protons and neutrons (i.e. to infinity).

$$E_B = \Delta m \cdot c^2$$

Equivalently, it is the energy *released* when the nucleus is assembled from separate nucleons.

Energy Released in a Nuclear Reaction

$$E = c^2\,\Delta m$$

where Δm is the difference between the total mass of reactants and the total mass of products.
If $\Delta m > 0$ (reactants heavier than products): energy is **released**.
If $\Delta m < 0$: energy must be **supplied**.

Common Mistake — Mass Defect vs Binding Energy

Students often confuse the *sign convention*. The mass defect is always defined as a positive quantity (how much mass is "missing"). The binding energy is the energy equivalent of this missing mass. A **larger** binding energy means a **more stable** nucleus — it takes more energy to pull it apart.

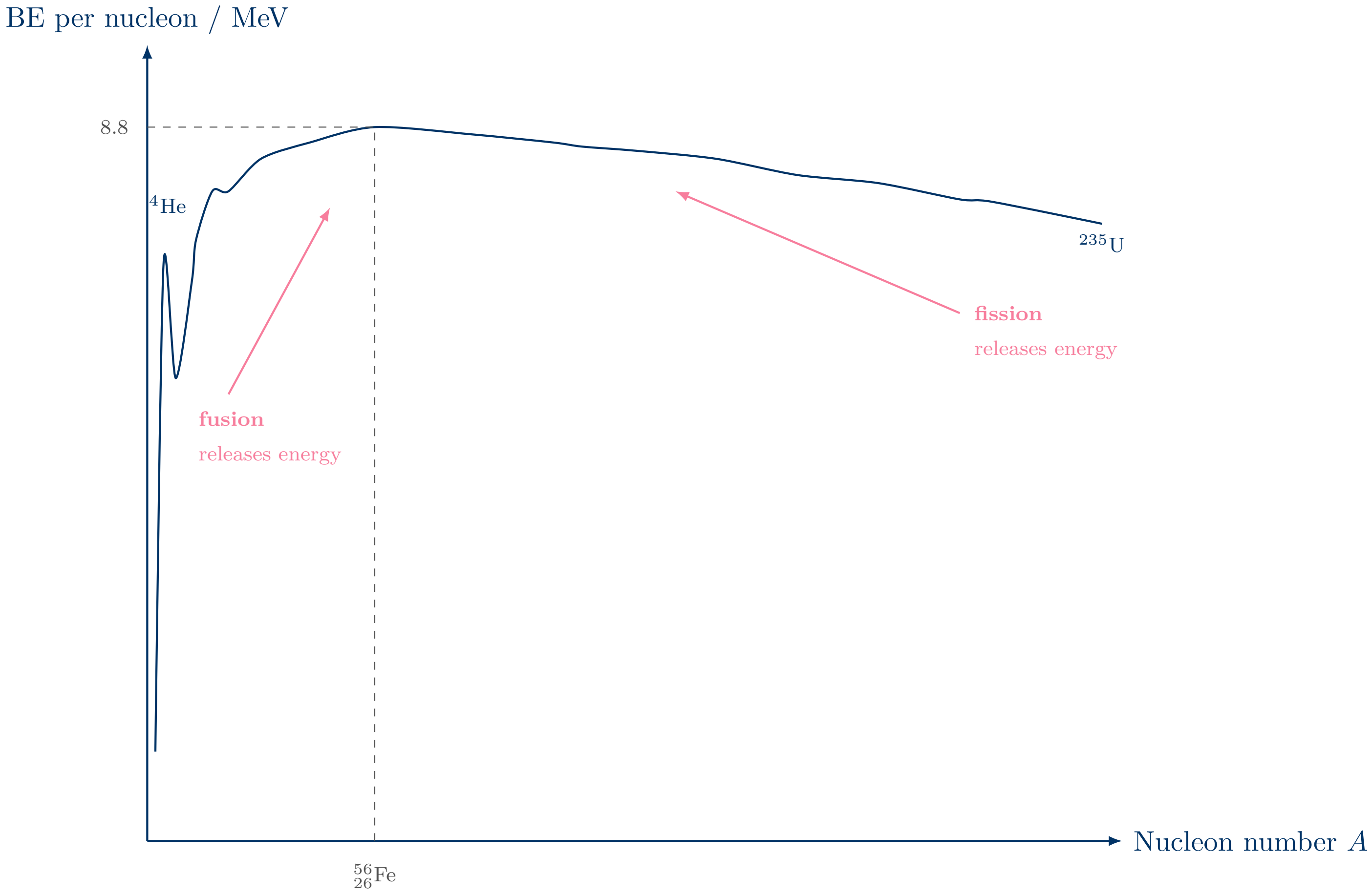
Binding Energy per Nucleon

Binding Energy per Nucleon

The **binding energy per nucleon** is the total binding energy of a nucleus divided by its nucleon number A. It is a measure of nuclear stability: the higher the value, the more stable the nucleus.

$$\text{BE per nucleon} = \frac{E_B}{A} = \frac{c^2\Delta m}{A}$$

Variation of Binding Energy per Nucleon with Nucleon Number



Key Features of the Graph

- The curve **rises steeply** for light nuclei, peaks near $^{56}_{26}$Fe at approximately 8.8 MeV per nucleon — the most stable nucleus.
- The curve **decreases gradually** for heavy nuclei ($A > 56$).
- **Fusion** of light nuclei (left of peak) moves up the curve ⇒ products are more stable ⇒ energy is released.
- **Fission** of heavy nuclei (right of peak) also moves up the curve ⇒ products are more stable ⇒ energy is released.
- $^{4}_{2}$He (helium-4) lies notably *above* the curve — it is exceptionally stable for its mass number.

Nuclear Fission and Fusion

Nuclear Fission

Nuclear fission is the splitting of a large, unstable nucleus into two smaller (daughter) nuclei of roughly equal mass, accompanied by the release of neutrons and energy.

- Induced fission: a slow (thermal) neutron is absorbed by a heavy nucleus (e.g. ^{235}U), which then splits.
- The products have greater binding energy per nucleon than the original nucleus $\Rightarrow$ energy is released.
- Typically releases 2–3 fast neutrons which can trigger further fissions (**chain reaction**).

Example: ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$

Nuclear Fusion

Nuclear fusion is the combining of two light nuclei to form a heavier nucleus, releasing energy.

- The product nucleus has greater binding energy per nucleon than the reactants $\Rightarrow$ energy is released.
- Requires **extremely high temperatures** ($\sim 10^7$ K) to overcome electrostatic repulsion between nuclei.
- Powers stars; the basis of proposed fusion reactors.

Example: ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$

Why These Reactions Release Energy

In both fission and fusion, the total mass of the **products** is less than the total mass of the **reactants**. This mass difference Δm is converted to kinetic energy of the products via $E = c^2\Delta m$. The reactions move nuclei *towards* the peak of the BE per nucleon curve (towards ^{56}Fe).

Radioactive Decay

Spontaneous and Random Decay

Radioactive decay is the spontaneous emission of radiation from an unstable nucleus.

- **Spontaneous**: the decay is not triggered by external conditions (temperature, pressure, chemical state); it cannot be predicted or controlled.
- **Random**: it is impossible to predict *when* any particular nucleus will decay. Each nucleus has the same probability of decaying per unit time.
- **Evidence for randomness**: fluctuations (statistical variation) in the measured count rate from a radioactive source.

Activity and Decay Constant

The **activity** A of a source is the number of nuclei that decay per unit time.

$$A = \lambda N$$

A = activity (Bq, where 1 Bq = 1 decay s^{-1})
λ = **decay constant** — the probability of decay of a nucleus per unit time (s^{-1})
N = number of undecayed nuclei present

Half-Life

The **half-life** $t_{1/2}$ is the time taken for the number of undecayed nuclei (or the activity) of a radioactive sample to fall to half its initial value.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

Half-life is constant for a given isotope — it does not depend on the number of nuclei present or external conditions.

Exponential Decay

Exponential Decay Equations

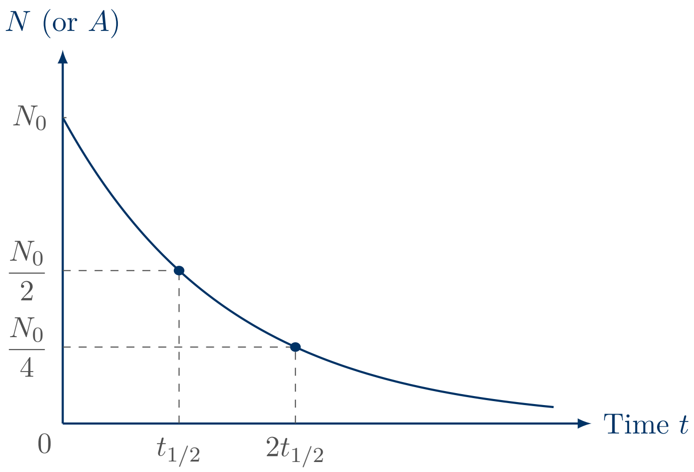
$$x = x_0\, e^{-\lambda t}$$

where x can represent:

- N = number of undecayed nuclei: $N = N_0\, e^{-\lambda t}$
- A = activity of the source: $A = A_0\, e^{-\lambda t}$
- C = received count rate: $C = C_0\, e^{-\lambda t}$

x_0 = initial value at $t = 0$
λ = decay constant (s^{-1})
t = time elapsed (s)

Graph of N against t



Linearising the Decay Equation

Taking the natural logarithm of $N = N_0 e^{-\lambda t}$:

$$\ln N = \ln N_0 - \lambda t$$

A graph of $\ln N$ (or $\ln A$) against t gives a **straight line** with:

- Gradient = $-\lambda$
- y-intercept = $\ln N_0$

This is the standard experimental method to determine λ and hence $t_{1/2}$.

Common Errors with Decay Calculations

- Using $t_{1/2}$ directly in the exponential formula — you must use λ, not $t_{1/2}$. Convert first: $\lambda = 0.693/t_{1/2}$.

- Forgetting to convert time units: if $t_{1/2}$ is in days, convert to seconds before finding λ in s^{-1}.
- Confusing **activity** $A = \lambda N$ (Bq) with **count rate** — the count rate is always less than activity due to detector efficiency and geometry.
- Applying the exponential formula to something that *increases* over time — it only applies to N, A, or count rate, which all decay.

Formula Summary Sheet

Formula	Quantity	Units
$E = mc^2$	Mass–energy equivalence	J
$E = c^2\Delta m$	Energy from mass change	J
$\Delta m = Zm_p + (A-Z)m_n - m_{\text{nuc}}$	Mass defect	kg
$E_B = \Delta m\, c^2$	Binding energy	J
$A = \lambda N$	Activity	Bq
$\lambda = 0.693/t_{1/2}$	Decay constant from half-life	s^{-1}
$N = N_0 e^{-\lambda t}$	Number of undecayed nuclei	—
$A = A_0 e^{-\lambda t}$	Activity	Bq
$\ln N = \ln N_0 - \lambda t$	Linearised decay	—

Constants: $c = 3.00 \times 10^8$ m s^{-1}, 1 u $= 1.661 \times 10^{-27}$ kg $\equiv$ 931.5 MeV, $m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg

Exam Technique and Problem-Solving Strategy

Step-by-Step Strategy for Nuclear Calculations

1. **Balance the equation** — check nucleon numbers and proton numbers sum correctly on both sides.
2. **Find** Δm — total reactant mass minus total product mass; work in kg or u.
3. **Apply** $E = c^2\Delta m$ to find energy released (convert u to kg if needed, or use 1 u = 931.5 MeV).
4. For decay: **convert** $t_{1/2}$ to seconds, find $\lambda = 0.693/t_{1/2}$, then apply $x = x_0e^{-\lambda t}$.

Common Errors — Avoid These!

- Using **atomic masses** (including electron masses) rather than nuclear masses without accounting for electron masses — take care with data provided in exam questions.
- Forgetting that Δm must be in **kg** when using $E = mc^2$ in SI units.
- Confusing **binding energy** with **ionisation energy** — binding energy refers to the nucleus.
- Stating that a higher binding energy per nucleon means **less** stable — it means **more** stable.
- Mixing up the direction of fusion and fission on the BE/nucleon graph.

Worked Examples

Example 1 — Mass Defect and Binding Energy

Question: Calculate the mass defect and binding energy of a helium-4 nucleus ($^4_2$He).
($m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg, $m_{^4\text{He}} = 6.6447 \times 10^{-27}$ kg)

Solution

Solution:
Mass of constituents: $2m_p + 2m_n = 2(1.6726) + 2(1.6749) = 6.6950 \times 10^{-27}$ kg
Mass defect:

$$\Delta m = 6.6950 \times 10^{-27} - 6.6447 \times 10^{-27} = \mathbf{5.03 \times 10^{-29}} \text{ kg}$$

Binding energy:

$$E_B = \Delta m\, c^2 = 5.03 \times 10^{-29} \times (3.00 \times 10^8)^2 = \mathbf{4.53 \times 10^{-12}} \text{ J } (= 28.3 \text{ MeV})$$

Binding energy per nucleon: $4.53 \times 10^{-12}/4 = 1.13 \times 10^{-12}$ J = 7.07 MeV per nucleon

Example 2 — Energy Released in Fission

Question: In a fission reaction, the total mass of products is 3.09×10^{-28} kg less than the total mass of reactants. Calculate the energy released in MeV.

Solution

Solution:

$$E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.09 \times 10^{-28} = 2.78 \times 10^{-11} \text{ J}$$

$$E = \frac{2.78 \times 10^{-11}}{1.60 \times 10^{-13}} = \mathbf{174} \text{ MeV}$$

Example 3 — Radioactive Decay Calculation

Question: A radioactive isotope has a half-life of 12.0 hours. A sample initially contains 8.00×10^{20} undecayed nuclei. Calculate (a) the decay constant, (b) the initial activity, (c) the number of undecayed nuclei after 30.0 hours.

Solution

Solution:
(a) Convert: $t_{1/2} = 12.0 \times 3600 = 4.32 \times 10^4$ s

$$\lambda = \frac{0.693}{4.32 \times 10^4} = \mathbf{1.60 \times 10^{-5}} \text{ s}^{-1}$$

(b) $A_0 = \lambda N_0 = 1.60 \times 10^{-5} \times 8.00 \times 10^{20} = \mathbf{1.28 \times 10^{16}}$ Bq
(c) $t = 30.0 \times 3600 = 1.08 \times 10^5$ s

$$N = N_0\, e^{-\lambda t} = 8.00 \times 10^{20} \times e^{-1.60\times10^{-5}\times1.08\times10^5}$$

$$N = 8.00 \times 10^{20} \times e^{-1.728} = 8.00 \times 10^{20} \times 0.178 = \mathbf{1.42 \times 10^{20}}$$

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define the terms *mass defect* and *binding energy* of a nucleus.

[4 marks]

Q2. State two features of the binding energy per nucleon graph that explain why both nuclear fusion and nuclear fission can release energy.

[2 marks]

Q3. A radioactive source has an activity of 6.4×10^5 Bq and a decay constant of $2.0 \times 10^{-3}\ \text{s}^{-1}$. Calculate the number of undecayed nuclei present and the half-life of the source.

[4 marks]

Q4. Explain what is meant by saying that radioactive decay is *spontaneous* and *random*.

[2 marks]

Q5. Complete and balance the following nuclear equation, identifying the unknown particle X:

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + \text{X}$$

[2 marks]

Section B — Longer Structured Questions

Q6. The fusion reaction between deuterium and tritium is:

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$

Relevant masses: $m(^2\text{H}) = 2.01410$ u, $m(^3\text{H}) = 3.01605$ u, $m(^4\text{He}) = 4.00260$ u, $m_n = 1.00867$ u.

(a) Calculate the mass defect of this reaction in kg.

[3 marks]

(b) Calculate the energy released in this reaction in joules and in MeV.

[2 marks]

(c) Explain, with reference to the binding energy per nucleon graph, why this reaction releases energy.

[3 marks]

Q7. The isotope iodine-131 ($^{131}_{53}\text{I}$) is used in medical treatment. It has a half-life of 8.04 days.

(a) Calculate the decay constant of iodine-131 in s^{-1}.

[2 marks]

(b) A patient is given a dose with an initial activity of 4.0×10^8 Bq. Calculate the activity after 24 days.

[2 marks]

(c) Sketch a graph of $\ln A$ against t for this source, labelling the y-intercept and stating the gradient in terms of λ.

[3 marks]

Mark Scheme and Answers

Q1. *Mass defect*: the difference between the total mass of the separate nucleons (protons and neutrons) and the actual mass of the nucleus [2]. *Binding energy*: the energy required to completely separate a nucleus into its constituent protons and neutrons [2].

Q2. The curve rises steeply for light nuclei — fusion of light nuclei produces a product with greater BE/nucleon, so energy is released [1]. The curve falls for heavy nuclei — fission of a heavy nucleus produces fragments with greater BE/nucleon, so energy is released [1].

Q3. $N = A/\lambda = 6.4 \times 10^5/2.0 \times 10^{-3} = \mathbf{3.2 \times 10^8}$ [2]. $t_{1/2} = 0.693/\lambda = 0.693/2.0 \times 10^{-3} = \mathbf{347}$ s [2].

Q4. *Spontaneous*: the decay is not triggered or affected by external conditions; it cannot be induced or prevented [1]. *Random*: it is impossible to predict which nucleus will decay next, or when; each nucleus has the same fixed probability of decaying per unit time [1].

Q5. Nucleon: $238 = 234 + A \Rightarrow A = 4$; proton: $92 = 90 + Z \Rightarrow Z = 2$. X is ${}^4_2$He (an alpha particle) [2].

Q6(a). $\Delta m = (2.01410+3.01605)-(4.00260+1.00867) = 5.03015-5.01127 = 0.01888$ u [1]; $= 0.01888 \times 1.661 \times 10^{-27} = \mathbf{3.14 \times 10^{-29}}$ kg [2].

Q6(b). $E = c^2\Delta m = (3.00 \times 10^8)^2 \times 3.14 \times 10^{-29} = \mathbf{2.82 \times 10^{-12}}$ J [1]; $= 2.82 \times 10^{-12}/1.60 \times 10^{-13} = \mathbf{17.6}$ MeV [1].

Q6(c). The reactants (^{2}H and ^{3}H) lie to the left of the peak of the BE/nucleon curve [1]; the product ^{4}He has a higher binding energy per nucleon than the reactants [1]; since the products are more tightly bound, mass is converted to energy and released [1].

Q7(a). $t_{1/2} = 8.04 \times 24 \times 3600 = 6.95 \times 10^5$ s; $\lambda = 0.693/6.95 \times 10^5 = \mathbf{9.97 \times 10^{-7}}$ $\mathbf{s^{-1}}$ [2].

Q7(b). 24 days $= 3 \times t_{1/2}$; $A = 4.0 \times 10^8 \times (1/2)^3 = 4.0 \times 10^8/8 = \mathbf{5.0 \times 10^7}$ Bq [2].

Q7(c). Straight line [1]; y-intercept at $\ln(4.0 \times 10^8) = 19.8$ labelled $\ln A_0$ [1]; gradient $= -\lambda$ (negative slope) [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

Learning Objective	Confidence (1–3)
☐ State and apply Einstein's mass–energy relation $E = mc^2$	
☐ Write and balance nuclear equations, conserving A and Z	
☐ Define mass defect and calculate it from nuclear masses	
☐ Define binding energy and use $E_B = \Delta m\, c^2$	
☐ Sketch the binding energy per nucleon vs nucleon number graph	
☐ Identify the most stable nucleus and the peak of the curve	
☐ Explain using the graph why fusion of light nuclei releases energy	
☐ Explain using the graph why fission of heavy nuclei releases energy	
☐ Calculate energy released in a nuclear reaction using $E = c^2 \Delta m$	
☐ Explain what is meant by spontaneous and random decay	
☐ Define activity and decay constant; use $A = \lambda N$	
☐ Define half-life and use $\lambda = 0.693/t_{1/2}$	
☐ Apply $x = x_0 e^{-\lambda t}$ to N, A, or count rate	
☐ Linearise the decay equation and interpret the graph of $\ln N$ vs t	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Nuclear physics connects the very small to the very large — from the stability of a single nucleus to the energy source of stars. Master the binding energy curve and the exponential decay equation and most of this topic follows naturally.

CIE International A Level Physics

Topic 24

Medical Physics

Revision Booklet

This booklet covers:

- Ultrasound: Generation, Detection and Imaging
- Acoustic Impedance and Reflection
- Attenuation of Ultrasound
- Production and Use of X-Rays
- Attenuation of X-Rays and CT Scanning
- PET Scanning and Annihilation

Production and Use of Ultrasound

Ultrasound

Ultrasound is sound with a frequency above the upper limit of human hearing (> 20 kHz). In medical imaging, frequencies of 1–20 MHz are typical.

Piezoelectric Transducer

A **piezoelectric crystal** exhibits two related effects:

- When a **p.d. is applied** across the crystal, it changes shape (contracts or expands). Applying an alternating p.d. at the crystal's resonant frequency causes it to **vibrate and emit ultrasound**.
- Conversely, when the crystal's shape **changes** (e.g. due to an incoming pressure wave), it **generates an e.m.f.** — it acts as a detector.

The same transducer can therefore act as both **emitter and receiver**.

A-Scan Imaging (Pulse-Echo)

- A short pulse of ultrasound is emitted into the body.
- At each **boundary between tissues** of different acoustic impedance, part of the pulse is **reflected** (echo) and part is **transmitted**.
- The time delay between emission and detection of each echo gives the **depth** of the boundary: $d = \frac{1}{2}vt$.
- The amplitude of each echo gives information about the nature of the boundary.

Acoustic Impedance and Reflection

Specific Acoustic Impedance

The **specific acoustic impedance** Z of a medium is defined as:

$$Z = \rho c$$

Z = specific acoustic impedance (kg m^{-2} s^{-1})
ρ = density of the medium (kg m^{-3})
c = speed of sound in the medium (m s^{-1})

Intensity Reflection Coefficient

The fraction of intensity reflected at a boundary between two media with impedances Z_1 and Z_2:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

- If $Z_1 = Z_2$ (matched impedances): $I_R/I_0 = 0$ — no reflection, all transmitted.
- If $Z_1 \gg Z_2$ or $Z_1 \ll Z_2$ (large mismatch): $I_R/I_0 \approx 1$ — almost all reflected.
- Air–tissue boundary: huge impedance mismatch $\Rightarrow$ almost complete reflection.

Coupling Gel

Because the acoustic impedance of air is much lower than that of tissue, a large fraction of ultrasound would be reflected at the skin–air boundary if no gel were used. A **coupling gel** (with impedance close to that of tissue) is applied between the transducer and the skin to **minimise reflection** and allow ultrasound to enter the body efficiently.

Attenuation of Ultrasound

Attenuation of Ultrasound in Matter

As ultrasound travels through a medium, its intensity decreases exponentially:

$$I = I_0 \, e^{-\mu x}$$

I = intensity at depth x ($\mathrm{W\ m^{-2}}$)
I_0 = initial intensity ($\mathrm{W\ m^{-2}}$)
μ = **absorption (attenuation) coefficient** of the medium ($\mathrm{m^{-1}}$)
x = distance travelled in the medium (m)

A larger μ means the medium absorbs ultrasound more strongly. Higher frequency ultrasound has a larger μ (greater attenuation) but better resolution.

Resolution vs Penetration Trade-off

- **Higher frequency**: shorter wavelength $\Rightarrow$ better resolution, but higher attenuation $\Rightarrow$ less depth penetration.
- **Lower frequency**: greater penetration but poorer resolution.
- The choice of frequency is a compromise depending on the depth of the structure being imaged.

Production and Use of X-Rays

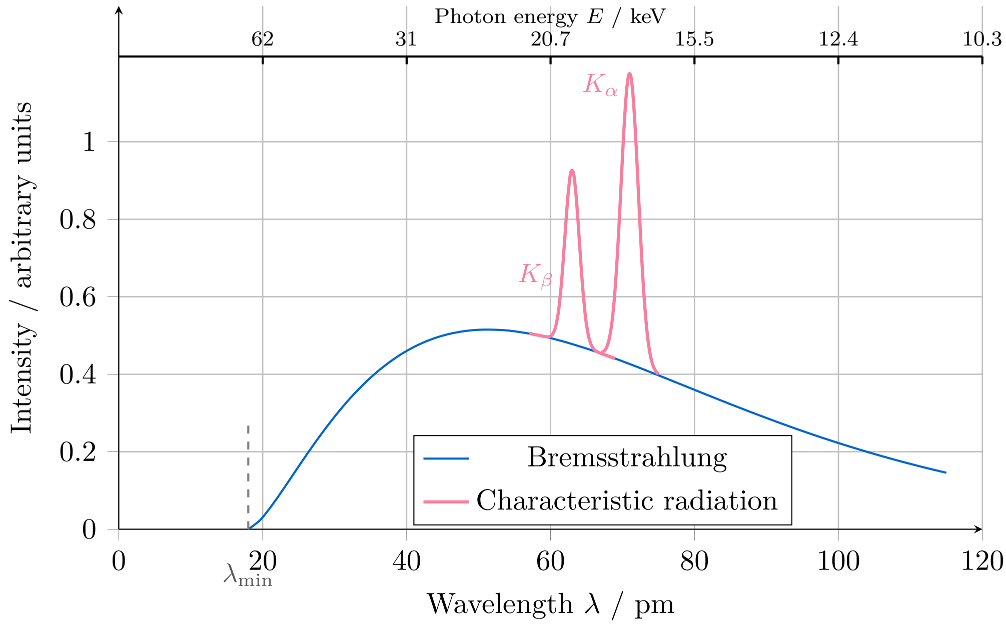
Production of X-Rays

X-rays are produced in an X-ray tube when fast-moving electrons are rapidly decelerated by a metal target (anode).

- Electrons are accelerated from a heated cathode through a high potential difference V.
- On striking the target, electrons lose kinetic energy, producing X-rays by two processes:

- **Bremsstrahlung** (braking radiation): continuous spectrum from deceleration.
- **Characteristic radiation**: discrete lines from inner-shell electron transitions in target atoms.

X-ray Emission Spectrum (Tungsten Target, 70 kV)



Minimum X-Ray Wavelength

The maximum photon energy (minimum wavelength) occurs when an electron gives all its kinetic energy to a single photon:

$$eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV}$$

Increasing the accelerating voltage V decreases λ_{min} and increases the penetrating power of the X-rays.

Contrast in X-Ray Imaging

Contrast refers to the difference in image intensity between adjacent structures, allowing them to be distinguished.

- Dense materials (e.g. bone, containing calcium) absorb X-rays strongly ⇒ appear **white** on the image.
- Soft tissues absorb less ⇒ appear **grey**.
- Air absorbs very little ⇒ appears **black**.

- **Contrast agents** (e.g. barium meal, iodine compounds) can be introduced to increase contrast for soft tissue structures such as the gut or blood vessels.

Attenuation of X-Rays in Matter

$$I = I_0\, e^{-\mu x}$$

The same exponential attenuation law applies as for ultrasound, where μ is now the **linear attenuation coefficient** for X-rays in the medium. Dense materials have larger μ.

CT Scanning (Computed Tomography)

A **CT scanner** produces a three-dimensional image of internal structures:

- Multiple X-ray images of the **same cross-sectional slice** are taken from **different angles**.
- These are combined computationally to produce a **2D image of one slice**.
- The process is **repeated along the body's axis**, producing multiple 2D slice images.
- The 2D slice images are **combined** to build a full **3D image**.

CT gives much better contrast for soft tissues than a plain X-ray, but involves a significantly higher radiation dose.

PET Scanning

Radioactive Tracers

A **tracer** is a substance containing radioactive nuclei that is introduced into the body and absorbed by the tissue under investigation. The emitted radiation is detected externally to produce an image of the tracer distribution.

- In PET scanning, a tracer that decays by β^+ **(positron) emission** is used.
- A common example is fluorine-18 labelled glucose (^{18}F-FDG), which is preferentially absorbed by metabolically active tissue (e.g. tumours).

Annihilation

Annihilation occurs when a particle meets its **antiparticle**: both are destroyed and their combined mass–energy is converted entirely into radiation.

- A positron (β^+) emitted by the tracer quickly meets an electron in the surrounding tissue.
- Both are annihilated, producing **two gamma-ray photons** travelling in **exactly opposite directions** (to conserve momentum).

- **Conservation laws**: mass–energy and momentum are both conserved in the process.

Energy of Annihilation Photons

Each photon carries energy equal to the rest-mass energy of one electron (the two particles have negligible kinetic energy at annihilation):

$$E_\gamma = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}\ \text{J} = \mathbf{0.511}\ \text{MeV}$$

Both photons have this energy (total energy released $= 2m_e c^2 = 1.02$ MeV).

How PET Produces an Image

- Detectors arranged around the patient detect the **coincident arrival** of the two gamma photons.
- Because the photons travel in opposite directions, the annihilation event must have occurred **somewhere along the line** joining the two detectors.
- By processing the **arrival times** from many such coincidences, the positions of annihilation events are reconstructed, producing a map of **tracer concentration** in the tissue.
- High tracer uptake indicates high metabolic activity — useful for identifying tumours, heart disease, and neurological conditions.

PET vs CT vs Ultrasound

- **Ultrasound**: no ionising radiation; good for soft tissue and real-time imaging (e.g. foetal scans); cannot penetrate bone or air.
- **X-ray / CT**: high contrast for bone; CT gives 3D information; ionising radiation dose (CT significantly higher than plain X-ray).
- **PET**: images *function* (metabolic activity), not just structure; requires a cyclotron to produce the short-lived tracer; relatively high radiation dose.

Formula Summary Sheet

Formula	Quantity	Units
$Z = \rho c$	Specific acoustic impedance	kg m^{-2} s^{-1}
$I_R/I_0 = (Z_1 - Z_2)^2/(Z_1 + Z_2)^2$	Intensity reflection coefficient	—
$I = I_0 e^{-\mu x}$	Attenuation (ultrasound or X-ray)	W m^{-2}
$d = \frac{1}{2}vt$	Depth from echo time	m
$\lambda_{\text{min}} = hc/eV$	Minimum X-ray wavelength	m
$E_\gamma = m_e c^2$	Energy of annihilation photon	J

Constants: $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^8$ m s^{-1}, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

Exam Technique and Problem-Solving Strategy

Key Strategies

1. For **attenuation** questions: identify I_0, μ and x; substitute into $I = I_0 e^{-\mu x}$.
2. For **reflection coefficient**: identify Z_1 and Z_2; substitute directly — the formula is symmetric in Z_1 and Z_2.
3. For **echo depth**: $d = \frac{1}{2}vt$ (factor of $\frac{1}{2}$ because pulse travels to the boundary *and back*).
4. For **annihilation photon energy**: always 0.511 MeV per photon; quote this or calculate from $m_e c^2$.

Common Errors — Avoid These!

- Forgetting the **factor of 2** in $d = vt/2$ for pulse-echo depth calculations.
- Confusing the **attenuation coefficient** μ with decay constant λ from Topic 23 — both appear in exponential decay equations but refer to completely different physical processes.
- Stating that annihilation produces **one** photon — it must produce **two** travelling in opposite directions to conserve momentum.
- Confusing **CT** (multiple X-ray angles $\Rightarrow$ 3D image) with a plain X-ray (single image, 2D projection).

- Applying the reflection coefficient formula with **intensities** instead of **impedances**.

Worked Examples

Example 1 — Acoustic Impedance and Reflection

Question: The specific acoustic impedance of muscle is 1.70×10^6 kg m^{-2}s^{-1} and of bone is 7.80×10^6 kg m^{-2}s^{-1}. Calculate the intensity reflection coefficient at a muscle–bone boundary.

Solution

Solution:

$$\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2} = \frac{(1.70 \times 10^6 - 7.80 \times 10^6)^2}{(1.70 \times 10^6 + 7.80 \times 10^6)^2}$$

$$= \frac{(-6.10 \times 10^6)^2}{(9.50 \times 10^6)^2} = \frac{3.72 \times 10^{13}}{9.02 \times 10^{13}} = \mathbf{0.413}$$

About 41% of the ultrasound intensity is reflected at this boundary.

Example 2 — Ultrasound Attenuation

Question: Ultrasound with an initial intensity of 250 W m^{-2} passes through 4.0 cm of tissue with attenuation coefficient $\mu = 23$ m^{-1}. Calculate the transmitted intensity.

Solution

Solution:

$$I = I_0 e^{-\mu x} = 250 \times e^{-23 \times 0.040} = 250 \times e^{-0.92}$$

$$I = 250 \times 0.399 = \mathbf{99.7} \text{ W m}^{\mathbf{-2}}$$

Example 3 — PET Scanning Annihilation Energy

Question: In a PET scan, a positron emitted by the tracer annihilates with an electron. Calculate the energy and frequency of each gamma-ray photon produced.

Solution

Solution:
Energy of each photon:

$$E = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14} \text{ J}$$

$$= 8.20 \times 10^{-14} / 1.60 \times 10^{-13} = \mathbf{0.511} \text{ MeV}$$

Frequency:

$$f = \frac{E}{h} = \frac{8.20 \times 10^{-14}}{6.63 \times 10^{-34}} = \mathbf{1.24 \times 10^{20}} \text{ Hz}$$

(This lies in the gamma-ray region of the electromagnetic spectrum.)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Describe how a piezoelectric transducer can act as both an emitter and a detector of ultrasound.

[4 marks]

Q2. The specific acoustic impedance of soft tissue is 1.63×10^6 kg m^{-2}s^{-1} and of air is 430 kg m^{-2}s^{-1}. Show that almost all ultrasound is reflected at an air–tissue boundary.

[3 marks]

Q3. An ultrasound pulse is emitted and an echo is received 85 μs later. The speed of ultrasound in tissue is 1500 m s^{-1}. Calculate the depth of the reflecting boundary.

[2 marks]

Q4. An X-ray beam of initial intensity I_0 passes through 8.0 cm of tissue with linear attenuation coefficient $\mu = 12$ m^{-1}. Calculate the ratio I/I_0.

[2 marks]

Q5. State **two** conservation laws that apply during electron–positron annihilation in PET scanning.

[2 marks]

Section B — Longer Structured Questions

Q6. A medical ultrasound system uses a piezoelectric transducer operating at 5.0 MHz.

(a) Explain why a coupling gel is applied between the transducer and the patient's skin.

[3 marks]

(b) The attenuation coefficient of soft tissue at this frequency is 40 m^{-1}. Calculate the depth at which the intensity has fallen to 5.0% of its initial value.

[3 marks]

(c) Suggest why a lower frequency might be chosen when imaging deep structures, and state one disadvantage.

[2 marks]

Q7. PET scanning uses a tracer that emits positrons.

(a) Explain what happens when a positron emitted by the tracer encounters an electron in the body tissue.

[3 marks]

(b) Explain why two detectors placed on opposite sides of the patient must detect photons simultaneously for the event to be recorded.

[2 marks]

(c) Calculate the wavelength of the gamma-ray photons produced in the annihilation.

[2 marks]

Mark Scheme and Answers

Q1. Emitter: an alternating p.d. at the resonant frequency is applied across the crystal [1]; the crystal vibrates at that frequency [1]; emitting ultrasound waves. Detector: incoming pressure wave causes the crystal to change shape [1]; this generates an e.m.f. which is detected as an electrical signal [1].

Q2. $I_R/I_0 = (1.63 \times 10^6 - 430)^2/(1.63 \times 10^6 + 430)^2$ [1] $\approx (1.63 \times 10^6)^2/(1.63 \times 10^6)^2$ [1] ≈ 0.9995 (i.e. ≈ 1, almost total reflection) [1].

Q3. $d = \frac{1}{2}vt = \frac{1}{2} \times 1500 \times 85 \times 10^{-6} = \mathbf{6.4 \times 10^{-2}}$ m (= 6.4 cm) [2].

Q4. $I/I_0 = e^{-\mu x} = e^{-12 \times 0.080} = e^{-0.96} = \mathbf{0.383}$ [2].

Q5. Any two of: conservation of mass–energy [1]; conservation of momentum [1]; conservation of charge [1].

Q6(a). The acoustic impedance of air is much lower than that of tissue [1]; this large mismatch means nearly all ultrasound would be reflected at the skin–air interface [1]; the gel has impedance close to that of tissue, minimising reflection and allowing ultrasound to enter the body [1].

Q6(b). $I/I_0 = 0.050$, so $e^{-40x} = 0.050$ [1]; $-40x = \ln(0.050) = -3.00$; $x = 3.00/40 = \mathbf{0.075}$ m (= 7.5 cm) [2].

Q6(c). Lower frequency has a smaller attenuation coefficient, so ultrasound penetrates more deeply [1]; disadvantage: lower frequency has a longer wavelength, giving **poorer spatial resolution** [1].

Q7(a). The positron meets an electron [1]; both are annihilated (annihilation) [1]; two gamma-ray photons are produced travelling in exactly opposite directions [1].

Q7(b). The two photons travel in exactly opposite directions (to conserve momentum) [1]; simultaneous detection at opposite detectors confirms the annihilation occurred on the line joining the two detectors [1].

Q7(c). $E = m_e c^2 = 8.20 \times 10^{-14}$ J; $\lambda = hc/E = (6.63 \times 10^{-34} \times 3.00 \times 10^8)/8.20 \times 10^{-14} = \mathbf{2.42 \times 10^{-12}}$ m [2].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Explain the piezoelectric effect in both directions (emitter and detector)	
☐	Describe how pulse-echo ultrasound produces diagnostic information	
☐	Define specific acoustic impedance using $Z = \rho c$	
☐	Use the intensity reflection coefficient formula	
☐	Explain the purpose of coupling gel	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of ultrasound	
☐	Explain the resolution vs penetration trade-off for ultrasound frequency	
☐	Explain how X-rays are produced (bremsstrahlung and characteristic)	
☐	Use $\lambda_{\text{min}} = hc/eV$ for X-ray tube calculations	
☐	Explain contrast in X-ray imaging	
☐	Apply $I = I_0 e^{-\mu x}$ for attenuation of X-rays	
☐	Describe how CT scanning builds a 3D image from multiple 2D slices	
☐	Explain what a radioactive tracer is and why a β^+ emitter is used in PET	
☐	Describe electron–positron annihilation and apply conservation laws	
☐	Calculate the energy of annihilation photons using $E = m_e c^2$	
☐	Explain how coincidence detection produces a PET image	

Key: 1 = Need more work 2 = Getting there 3 = Confident

Good luck with your revision!

Medical physics is where fundamental physics saves lives. The same exponential attenuation, the same $E = mc^2$, the same wave properties — seen in a new and important context. Make sure you can explain *why* each technique works, not just apply the formulas.

CIE International A Level Physics

Topic 25

Astronomy and Cosmology

Revision Booklet

This booklet covers:

- Luminosity and Radiant Flux Intensity
- Standard Candles and Distance Measurement
- Wien's Displacement Law and Stellar Temperature
- The Stefan–Boltzmann Law and Stellar Radii
- Redshift and the Expanding Universe
- Hubble's Law and the Big Bang

Luminosity and Radiant Flux Intensity

Luminosity

The **luminosity** L of a star is the total power of electromagnetic radiation emitted by the star in all directions.

L units: W (watts)

Luminosity depends on the star's **surface temperature** and **surface area** — not on its distance from us.

Radiant Flux Intensity

The **radiant flux intensity** F (sometimes called apparent brightness) is the power of radiation received per unit area at a detector (e.g. a telescope on Earth).

F units: W m^{-2}

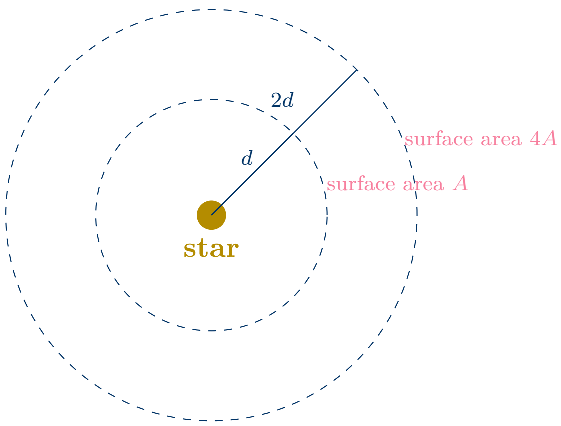
F depends on both the luminosity of the source and its distance d.

Inverse Square Law for Radiant Flux

Assuming the star radiates uniformly in all directions and there is no absorption by the intervening medium, the radiation spreads over a sphere of area $4\pi d^2$:

$$F = \frac{L}{4\pi d^2}$$

F = radiant flux intensity at distance d (W m^{-2})
L = luminosity of the star (W)
d = distance from the star (m)



$F \propto 1/d^2$: double the distance, quarter the flux

Standard Candles

Standard Candle

A **standard candle** is an astronomical object whose **luminosity is known** (or can be determined independently of distance). By measuring the radiant flux intensity F received from the object and using $F = L/(4\pi d^2)$, its distance d can be calculated:

$$d = \sqrt{\frac{L}{4\pi F}}$$

Standard Candles Used in Practice

- **Cepheid variable stars**: pulsating stars whose *period of variation* is related to their luminosity (period–luminosity relation). Measure the period ⇒ know L ⇒ measure F ⇒ find d.
- **Type Ia supernovae**: thermonuclear explosions of white dwarf stars that all reach approximately the same peak luminosity. Visible across enormous distances (billions of light-years), making them useful for measuring distances to distant galaxies.

Assumptions and Limitations

The inverse square law assumes:

- No absorption of radiation between source and detector (no dust or gas in the way).
- The source radiates **isotropically** (equally in all directions).

Dust absorption causes stars to appear **dimmer** than expected, which would lead to an **overestimate** of distance if uncorrected.

Wien's Displacement Law and Stellar Temperature

Black-Body Radiation

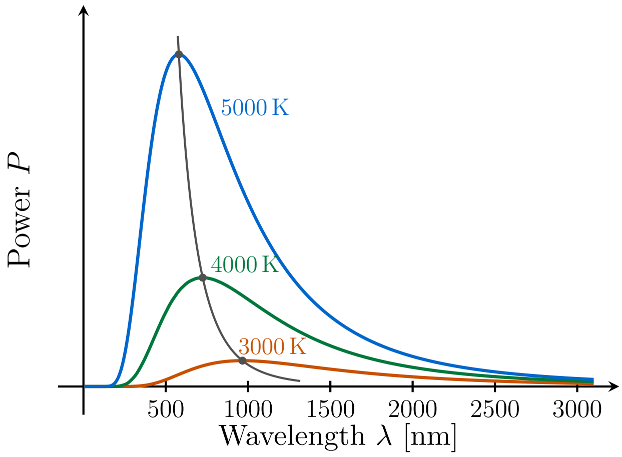
Stars approximate **black bodies** — objects that absorb all incident radiation and emit a characteristic continuous spectrum that depends only on temperature. The peak wavelength of this spectrum shifts with temperature.

Wien's Displacement Law

$$\lambda_{\max} \propto \frac{1}{T} \qquad \text{equivalently} \qquad \lambda_{\max} T = b$$

$\lambda_{\max}$ = wavelength at peak intensity of the spectrum (m)
T = surface temperature of the star (K)
b = Wien's constant = 2.90×10^{-3} m K

A **hotter** star has its peak at a **shorter** wavelength (bluer colour). A **cooler** star peaks at a **longer** wavelength (redder colour).



The Stefan–Boltzmann Law and Stellar Radii

Stefan–Boltzmann Law

For a black-body sphere (approximating a star) of radius r and surface temperature T:

$$L = 4\pi r^2 \sigma T^4$$

L = luminosity (W)
r = radius of the star (m)
σ = Stefan–Boltzmann constant = 5.67×10^{-8} W m^{-2}K^{-4}
T = surface temperature (K)

Estimating Stellar Radius

By combining Wien's law and the Stefan–Boltzmann law:

1. Measure λ_{max} from the star's spectrum $\Rightarrow$ use Wien's law to find T.
2. Measure F (radiant flux intensity) and find d (e.g. via a standard candle or parallax).
3. Find L from $L = 4\pi d^2 F$.
4. Rearrange Stefan–Boltzmann: $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Using Ratios

Exam questions often ask you to *compare* two stars rather than calculate absolute values. In that case, form a ratio to cancel constants:

$$\frac{L_1}{L_2} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4}$$

This avoids large numbers and reduces the risk of errors.

Redshift and the Expanding Universe

Redshift

When a source of light moves **away** from an observer, the observed wavelength is **longer** (shifted towards the red end of the spectrum) than the wavelength emitted. This is the **Doppler effect** applied to light.
The lines in the emission or absorption spectrum of a distant galaxy are observed at **longer wavelengths** than those of the same element measured in the laboratory.

Redshift Formula

For a source moving at speed $v \ll c$ relative to an observer:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

z = redshift (dimensionless)
$\Delta\lambda$ = shift in wavelength (= observed λ − emitted λ) (m)
λ = emitted (rest-frame) wavelength (m)
Δf = shift in frequency (Hz)
v = recession speed of the source (m s^{-1})

Redshift as Evidence for Expansion

- Observations of distant galaxies show that their spectral lines are **all redshifted**.
- The greater the distance of a galaxy, the greater its redshift.
- This indicates that distant galaxies are moving **away** from us, and the further away they are, the faster they recede.
- This is consistent with the **Universe expanding**: it is space itself that is stretching, carrying galaxies apart, rather than the galaxies moving through space.

Hubble's Law and the Big Bang Theory

Hubble's Law

$$v \approx H_0\, d$$

v = recession speed of a galaxy (m s^{-1})
H_0 = Hubble constant (s^{-1}, though often quoted in km s^{-1} Mpc^{-1})
d = distance of the galaxy from Earth (m)

In CIE examinations, SI units are used: H_0 in s^{-1} and d in metres.

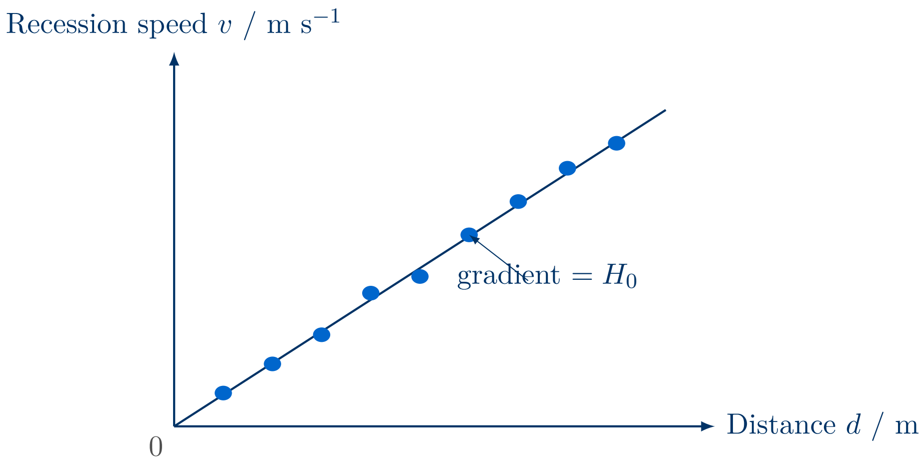
Hubble's Law and the Big Bang Theory

- Hubble's law states that the recession speed of a galaxy is proportional to its distance.
- If all galaxies are currently moving apart, then in the past they must have been **closer together**.
- Extrapolating back in time, all matter was once concentrated in an extremely hot, dense state — the **Big Bang**.
- An estimate of the **age of the Universe** can be obtained from:

$$t \approx \frac{1}{H_0}$$

(This assumes a constant rate of expansion, which is a simplification.)

Hubble Plot: Recession Speed vs Distance



Determining H_0 from a Graph

A graph of recession speed v against distance d gives a straight line through the origin. The **gradient** of this line is the Hubble constant H_0. In practice, there is significant scatter due to the difficulty of measuring distances to distant galaxies accurately.

Formula Summary Sheet

Formula	Quantity	Units
$F = L/(4\pi d^2)$	Inverse square law / flux	W m^{-2}
$\lambda_{\text{max}}T = b$	Wien's displacement law	m K
$L = 4\pi r^2\sigma T^4$	Stefan–Boltzmann law	W
$\Delta\lambda/\lambda \approx \Delta f/f \approx v/c$	Redshift formula	—
$v \approx H_0 d$	Hubble's law	m s^{-1}, m
$t \approx 1/H_0$	Age of Universe estimate	s

Constants: $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4}, $b = 2.90 \times 10^{-3}$ m K, $c = 3.00 \times 10^{8}$ m s^{-1}

Exam Technique and Problem-Solving Strategy

Step-by-Step for Stellar Radius Problems

1. Find T: measure λ_{max} from spectrum $\Rightarrow T = b/\lambda_{\text{max}}$.
2. Find L: use standard candle or parallax to get d; then $L = 4\pi d^2 F$.
3. Find r: rearrange Stefan–Boltzmann: $r = \sqrt{L/(4\pi\sigma T^4)}$.

Common Errors — Avoid These!

- Forgetting the 4π in both $F = L/4\pi d^2$ and $L = 4\pi r^2\sigma T^4$.
- Confusing **luminosity** (intrinsic power, independent of distance) with **flux intensity** (observed brightness, depends on distance).
- Using $\Delta\lambda = \lambda_{\text{obs}} - \lambda_{\text{emitted}}$ but getting the **sign wrong**: for a receding source, $\Delta\lambda > 0$ (observed wavelength is longer).
- Applying $v = H_0 d$ with d in Mpc and H_0 in km s^{-1} Mpc^{-1} — in CIE exams **always convert to SI** (metres and s^{-1}).
- Stating that the Big Bang means galaxies are moving **through space** — more accurately, **space itself is expanding**.

Worked Examples

Example 1 — Stellar Radius from Wien and Stefan–Boltzmann

Question: A star has a peak emission wavelength of 480 nm and a luminosity of 5.2×10^{26} W. Estimate its radius. ($b = 2.90 \times 10^{-3}$ m K, $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4})

Solution

Solution:
Step 1 — Surface temperature:

$$T = \frac{b}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-3}}{480 \times 10^{-9}} = 6040 \text{ K}$$

Step 2 — Rearrange Stefan–Boltzmann:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times (6040)^4}}$$

$$T^4 = (6.04 \times 10^3)^4 = 1.33 \times 10^{15} \text{ K}^4$$

$$r = \sqrt{\frac{5.2 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times 1.33 \times 10^{15}}} = \sqrt{\frac{5.2 \times 10^{26}}{9.47 \times 10^{8}}} = \sqrt{5.49 \times 10^{17}} = \mathbf{7.4 \times 10^{8}} \text{ m}$$

Example 2 — Distance Using Flux and Luminosity

Question: A type Ia supernova has a peak luminosity of 2.0×10^{36} W. It is observed with a radiant flux intensity of 3.5×10^{-14} W m^{-2}. Calculate its distance.

Solution

Solution:

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{2.0 \times 10^{36}}{4\pi \times 3.5 \times 10^{-14}}}$$

$$d = \sqrt{\frac{2.0 \times 10^{36}}{4.40 \times 10^{-13}}} = \sqrt{4.55 \times 10^{48}} = \mathbf{2.1 \times 10^{24}} \text{ m}$$

Example 3 — Recession Speed and Age of Universe

Question: A galaxy shows a spectral line at 656 nm that is observed at 689 nm. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}. Calculate (a) the recession speed of the galaxy, (b) its distance, and (c) an estimate of the age of the Universe.

Solution

Solution:
(a) $\dfrac{\Delta\lambda}{\lambda} = \dfrac{689 - 656}{656} = \dfrac{33}{656} = 0.0503$
$v = 0.0503 \times c = 0.0503 \times 3.00 \times 10^8 = \mathbf{1.51 \times 10^{7}}$ m s^{-1}

(b) $d = v/H_0 = 1.51 \times 10^7 / 2.2 \times 10^{-18} = \mathbf{6.9 \times 10^{24}}$ m
(c) $t \approx 1/H_0 = 1/(2.2 \times 10^{-18}) = \mathbf{4.5 \times 10^{17}}$ s ($\approx$ 14 billion years)

Practice Exam Questions

Section A — Short Answer Questions

Q1. Define (a) luminosity and (b) radiant flux intensity. State the relationship between them and the distance d to the source.

[4 marks]

Q2. The Sun has a surface temperature of 5800 K. Calculate the peak wavelength of its emission spectrum and state what colour this corresponds to.

[2 marks]

Q3. Explain what is meant by a *standard candle* and describe one example of a standard candle used in astronomy.

[3 marks]

Q4. A hydrogen spectral line has a rest wavelength of 434 nm. In the spectrum of a distant galaxy it is observed at 461 nm. Calculate the recession speed of the galaxy.

[3 marks]

Q5. Explain how observations of redshift from distant galaxies lead to the conclusion that the Universe is expanding.

[3 marks]

Section B — Longer Structured Questions

Q6. Star A has surface temperature 12 000 K and radius 3.5×10^9 m. Star B has the same luminosity as Star A but a surface temperature of 4500 K.

(a) Calculate the luminosity of Star A.

[2 marks]

(b) Calculate the radius of Star B.

[3 marks]

(c) State which star would appear bluer and explain why.

[2 marks]

Q7. A galaxy is observed to have a recession speed of 4.8×10^6 m s^{-1}. The Hubble constant is $H_0 = 2.2 \times 10^{-18}$ s^{-1}.

(a) Calculate the distance of the galaxy.

[2 marks]

(b) A spectral line in the galaxy's spectrum has an emitted wavelength of 589 nm. Calculate the observed wavelength.

[2 marks]

(c) Explain how Hubble's law provides evidence for the Big Bang theory.

[3 marks]

Mark Scheme and Answers

Q1. (a) Luminosity: the total power of electromagnetic radiation emitted by a star (in all directions) [2]. (b) Radiant flux intensity: the power of radiation received per unit area at the observer's location [1]; $F = L/(4\pi d^2)$ [1].

Q2. $\lambda_{\text{max}} = b/T = 2.90 \times 10^{-3}/5800 = \mathbf{500}$ nm [1]; this corresponds to green light (near the centre of the visible spectrum) [1].

Q3. A standard candle is an astronomical object of known luminosity [1]; example: type Ia supernova (all reach the same peak luminosity) [1]; or Cepheid variable (period of brightness variation gives luminosity via period–luminosity relation) [1].

Q4. $\Delta\lambda = 461 - 434 = 27$ nm [1]; $v = c\Delta\lambda/\lambda = 3.00 \times 10^8 \times 27/434$ [1] $= \mathbf{1.87 \times 10^7}$ m s^{-1} [1].

Q5. Spectral lines from distant galaxies are observed at longer wavelengths (redshifted) than expected [1]; this indicates the galaxies are moving away from us [1]; the greater the distance, the greater the redshift/recession speed — consistent with all space expanding uniformly [1].

Q6(a). $L = 4\pi r^2 \sigma T^4 = 4\pi \times (3.5 \times 10^9)^2 \times 5.67 \times 10^{-8} \times (12000)^4$ [1]; $= 4\pi \times 1.225 \times 10^{19} \times 5.67 \times 10^{-8} \times 2.07 \times 10^{16} = \mathbf{1.80 \times 10^{29}}$ W [1].

Q6(b). Same L; $r_B^2 T_B^4 = r_A^2 T_A^4$ [1]; $r_B = r_A(T_A/T_B)^2 = 3.5 \times 10^9 \times (12000/4500)^2 = 3.5 \times 10^9 \times 7.11$ [1] $= \mathbf{2.49 \times 10^{10}}$ m [1].

Q6(c). Star A is bluer [1]; it has a higher surface temperature, so by Wien's law its peak wavelength is shorter (towards the blue end of the spectrum) [1].

Q7(a). $d = v/H_0 = 4.8 \times 10^6/2.2 \times 10^{-18} = \mathbf{2.18 \times 10^{24}}$ m [2].

Q7(b). $\Delta\lambda = \lambda v/c = 589 \times 10^{-9} \times 4.8 \times 10^6/3.00 \times 10^8 = 9.42 \times 10^{-9}$ m [1]; observed $\lambda = 589 + 9.4 = \mathbf{598}$ nm [1].

Q7(c). Hubble's law shows recession speed is proportional to distance — the further away a galaxy, the faster it recedes [1]; this means all galaxies are moving apart from one another, implying the Universe is expanding [1]; extrapolating back in time, all matter must have originated from a single point — the Big Bang [1].

Revision Checklist

Use this checklist to track your understanding. Tick each box when you are confident:

	Learning Objective	Confidence (1–3)
☐	Define luminosity and radiant flux intensity	
☐	Use the inverse square law $F = L/(4\pi d^2)$	
☐	Define a standard candle and explain how it is used to find distance	
☐	Give examples of standard candles (Cepheid variables, Type Ia supernovae)	
☐	State and apply Wien's displacement law $\lambda_{\max} T = b$	
☐	State and apply the Stefan–Boltzmann law $L = 4\pi r^2 \sigma T^4$	
☐	Combine Wien and Stefan–Boltzmann to estimate stellar radius	
☐	Explain what redshift is and how it is observed in galaxy spectra	
☐	Use $\Delta\lambda/\lambda \approx v/c$ to find recession speeds	
☐	Explain how redshift provides evidence for an expanding Universe	
☐	State and apply Hubble's law $v \approx H_0 d$ (SI units)	
☐	Use $t \approx 1/H_0$ as an estimate for the age of the Universe	
☐	Explain how Hubble's law leads to the Big Bang theory	

Key: 1 = Need more work   2 = Getting there   3 = Confident

Good luck with your revision!

Cosmology asks the biggest questions in physics — and answers them with the same tools you've used all year. A few formulas, careful unit conversions, and the ability to interpret a graph are all you need to explore the scale of the Universe itself.